2-Channel Kondo Scaling in Conductance Signals from 2-Level Tunneling Systems

D. C. Ralph,[1,*] A. W. W. Ludwig,[2] Jan von Delft,[3] and R. A. Buhrman[1]
[1] *School of Applied and Engineering Physics, Cornell University, Ithaca, New York 14853*
[2] *Joseph Henry Laboratories of Physics, Princeton University, Princeton, New Jersey 08544*
[3] *Laboratory of Atomic and Solid State Physics, Cornell University, Ithaca, New York 14853*
(Received 1 November 1993)

The temperature (T) and voltage (V) dependence of conductance signals in metal point contacts, previously asserted to be due to 2-channel Kondo scattering from atomic 2-level tunneling systems, collapse onto a universal scaling curve dependent on eV/k_BT, for all T and eV/k_B below a characteristic Kondo scale. Measurements determine a conductance exponent of 1/2, as expected from conformal field theory solution of the 2-channel Kondo model. The magnetic field (H) dependence at low T is nonanalytic at $H = 0$ ($\propto |H|$) in contrast to Fermi-liquid theory but in agreement with conformal field theory.

PACS numbers: 72.15.Qm, 63.50.+x, 71.25.Mg, 72.10.Fk

The multichannel Kondo model [1] has recently attracted significant interest due to its non-Fermi-liquid properties. Its exact conformal field theory (CFT) solution (see [2,3] and references therein) provides a physical understanding of its unusual low temperature (T) behavior in terms of a generalization of spin-charge separation. The 2-channel model exhibits properties reminiscent of the marginal Fermi-liquid phenomenology [4] of high-T_c superconductors [5–7] and has been proposed as a model for the observed non-Fermi-liquid features in certain U-containing heavy-fermion materials [8–10]. It is also predicted to describe the interaction between conduction electrons and symmetric atomic 2-level tunneling systems (TLSs) in metals [11–15]. To date, however, no experimental system has been unambiguously shown to realize the 2-channel Kondo model.

Previously, some of us have suggested that conductance signals observed in ballistic metal point contacts may be due to 2-channel Kondo scattering from TLSs in the constriction, and that such devices may thus provide a realization of the 2-channel Kondo fixed point at T=0 [15]. In this Letter we confirm this suggestion by showing that the conductance signals as a function of T and voltage (V) may be rescaled at low T and low V to collapse onto a single universal curve, with a scaling exponent of $\frac{1}{2}$, expected from CFT [2]. We thus report the first transport measurements in accord with 2-channel Kondo scattering in the low-T regime. We determine Taylor coefficients of the universal scaling curve for future comparison with theory. Furthermore, we show that the magnetic-field dependence of the signals, very different from the ordinary Fermi liquid, scales precisely as predicted by CFT [16].

The devices we analyze are ballistic Cu constrictions, with diameters less than 10 nm. Details of the fabrication and properties of these samples have been described previously [15,17]. The samples exhibit zero-bias minima in the differential conductance ($G = dI/dV$) and V-symmetric spikes in G at larger V [15]. We have argued previously that the signals are due to structural defects, likely dislocation kinks, which act as TLSs in the constriction region [15,17]. We have detailed how G is approximately logarithmically dependent on T and eV/k_B above 2 K, indicative of a high-T Kondo regime. In this Letter we examine closely the lower-T,V regime. Figure 1(a) displays the V dependence of $G(V,T)$ for one Cu sample, for T ranging from 400 mK to 5.6 K.

