Dissipative spin-boson model and Kondo effect in low dimensional quantum gases

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We show that ultracold atoms confined in a quasi-1d trap can be used to study the dissipative spin-boson model with *tunable* tunneling splitting and strength of dissipation. It is shown that with realistic parameters this system allows to study the crossover from coherent Rabi dynamics to incoherent tunneling.

An effective two state system (spin 1/2) coupled to a bath of harmonic oscillators is one of the most important models to describe the effect of dissipation in quantum mechanics [1,2]. The associated spin-Boson Hamiltonian was extensively studied for a wide range of problems, most notably the issue of decoherence for superpositions of macroscopically distinct states [1] or various condensed matter realizations like the tunneling of interstitials in solids which are coupled to phonons or to electron-hole excitations in a metal [5]. In spite of its apparent simplicity, the spin-Boson model exhibits a very rich behavior, which crucially depends on the low frequency weight of the effective environment spectrum. It ranges from a simple damped oscillation described by the phenomenological Bloch equations well known e.g. from NMR to complete localization in the so called ohmic dissipation case [1]. The latter is in fact closely related to a generalized Kondo-Hamiltonian, a paradigmatic model for quantum impurity problems.

In most of these applications, the precise strength and frequency spectrum of the environment depends on microscopic details and cannot be changed externally. This is the case e.g. in the context of macroscopic quantum coherence, where the final observation of coherent superpositions of counter-propagating currents in a SQUID [3,4] requires extremely low dissipation whose origin is still not completely understood on a microscopic level. Similarly, charged impurities in a metal are subject to ohmic dissipation through electron-hole excitations, however their strength is typically very small, depending on the precise scattering phase shifts at the Fermi energy.

Taking advantage of recent experimental progress in cooling and trapping of low dimensional quantum gases [6,7], we show that cold atoms can be used to realize an ohmic spin-Boson model with a *tunable* strength of both the interaction and the effective tunneling amplitude. This allows to study the complete range of dynamics from coherent oscillations to incoherent dynamics in the Kondo regime and finally to localization. The oscillator bath in this case arises from the coupling to the low energy excitations of a Bose condensate, which produces an ohmic spectrum in the one-dimensional case, where gapless quantum liquids exhibit universal low energy properties.

The spin-boson model describes a two-level system (a spin-1/2, characterized by the Pauli matrix operators σ), interacting with a bath of harmonic oscillators (phonons) according to the following Hamiltonian:

$$H = -\frac{\Delta}{2}\sigma_x + \sum_q \omega_q b_q^\dagger b_q + \frac{\sigma_z}{2}\sum_q \lambda_q (b_q + b_q^\dagger), \qquad (1)$$

where Δ is the "tunneling" amplitude (the Rabi frequency in the case of a "free" two-level system), b_q and $b_q^\dagger$ are the annihilation and creation operators of the phonon modes characterized by a momentum q and the dispersion relation $\omega_q = uq$, where u is the velocity of sound. Formally eliminating the oscillator part of the Hamiltonian, it can be shown that the reduced dynamics of the spin is completely determined by the effective density of states

$$J(\omega) = \sum_q \lambda_q^2 \delta(\omega - \omega_q) \sim \omega^s$$

for sufficiently small values of ω. Depending on the value of the exponent s there are several different regimes. The most interesting situations arises in the so called ohmic case $s = 1$. The latter is is equivalent to $1/r^2$-Ising model, which is known to have a phase transition even in 1d [10].

The spin-boson Hamiltonian (1) can be implemented in a setup involving a sample of trapped cold atoms. Consider the two distinct hyperfine states (let us call them the state a and b respectively) of the same atom confined within a quasi-1d trap (see the Fig. 1). The atoms of the two bosonic species are trapped by the two different external potentials, $V_a(x)$ and $V_b(x)$, respectively. Both potentials are assumed to have a high frequency of the radial motion, so that the motion of the particles can be considered as quasi-1d (this implies that both the temperature T and the characteristic interparticle interaction do not exceed the characteristic frequency of the transverse confinement, $\omega_\perp$). Most of the atoms are in the state a and form a dense quantum liquid (the number of atoms in the state a is large $N_a \gg 1$). The potential V_b is assumed to have the form of an optical lattice with very well separated and tightly confining potential wells. Both potentials should spatially overlap to allow the interaction between the atomic species. The optical lattice should have fairly low filling: $N_b \ll N_a$, so

that the wavepackets of atoms b do not overlap with each other and hence have no cooperative interactions with the quantum liquid. At the same time, N_b should be sufficiently large, to facilitate (ideally non-destructive) measurement.

