Evidence for Non-planar Amplituhedron

Based on 1512.08591
with Zvi Bern, Enrico Herrmann, Sean Litsey, James Stankowicz

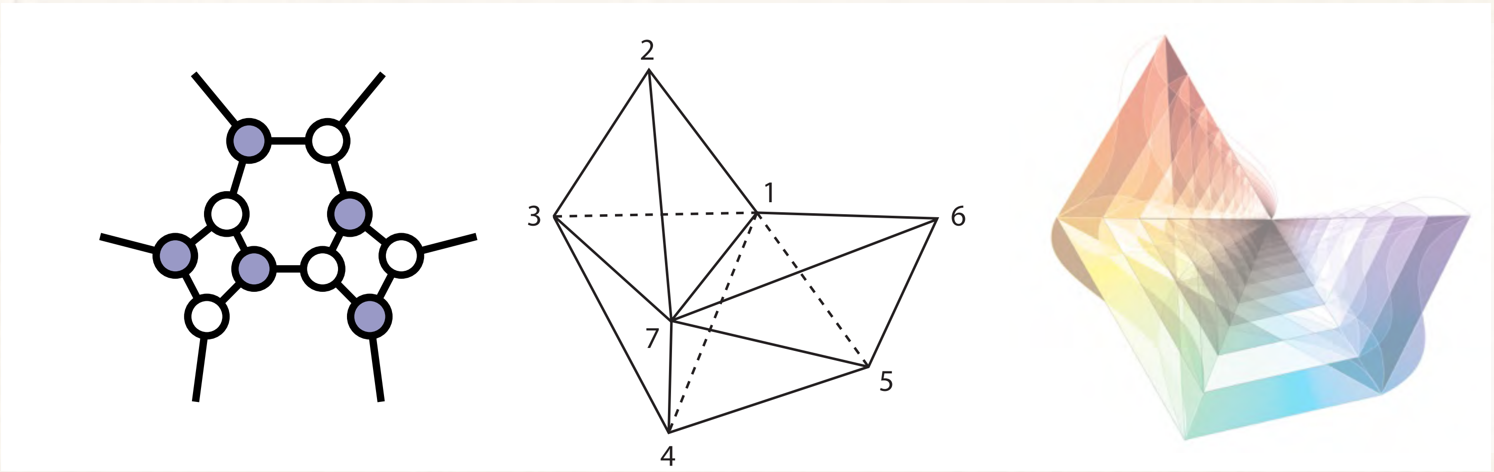
also 1604.03479 with Enrico Herrmann and older work
1410.0354 with Nima Arkani-Hamed, Jake Bourjaily, Freddy Cachazo

Jaroslav Trnka (QMAP, UC Davis)

LMU Munich, September 5, 2016

Goal

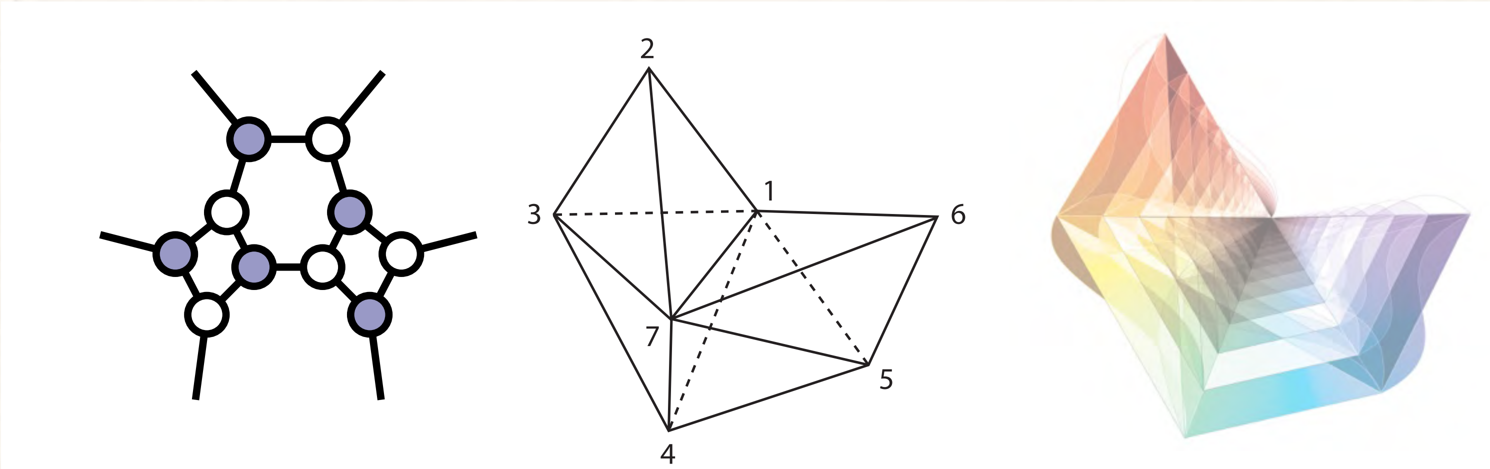
Mathematical structures in planar N=4 SYM