Scaling analysis.—Consider the conductance signal $G(V,T)$ due to scattering off an individual 2-channel Kondo defect with energy asymmetry Δ. First put $\Delta = 0$. To be in the scaling regime of the T=0 fixed point, T and eV/k_B must both be well below the Kondo temperature T_K, but the *ratio* eV/k_BT may be arbitrary. In the scaling regime G should obey a homogeneous scaling relation, governed by the fixed point:

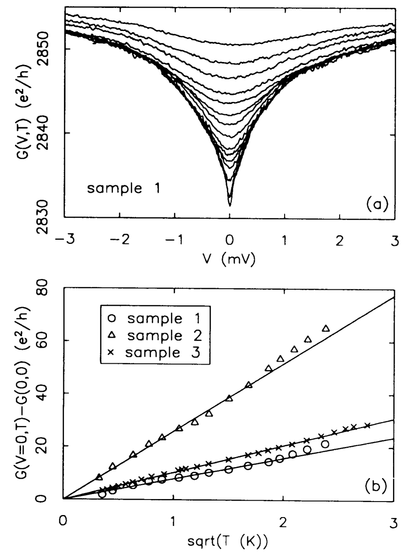

FIG. 1. (a) Differential conductance of sample 1 at various T, from 0.4 K (bottom) to 5.6 K (top). (Specific values are listed in Fig. 2.) (b) Zero-bias conductance for 3 samples. Extrapolated values of $G(0,0)$ are for sample 1: $2830e^2/h$, sample 2: $3970e^2/h$, and sample 3: $30.8e^2/h$.

0031-9007/94/72(7)/1064(4)$06.00

$$G(V,T) - G(0,0) = BT^{\alpha}\Gamma\left(\frac{AeV}{(k_BT)^{\alpha/\beta}}\right). \quad (1)$$

The parameters A and B are nonuniversal constants which may vary, for instance, as a function of the distance of the defect from the narrowest point of the constriction. However, $\Gamma(x)$ should be a universal function of x, a fingerprint applying to *any* microscopic realization of the 2-channel Kondo model. It must have the asymptotic form $\Gamma(x) \propto x^{\beta}$ as $x \to \infty$, so that $G(V,T)$ is independent of T for $eV \gg k_BT$. We use the normalization conventions that $\Gamma(0) = 1$ and $\Gamma(x)$ vs x^{β} has slope of 1 at large x^{β}. A small applied V enters only in the argument of Fermi functions for the leads, i.e., in the combination V/T, implying $\alpha=\beta$ [18].

For a bulk metal, an exact CFT calculation has found such a scaling form due to the $T=0$ fixed point, with $\alpha = 1/2$, for the electron self-energy $\Sigma(\omega,T)$, which directly determines the bulk conductivity [2(b)]. This result for α is in contrast to the exponent $\alpha=2$ that holds for the (Fermi-liquid) 1-channel Kondo problem [1]. In a point contact geometry, a homogeneous scaling relation should also describe $G(V,T)$ due to the $T=0$ fixed point, and we do not not expect the value of the 2-channel conductance exponent, $\alpha=1/2$, to differ from the bulk case. The calculation of the exact universal scaling function for $G(V,T)$ from CFT is in progress.

In the presence of a small asymmetry Δ, crossover to a Fermi liquid is predicted [16] below a characteristic temperature $T_X \sim \Delta^2/k_B^2T_K$ which may be very small for small Δ.

Comparison with experiment.—The zero-bias conductance signals in the devices we discuss are large enough that they are likely due to several defects contributing simultaneously [15]. If interaction effects among the defects are irrelevant [19], the conductance signal is additive (now using $\alpha=\beta$):

$$G(V,T) - G(0,0) = T^{\alpha}\sum_i B_i\Gamma\left(\frac{A_ieV}{k_BT}\right). \quad (2)$$

As a first test of this relation, we plot $G(V=0,T)$ vs T^{α}, using the expected value $\alpha = 1/2$, for 3 Cu samples in Fig. 1(b). Agreement with $T^{1/2}$ behavior is quite good. The size of deviations is consistent with amplifier drift. Fits to a straight line of the data below 2 K in Fig. 1(b) determine values for the parameter $B_{\Sigma}\equiv\sum_i B_i$, listed in Table I.