The interatomic interaction is described by the three interaction constants $g_{\beta\beta'}$, with $\beta, \beta' = a, b$ characterizing the interaction between the atoms a, b and between the atoms a and b. All the interactions are assumed to be short-range. The values of coupling constants are then related to the 3d scattering amplitude in a fairly complicated way and may contain resonances. In the simplest case, when the scattering length, corresponding to the scattering of the components β and β', $a_{\beta\beta'}$, is much smaller than the radial ground state size $l_\perp \sim \omega_\perp^{-1/2}$ (hereafter we use units such that $\hbar = m = 1$, where m is the mass of an atom), the effective 1d "scattering length" is given by a simple relation: $g_{\beta\beta'} = 2\pi a_{\beta\beta'}/l_\perp^2$. The value and even the sign of the interaction constants $g_{\beta\beta'}$ may also be changed by applying external magnetic field (Feshbach resonances, see [11] and references therein), or simply by changing the trap aspect ratio (i.e. by varying $\omega_\perp$, see [8]).

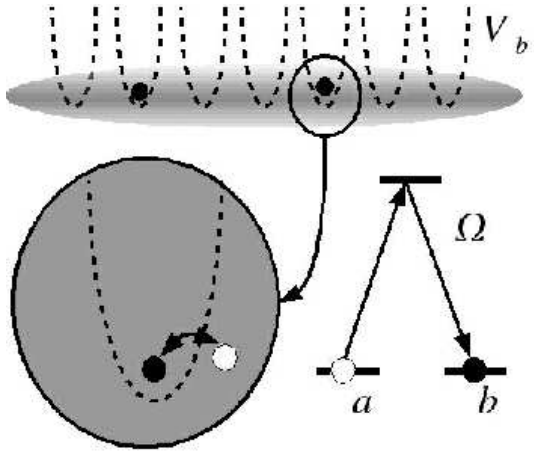

FIG. 1. The suggested experimental setup. The Bose-liquid of atoms "a" (white circles) is confined in a shallow quasi-1d trap (gray). The atoms "b" (black circles) are localized by a separate trap (dashed line). The transition between the atoms is performed by a laser light.

The on-site repulsion between the atoms b is $U_{bb} \sim g_{bb}/l_b$, where l_b is the characteristic size of the ground state wavefunction $\psi_b(x)$ localized within a single potential well of potential V_b. The value of U_{bb} should be large compared with the characteristic energy of interaction within and with the Bose-liquid of atoms a: $g_{aa}\rho, |g_{ab}\rho| \ll U_{bb}$ (but still $U_{bb} \lesssim \omega_\perp$). Then, only one atom of the type b can be trapped on the same lattice site at the same time. Accordingly, any state of the atoms b can be modeled by the corresponding state of a spin-1/2 system, namely: the spin-up state corresponding to, say, the state with a single atom b, whereas the spin-down state corresponds to the situation with no atoms b on a certain site.

To model the exchange interaction we suggest to put the atomic system into the far-detuned laser light, providing a transition between the internal states a and b. The amplitude of the Rabi flip Ω can, in principle, be spatially inhomogeneous. The part of the Hamiltonian describing the interaction of atoms b with the Bose-liquid can be presented in the form of the following spin Hamiltonian:

$$H_b = \frac{g_{ab}}{2}(1+\sigma_z)\int dx \psi_b^2 \rho_a + \int dx \Omega(\Psi_a \psi_b \sigma_+ + h.c.), \tag{2}$$

where $\hat{\Psi}_a(x)$ is the annihilation operator valued-field of an atom a at the point x, $\rho_a = \hat{\Psi}_a^\dagger \hat{\Psi}_a$ is the liquid density operator. We note, that in order to ensure a single occupancy of the lattice site the interaction between the atoms b should be repulsive, $g_{bb} > 0$ and sufficiently large. At the same time, the interaction of the atoms a and b can have arbitrary sign (provided that the number of atoms b is sufficiently small). The first term of (2) contains a time-independent phonon displacement operator (the identity part of $1 + \sigma_z$ operator), which can be always integrated out from the system's action and only shifts the effective Hamiltonian by an additive constant.