Other theories

Goal

Mathematical structures in planar N=4 SYM



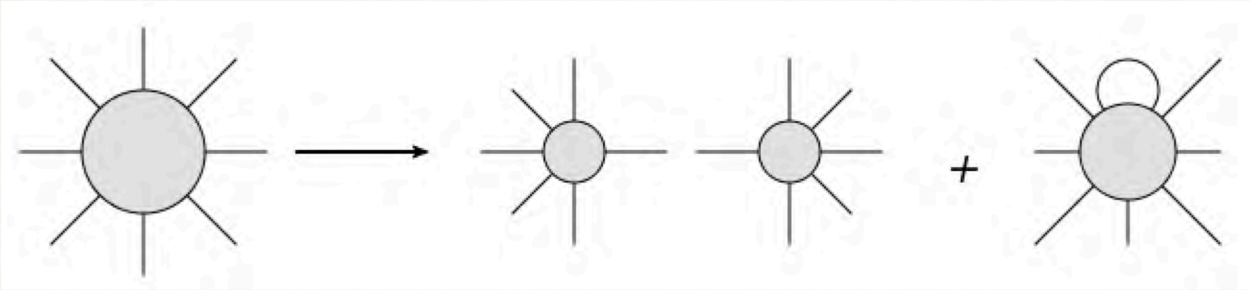
Non-planar N=4 SYM / N=8 SUGRA

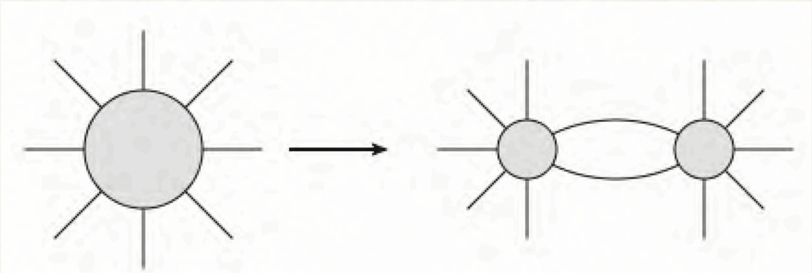
Plan of the talk

- On-shell diagrams, Amplituhedron in planar N=4 SYM
- Evidence for non-planar Amplituhedron in N=4 SYM
- Partial progress in N=8 SUGRA

Object of interest

- Massless maximal susy scattering amplitudes in D=4
- Integrands: no divergencies, only simple pole $\dfrac{1}{P^2}$
- Simple singularity structure:





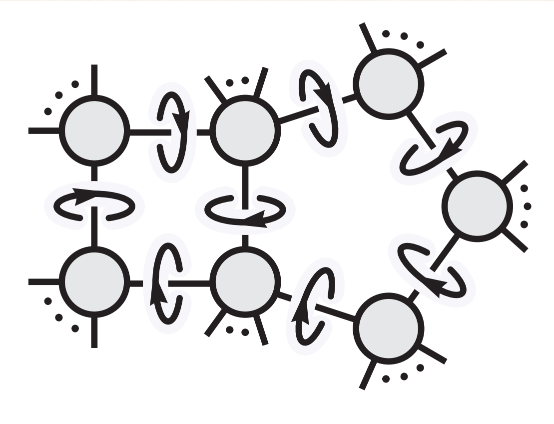
- Recursion relations: “integrating” this equation