A much more stringent test of the exponent α of the conductance signals is provided by the scaling properties of the *combined* V and T dependence. It is convenient to rewrite the scaling ansatz [Eq. (2)] to eliminate $G(0,0)$, which is not measured directly:

$$\frac{G(V,T) - G(0,T)}{T^{\alpha}} = \sum_i B_i\left[\Gamma(A_ix) - 1\right] \equiv F(x), \quad (3)$$

where $x = eV/k_BT$. For $\alpha = 1/2$, this implies that if one plots the left hand side vs $(eV/k_BT)^{1/2}$, then with no adjustment of free parameters the low-T curves for a given sample should all collapse onto the sample-specific scaling curve $F(x)$ vs $x^{1/2}$, which is expected be linear for large $x^{1/2}$ [indicating T independence of $G(V,T)$].

Figure 2(a) shows the result of this procedure for the data of Fig. 1(a). The data at low V and low T, but *varying ratio*, collapse remarkably well onto one curve (with linear asymptote). As a quantitative measure of the quality of scaling, we plot in the inset of Fig. 2(a), as a function of α, the mean square deviation from the average scaling curve for the data at the 5 lowest T ($\leq$1.4 K), integrated over small values of eV/k_BT (between -8 and 8). These are the data which *a priori* would be expected to be most accurately within the scaling regime about a $T=0$ fixed point. The best scaling of the data requires $\alpha=0.48\pm0.05$, in agreement with the CFT prediction for the 2-channel Kondo model in bulk metal, $\alpha=1/2$ [20]. Figure 2(b) displays an example of the poor collapse of the low-V, low-T data using a value for α different than 1/2.

When either V or T becomes too large ($\geq T_K$), the scaling ansatz is expected to break down. This explains the downward deviations from the scaling curve seen in Fig. 2(a) for the higher-T curves. We estimate T_K as that T for which the rescaled data already deviate from the scaling curve at $eV/k_BT \leq 1$. This gives $T_K \geq 5$ K for the defects of sample 1.

We have also performed the test of the scaling ansatz on two other Cu samples. The rescaled data for sample 2 [Fig. 2(c)] collapse well onto a single curve at low V and T, for $\alpha=0.52\pm0.05$ and with $T_K \geq 3.5$ K. At high V and high T the nonuniversal conductance spikes, discussed previously [15,17], are visible. The data for sample 3 do not seem to collapse as well [Fig. 2(d)] (illustrating how impressively accurate by comparison the scaling is for samples 1 and 2). However, we suggest that this sample in fact displays two separate sets of scaling curves (see arrows), one for $T \leq 0.4$ K and one for 0.6 K$\leq T \leq 5$ K, with interpolating curves in between. This

TABLE I. Measured parameters of the scaling functions for the Kondo signals in 3 Cu samples. B_{Σ}, $F''(0)$, F_0, and F_1 have units $K^{-1/2}\,e^2/h$. Γ_1 and $\Gamma''(0)$ are dimensionless.

Sample	B_{Σ}	$F''(0)$	F_0	F_1	$\Gamma_1 = \frac{F_1}{B_{\Sigma}}$	$\frac{F''(0)B_{\Sigma}^3}{F_0^4} \geq \Gamma''(0)$
1	7.8 ± 0.2	0.55 ± 0.04	4.2 ± 0.3	-5.7 ± 0.9	-0.73 ± 0.11	0.8 ± 0.3
2	25.2 ± 0.7	1.03 ± 0.09	12.8 ± 0.8	-19.7 ± 1.5	-0.78 ± 0.06	0.6 ± 0.2
3	10.3 ± 0.4	0.82 ± 0.06	6.0 ± 0.6	-7.7 ± 1.6	-0.75 ± 0.16	0.7 ± 0.3