Now we turn to the description of the atoms a. To implement the Hamiltonian (1) we need to exploit a situation when the dispersion relation for the low energy excitations of the Bose liquid of atoms a is linear in momentum, i.e. has no gap. In the long wavelength limit a Bose-gas is a Luttinger Liquid (LL), and is described by the Haldane hydrodynamics Hamiltonian [12]:

$$H = \frac{1}{2\pi}\int dx (v_J \partial\phi^2 + v_N(\partial\theta - \pi\rho_a)^2), \tag{3}$$

where ρ_a is the equilibrium liquid density, $v_J = \pi\rho_a$ and $v_N = \kappa/\pi\rho_a^2$ where κ is the compressibility of the liquid. The fields ϕ and θ describe the superfluid velocity and the density fluctuations, respectively.

The LL model (3) can be diagonalized by the following transformation:

$$\phi(x) = -i\sum_q \left|\frac{2\pi}{qLK}\right|^{1/2} e^{iqx}(b_q - b_{-q}^\dagger) \tag{4}$$

$$\theta(x) = -i\sum_q \left|\frac{2\pi K}{qL}\right|^{1/2} sign(q) e^{iqx}(b_q + b_{-q}^\dagger), \tag{5}$$

where L is the sample length. Accordingly, the Hamiltonian (3) takes the form:

$$H_{LL} = v_s \sum_q |q| b_q^\dagger b_q, \tag{6}$$

where: $v_s = (v_N v_J)^{1/2} = (\kappa/\rho_a)^{1/2}$. As expected, the spectrum of the lowest energy excitations is characterized by a single sound velocity $u = v_s$ (cf. Eq.(1)).

The actual values of the phenomenological parameters v_s and $K = (v_J/v_N)^{1/2}$ in the LL Hamiltonian (3) depend on ρ_a, the interaction g_{aa}, and on the parameters of the external potential V_a. The requirement of the absence of a gap in the excitation spectrum sets a number of restrictions. First of all, the interparticle interaction has to be repulsive ($g_{aa} > 0$). Second, the trapping potential should not allow the formation of the so-called Mott-insulator state. The Bose-field operator can be expressed as

$$\hat{\Psi}_a \sim \left(\rho_a + \frac{\partial\theta}{\pi}\right)^{1/2} \exp(i\phi) \sum_{m=even} \exp(im\theta), \qquad (7)$$

where the dimensionless proportionality coefficient depends on non-universal short interparticle separation properties of the Bose-liquid. On the contrary, the expression for the density in terms of the field Π does not have this ambiguity:

$$\rho_a(x) = (\rho_a + \Pi(x)) \sum_{m=even} \exp(i2m\theta(x)). \qquad (8)$$

In the case of a trapped Bose-gas, the Luttinger parameter K depends on the interaction in the combination $\gamma = g_{aa}/\rho_a$ [12]. In particular, for weak interactions ($\gamma \to 0$) $K(\gamma) \approx \pi/\gamma^{1/2}$ and thus can be very large. In the other limit, i.e. when the interaction is strong, $\gamma \to \infty$, $K(\gamma) \approx 1$ (Tonks gas limit, [9]).

Eqs.(7) and (8) for the Bose-field operator and the particle density can be used to rewrite the interaction Hamiltonian (2). The role of the terms with various values of m is quite different. Since the phase θ contains a quickly oscillating term $\pi\rho_a x$, all the contribution containing $\exp(im\theta)$ with $m \neq 0$ average out under the integral sign in Eq.(2), provided that the localization length l_b of the wavefunction ψ_b is sufficiently large: $\pi\rho_a l_b \gg 1$. Then, in the limit $q \to 0$ we can put $qx \approx 0$ everywhere in Eqs.(4) and (5). Finally, keeping only the terms with $m = 0$ in Eqs.(7) and (8) we can derive the following representation of the Hamiltonian (2):