Generalized unitarity

(Bern, Dixon, Kosower) (Britto, Cachazo, Feng 2004)

- Write the amplitude in the basis of integrals

$$\mathcal{A} = \sum_j a_j \int d\mathcal{I}_j$$



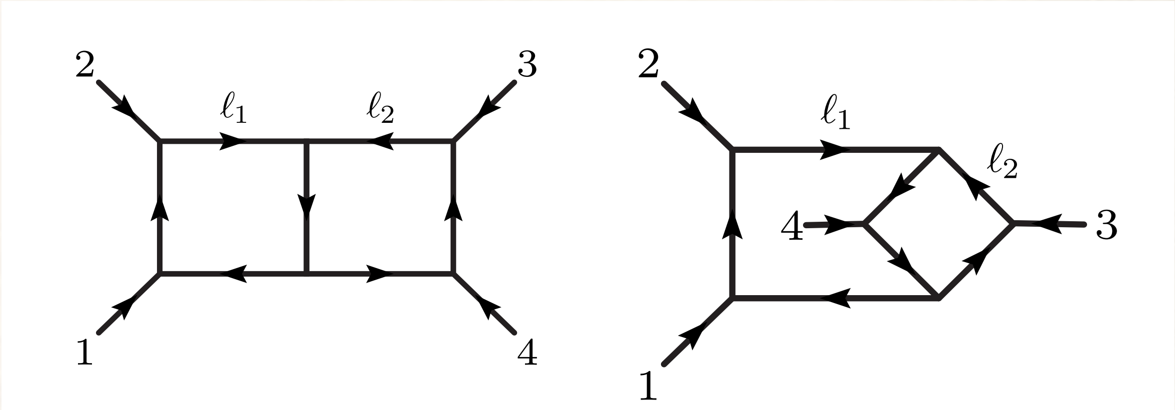
- Iterative use of cut equation
- Cuts of loops are products of tree-level amplitudes
- Very efficient method of calculation

Hydrogen atom of gauge theories

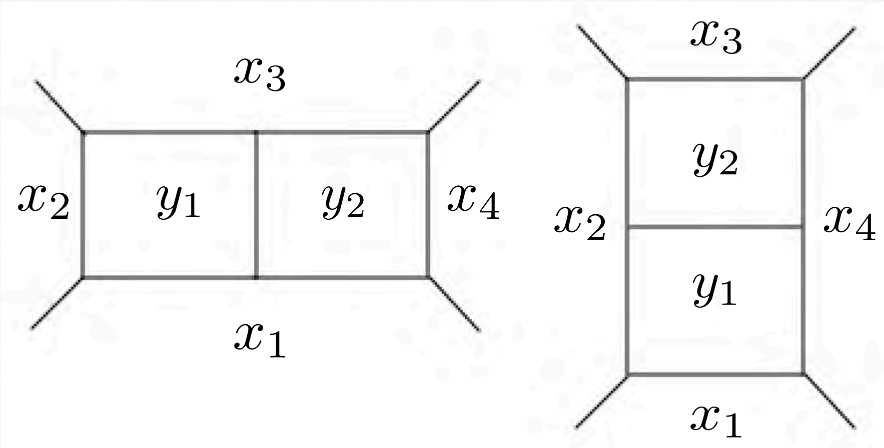
- N=4 Super Yang-Mills theory in the planar limit
- Convergent perturbative series, hidden symmetries
- Two ingredients:
  - N=4 susy: conformal symmetry, helicity book-keeping
  - Planarity: dual variables, dual conformal symmetry
- Loops: no renormalization, running of coupling

Dual variables

- Expand the amplitude as a sum of diagrams
  - No global loop momenta
  - Each diagram: its own labels



- Planar limit: dual variables $p_i = x_{i+1} - x_i$



$$k_1 = (x_1 - x_2) \quad k_2 = (x_2 - x_3)$$
$$\ell_1 = (x_3 - y_1) \quad \ell_2 = (y_2 - x_3)$$
etc