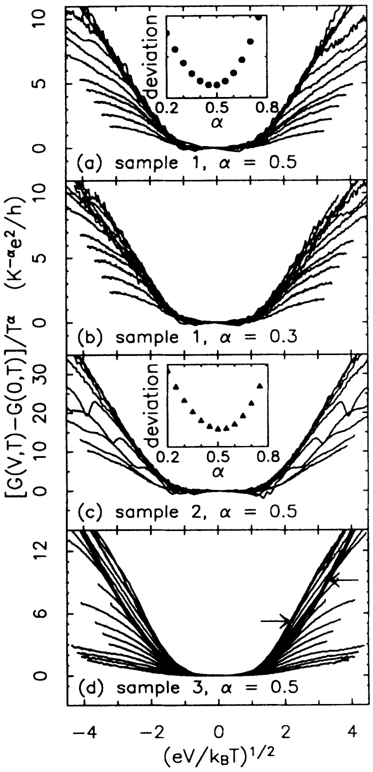

FIG. 2. (a),(b) Rescaled conductance of sample 1 for (top to bottom) $T = 0.4$, 0.6, 0.8, 1.1, 1.4, 1.75, 2.25, 2.8, 3.5, 3.9, 4.3, 4.9, and 5.6 K. (c) Rescaled conductance of sample 2 at same T up to 4.3 K. (d) Rescaled conductance of sample 3 from 125 mK to 7.6 K. Arrows show 2 separate scaling curves. Insets: Integrated mean square deviation from the average scaling curve for $T \leq 1.4$ K and $-8 \leq eV/k_BT \leq 8$. The scale of the deviation axis is in (a) from 0 to 4 $(e^2/h)^2$ and in (c) from 0 to 25 $(e^2/h)^2$. Residual deviations for α=0.5 are consistent with amplifier noise.

could be due to defects with a distribution of T_K's, some having $T_K \simeq 0.4$ K and others having $T_K \geq 5$ K. The second (higher-T) set of curves do not collapse onto each other as well as the first, presumably because there is still some (approximately logarithmic) contribution from the $T_K \simeq 0.4$ K defects.

Universality.—If for any sample all the A_i in Eq. (3) were equal, one could directly extract the universal scaling curve from the data. The curve obtained by plotting $[G(V,T)-G(0,T)]/B_\Sigma T^{1/2}$ vs $(AeV/k_BT)^{1/2}$, with A determined by the requirement that the asymptotic slope of 1, would be identical to the universal curve $[\Gamma(x)-1]$ vs $x^{1/2}$. Such plots are shown in Fig. 3(a). The fact that the scaling curves for all three samples are indistinguishable indicates that the distribution of A_i's in each sample is quite narrow, and is a measure of the universality of the observed behavior.

To make possible quantitative comparisons of the data with future calculations of the universal scaling function

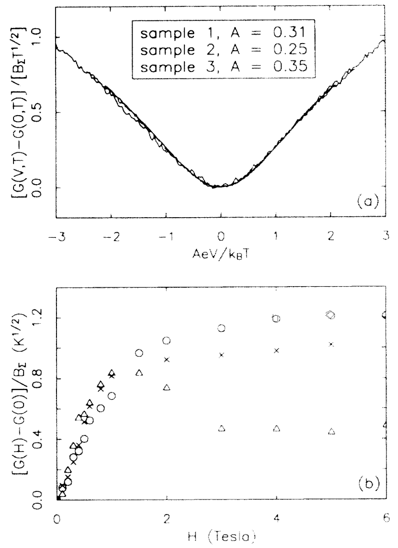

FIG. 3. (a) Scaling curves for the 3 samples. (b) Magnetic field dependence of T=0.1 K, V=0 conductance. The field dependence is symmetric about H=0.

(from CFT, in progress), we now proceed to extract from the data the value of one universal (sample-independent) constant and an upper bound on another [essentially Taylor coefficients of $\Gamma(x)$]. The procedure by which we extract these parameters is independent of the possible distribution of A_i's and B_i's.

Consider the sample-specific scaling function $F(x)$ defined in Eq. (3). By construction, $F(0) = 0$, and if $F(x)$ is symmetric and analytic at $x = 0$ (as the data suggest) one also has $F'(0) = 0$. The second derivative, $F''(0) = \Gamma''(0)\sum_i B_i A_i^2$, may be measured directly from the low eV/k_BT portion of the scaling curve.