$$H_b = \frac{g_{ab}}{2\pi}\sigma_z \sum_q \left|\frac{2\pi K}{L} q\right|^{1/2} (b_q + b_q^\dagger) +$$

$$+\tilde{\Omega}\rho_a^{1/2} l_b^{1/2} (\sigma_+ \exp(i\phi(0)) + h.c.), \qquad (9)$$

where

$$i\phi(0) = \sum_q \left|\frac{2\pi}{qLK}\right|^{1/2} (b_q - b_q^\dagger),$$

and $\tilde{\Omega} \sim \Omega$ (the unknown numerical coefficient is the combination of the two factors: explicit dependence of the result on the wavefunction ψ_b and the unknown factor in Eq.(7)).

To prove the equivalence of the Hamiltonians (1) and (9) we follow the unitary transformation: $H' = S^{-1}(H_{LL} + H_b)S$ with $S = \exp(\sigma_z i\phi(0)/2)$ and identify $u = v_s$, $\Delta = \tilde{\Omega}(\rho_a l_b)^{1/2}$, and

$$\lambda_q = u\left|\frac{\pi q}{L}\right|^{1/2} \left(\frac{g_{ab}}{2\pi u}(2K)^{1/2} - \frac{1}{(2K)^{1/2}}\right). \qquad (10)$$

We note though, that the equivalence of the initial spin-boson Hamiltonian (1) and its quantum gases version (9), (6) can only be justified if for $ql_b \ll 1$. For larger values of q the interaction becomes more sophisticated, but also decreases as soon as $ql_b \gtrsim 1$. One can still use Eq.(9), assuming that the summation over q is restricted by the condition $\omega(q) \lesssim \omega_c$, where the quantity $\omega_c \sim u/l_b$ plays the role of the high-frequency cutoff.

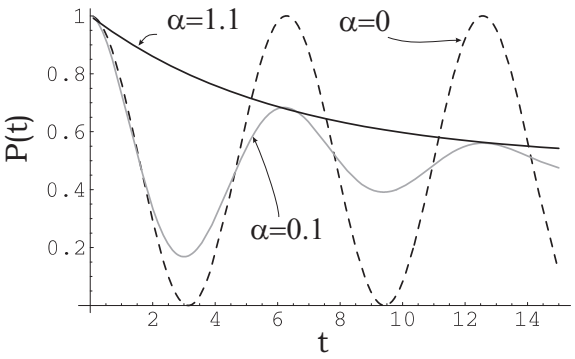

FIG. 2. The atoms b population relaxation. In the coherent regime, at zero temperature, for $\alpha = 0.1$ and $\alpha = 0$. In the incoherent regime, at low temperature, for $\alpha = 1.1$.

The spin-boson Hamiltonian (1) can also describe an isolated spin-1/2 impurity dynamics in a 3d Fermi system (the so called Kondo model). A slightly generalized (anisotropic) version of the model can be formulated with the help of the following Hamiltonian:

$$H = \sum_{q,\sigma} \epsilon(q) c_{\sigma q}^\dagger c_{\sigma q} + J_\parallel S_z s_z(0) + J_\perp S_\perp s_\perp(0), \qquad (11)$$

where $c_{\sigma q}$ is the annihilation of a fermion with (now 3d) momentum q and the spin projection σ, $\mathbf{S}$ is the spin of the impurity, $\mathbf{s}$ is the total spin of the electrons at a given point. The exchange couplings $J_\perp$and $J_\parallel$ describe the interaction of the different spin components ($S_\perp s_\perp = S_x s_x + S_y s_y$). The exact solution of the model is given in [13]. Mapping of the Hamiltonian (11) to the spin-boson model Hamiltonian (1) is described in every detail in [1,14], and is based on the fact that the spin density excitations of a free Fermi gas can serve as an oscillator bath. This leads to the identifications $\Delta \sim J_\perp$, $u = v_F$ with v_F being the Fermi velocity of the Fermi gas, and

$$\lambda_q = u\left(\frac{\pi q}{L}\right)^{1/2} \left(1 - \frac{J_\parallel}{4\pi u}\right). \qquad (12)$$

The behavior of the system (the character of the impurity spin dynamics) depends strongly on the sign of the coupling $J_\parallel$: the situations with $J_\parallel < 0$ and $J_\parallel > 0$ are commonly referred to as the ferromagnetic and antiferromagnetic cases respectively.