Global variables

Dual conformal symmetry

- Using these variables: define a single function

$$\mathcal{M} = \int d^4y_1 \ldots d^4y_L \, \mathcal{I}(x_i, y_j)$$

Planar integrand

- Tree-level amplitudes + integrand in planar N=4 SYM:

  Dual conformal symmetry (Drummond, Henn, Smirnov, Sokatchev 2007)

- Superconformal symmetry + Dual -> Yangian

  (Drummond, Henn, Plefka 2009)

Hidden structures

- Modern unitarity approach for planar N=4 SYM

$$\mathcal{A} = \sum_j a_j \int d\mathcal{I}_j$$

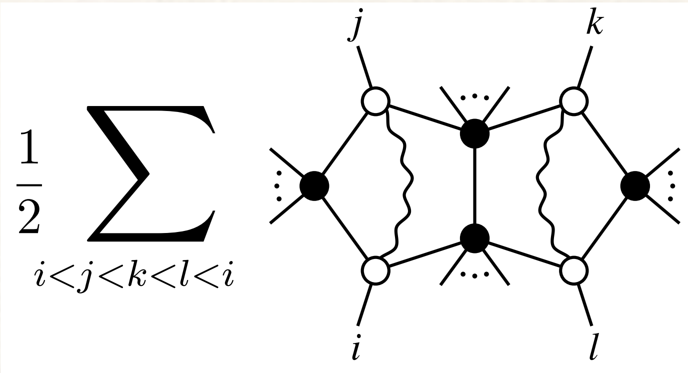
Yangian invariant coefficients fixed by leading singularities

Dual conformal invariant basis of pure integrals

- We obtained many results using this method
  - 2-loop for any n,k
  - 3-loop n-pt MHV

(Arkani-Hamed, Bourjaily, Cachazo, Trnka, 2010)

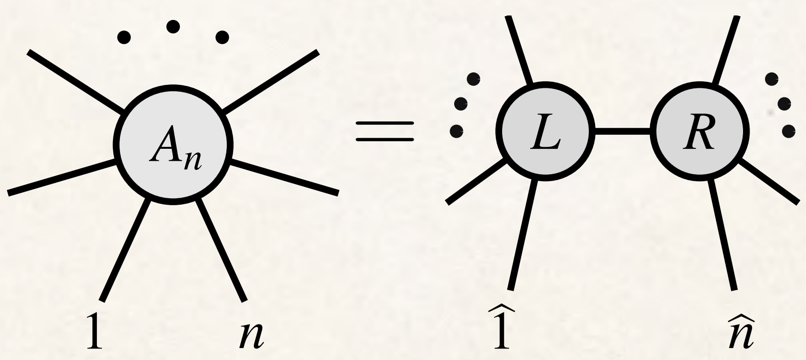
(Bourjaily, Trnka, 2015)



Why is planarity important?

- Unique planar integrand: one object, no expansions
- Search for new methods which reproduce the function
- Unphysical poles: well-defined cancelations

For example:
Generalization of BCFW
to loop amplitudes



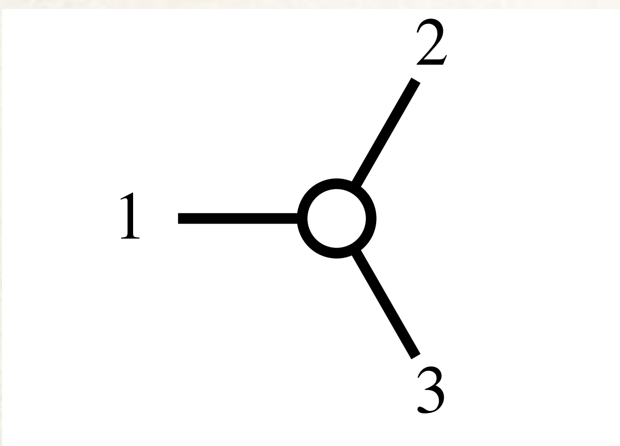
(Arkani-Hamed, Bourjaily, Cachazo, Caron-Huot, Trnka, 2010)