Next, consider the regime $x \gg 1$. As argued earlier, here $\Gamma(x) \simeq x^\beta$, and since β=α=1/2, with our normalization conventions we can write, asymptotically,

$$\Gamma(x) - 1 \equiv x^{1/2} + \Gamma_1 + O(x^{-1/2})\,. \qquad (4)$$

It follows from Eq. (3) that

$$F(x) = x^{1/2}F_0 + F_1 + O(x^{-1/2})\,, \qquad (5)$$

where $F_0 \equiv \sum_i B_i A_i^{1/2}$ and $F_1 \equiv \Gamma_1 B_\Sigma$. Values for F_0 and F_1 may be determined from the conductance data by plotting F versus $(eV/k_BT)^{1/2}$ and fitting the data for large $(eV/k_BT)^{1/2}$ to a straight line. For samples 1 and 2 we fit between $(eV/k_BT)^{1/2}$=2 and 3, and for sample 3 (using only the curves below 250 mK) between 2 and 2.5. Values for $F''(0)$, F_0, and F_1 are listed in Table I. The uncertainties listed are standard deviations of values determined at different T within the scaling regime for each sample.

From these quantities, we obtain an experimental determination of the universal number $\Gamma_1 = F_1/B_\Sigma$ and an upper bound on the universal number $\Gamma''(0)$:

$$\frac{F''(0)B_\Sigma^3}{F_0^4} = \frac{\Gamma''(0)[\sum_i B_i A_i^2]B_\Sigma^3}{[\sum_i B_i A_i^{1/2}]^4} \geq \Gamma''(0)\,. \qquad (6)$$

Values for Γ_1 and this ratio are listed in Table I and are consistent among all 3 samples.

Magnetic field dependence.—A magnetic field (H) breaks the spin up-down degeneracy for the conduction electrons, thus acting as a channel anisotropy of scaling dimension 1/2 [16]. Therefore, the scaling function Γ in Eq. (1) depends now on the argument $|H|/T^{1/2}$. Since $T^{1/2}\Gamma(V{=}0,|H|/T^{1/2})$ must be independent of T as $T \to 0$, it must be $\propto |H|$ in this limit. This nonanalyticity in H is precisely the behavior we observe in the $V=0$ conductance [Fig. 3(b)] for small H and very small T (=0.1 K). The field dependence beyond 1 T is influenced by the conductance spikes [15,17], and is clearly nonuniversal.

Asymmetry energy.—It is known [16] that if a TLS has an energy asymmetry Δ between its position states, a crossover to a Fermi liquid occurs at a temperature scale $T_X \sim \Delta^2/k_B^2 T_K$, below which the T dependence of $G(V{=}0,T)$ is much weaker. The fact that the data for samples 1 and 2 show pure $(T/T_K)^{1/2}$ scaling for 0.4 K< $T < T_K$ implies that any nonzero Δ must be rather small: for $T_K \approx 5$ K, good scaling down to 0.4 K implies $T_X < 0.4$ K and hence $\Delta < 1.4$ K. We suggest that the defects that are selected by (i.e., dominate) our transport measurements are those with a strong V,T dependence. Such TLSs must have significant tunneling amplitudes, and hence cannot have large Δ ([11(b)], pp. 1599–1600). Also, the microscopic origin of our TLSs, likely dislocations in clean metal, may well produce lower asymmetries than for TLSs in glassy materials.

Static disorder.—We have found that the introduction of impurities (e.g., by coevaporating 1% Au with Cu) eliminates rather than enhances the effect we observe. Along with previous arguments [15], this is further evidence that effects of static disorder cannot explain our data. We suggest that impurities pin the mobile dislocation kinks which may serve as TLSs in the metal.