The analysis above shows, that the spin-boson model (1) can describe both the Kondo and our quasi-1d Bose gas with an impurity. On the other hand, this also means that the cold atoms version of the spin-boson model can, among other things, be used to model the Kondo Hamiltonian and the related phenomena, provided that the values of the independent parameters g_{ab} and K in (10) are selected accordingly, i.e. the couplings (10) and (12) match each other. Both systems are characterized by $J(\omega) = 2\alpha\omega$ in the $\omega \to 0$ limit, where

$$\alpha = \frac{1}{2K}\left(\frac{g_{ab}K^2}{\pi^2\rho_a} - 1\right)^2. \qquad (13)$$

In the Kondo language, $\alpha > 1$ and $\alpha < 1$ correspond to the antiferromagnetic and ferromagnetic cases, respectively. We note, that since both K and g_{ab} are independent and practically unrestricted (g_{ab} can have arbitrary sign and practically any value, whereas $1 < K < \infty$), the parameter α can take arbitrary values, both small and large. For realistic values $g_{ab} \ll \rho_a$ $\alpha \approx 1/2K$ and is small. For example, for a potassium condensate of $N_a = 10^4$ particles the interaction parameter $g_{ab}/n_a \sim 10^{-2}$, and $K \sim 30$, which means that $\alpha \sim 0.02$ and can be made larger by a Feshbach resonance.

The dynamics of the "spin impurities" in our proposal can be observed by following the population dynamics of the atoms b in the presence of the laser light Ω. In the absence of interaction between the "impurity" b and the LL of atoms a, the atom of impurity should undergo familiar undamped Rabi oscillations with the laser field. The actual damping is determined by the width of the involved atomic levels and will be neglected here. On the other hand, the interaction between the atoms a collectivize the Rabi oscillations of the "spin" and the collective degrees of the LL of atoms a.

A particular interesting regime of the ohmic two-state system is the low T and $\alpha < 1/2$ weak damping case. In this limit the occupation $P(t)$ of, say, the state "b" exhibits damped Rabi oscillations [15]

$$P(t) = \cos\left(\Delta_r t\cos(\eta)\right)\exp(-\Delta_r t\sin(\eta)),$$

where $\eta = \pi\alpha/(2(1-\alpha))$, $\Delta_r = \Delta(\Delta/\omega_c)^{\alpha/(\alpha-1)}$ (see Fig. 2). This result holds as long as $\alpha T \lesssim \Delta_r$. At higher T the dynamics is incoherent (no oscillations should be visible).

For $1/2 < \alpha < 1$ the system shows no oscillations and $P(t)$ is a sum of exponentials [15]. This is the regime which is directly relevant for the Kondo model. The detailed description of the model dynamics is still unsolved.

The situation becomes simpler again for $\alpha > 1$, which is the ferromagnetic analogue of the Kondo problem. At $T = 0$ the spin is localized, whereas at larger T there are incoherent transitions between the states at a rate $\sim T^{2\alpha-1}$. This peculiar behavior was in fact observed in SQUIDs experiments [16] (see the Fig.2).

In fact the suggested experimental scheme is not confined to a case of a single spin impurity in a quantum liquid. If the N_b is sufficiently large, the pseudo-spins can interact with "host" liquid in a collective manner. This situation is described by Anderson model and is characteristic to a number of important solid state systems, like heavy fermionic compounds (see e.g. [13]). The spin-boson model is also a paradigmatic model for qubits damped by an environment. Understanding how such a system reacts under *tunable* conditions, is obviously important for all those interested in quantum computation.

Discussions with U. R. Fischer, J.I. Cirac, M. Cazalilla, D. Jaksch are gratefully acknowledged. Work at Innsbruck supported in part by the A.S.F., EU Networks, and the Institute for Quantum Information. P.Z. thanks the Alexander von Humboldt Award for support. A.R. has been supported by the European Commission Project Cold Quantum Gases RTN Network Contract No. HPRN-CT-2000-00125.

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