In summary, we have found that low-T, low-V conductance signals in point contacts, previously suggested to be due to 2-channel Kondo scattering from TLSs, obey a homogeneous scaling relationship with exponent 1/2, and a magnetic field exponent 1. This agrees with CFT predictions for 2-channel Kondo scattering in bulk metal. Deviations from scaling at high V and T indicate Kondo temperatures of order 0.5–5 K, depending on the individual TLS. We have determined Taylor coefficients of the universal scaling function for comparison with future analytic calculations.

It is a pleasure to acknowledge discussions with V. Ambegaokar and J. P. Sethna. D.C.R. and R.A.B. acknowledge support by the Office of Naval Research, Contract No. N00014-89-J-1692, and by the National Science Foundation through the Cornell Materials Science Center, DMR-9121654, and through use of the National Nanofabrication Facility at Cornell, ECS-8619049. J.v.D. was partially supported by the MRL Program of the National Science Foundation under Award No. DMR-9121654.

---

* Present address: Department of Physics, Harvard University, Cambridge, MA 01238.

[1] P. Nozieres and A. Blandin, J. Phys. (Paris) **41**, 193 (1980).

[2] (a) A. W. W. Ludwig and I. Affleck, Phys. Rev. Lett. **67**, 3160 (1991); (b) I. Affleck and A. W. W. Ludwig, Phys. Rev. B **48**, 7297 (1993).

[3] I. Affleck and A. W. W. Ludwig, Nucl. Phys. **B360**, 641 (1991).

[4] C. M. Varma *et al.*, Phys. Rev. Lett. **63**, 1996 (1989).

[5] D. L. Cox *et al.*, Phys. Rev. Lett. **62**, 2188 (1989).

[6] V. J. Emery and S. Kivelson, Phys. Rev. B **46**, 10 812 (1992); Phys. Rev. Lett. **71**, 3701 (1993).

[7] T. Giamarchi *et al.*, Phys. Rev. Lett. **70**, 3967 (1993).

[8] D. L. Cox, Phys. Rev. Lett. **59**, 1240 (1987); Physica (Amsterdam) **153-155C**, 1642 (1987); J. Magn. Magn. **76&77**, 53 (1988).

[9] C. L. Seaman *et al.*, Phys. Rev. Lett. **67**, 2882 (1991).

[10] B. Andraka and A. M. Tsvelik, Phys. Rev. Lett. **67**, 2886 (1991).

[11] (a) A. Zawadowski, Phys. Rev. Lett. **45**, 211 (1980); (b) K. Vladar and A. Zawadowski, Phys. Rev. B **28**, 1564 (1983); **28**, 1582 (1983); **28**, 1596 (1983).

[12] A. Muramatsu and F. Guinea, Phys. Rev. Lett. **57**, 2337 (1986).

[13] S. Katayama *et al.*, J. Phys. Soc. Jpn. **56**, 697 (1987).

[14] T. Ishiguro *et al.*, Phys. Rev. Lett. **69**, 660 (1992).

[15] D. C. Ralph and R. A. Buhrman, Phys. Rev. Lett. **69**, 2118 (1992).

[16] I. Affleck, A. W. W. Ludwig, H.-B. Pang, and D. L. Cox, Phys. Rev. B **45**, 7918 (1992).

[17] D. C. Ralph, Ph.D. dissertation, Cornell University, 1993.

[18] Alternatively, one expects $\alpha = \beta$ since V couples to a conserved current. In the equilibrium context, this is, e.g., discussed in Ref. [3] [see also S.Sachdev, Yale University report, 1993 (to be published)].

[19] One might expect this since breaking of particle-hole symmetry, present in any sample, leaves the 1-impurity 2-channel fixed point intact [2], while it likely destroys the 2-impurity 2-channel fixed point. Alternatively, defect interactions may set in only at very low T.

[20] We have also tested the more general scaling form of Eq. (1), and have observed scaling for $0.2 < \alpha < 0.8$, with $\beta - 0.5 \approx (\alpha - 0.5)/2$, with best scaling for $\alpha = 0.5 \pm 0.05$. But as argued earlier, one expects $\alpha = \beta$ on general grounds.