- Definition of the amplitude as single object: Amplituhedron

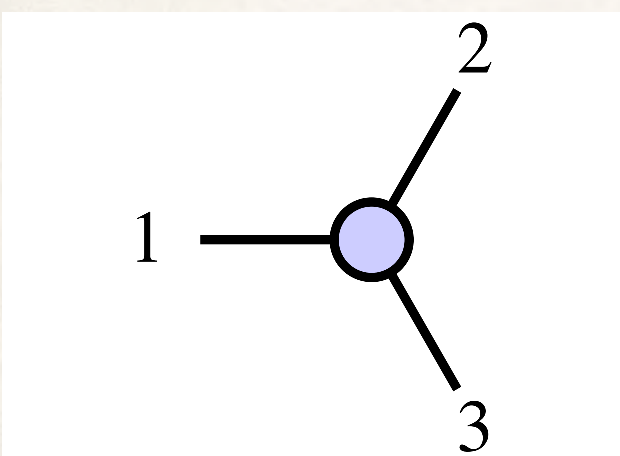
New picture for planar integrand

Three point kinematics

- Two options



$$\lambda_1 \sim \lambda_2 \sim \lambda_3$$



$$\widetilde{\lambda}_1 \sim \widetilde{\lambda}_2 \sim \widetilde{\lambda}_3$$

<u>Spinor helicity variables</u>

$$p^\mu = \sigma^\mu_{a\dot{a}} \lambda_a \widetilde{\lambda}_{\dot{a}}$$

$$\langle 12 \rangle = \epsilon_{ab} \lambda_{1a} \lambda_{2b}$$

$$[12] = \epsilon_{\dot{a}\dot{b}} \lambda_{1\dot{a}} \lambda_{2\dot{b}}$$

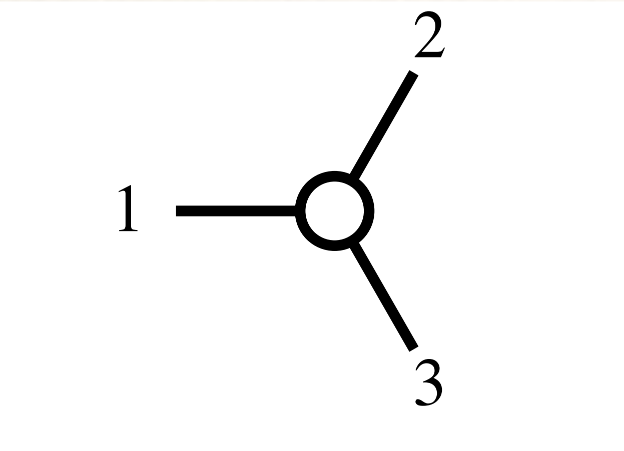
Two solutions for 3pt kinematics

$$p_1^2 = p_2^2 = p_3^2 = (p_1 + p_2 + p_3) = 0$$

Three point massless amplitudes fixed in any QFT

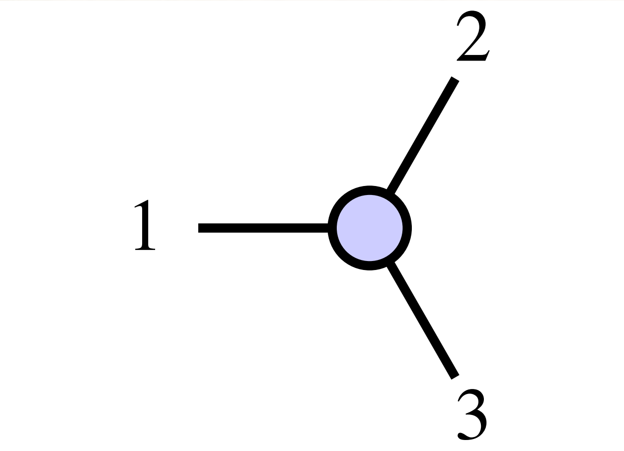
Three point amplitudes

- Three point amplitudes in N=4 SYM



$$\mathcal{A}_3^{(1)} = \frac{\delta^4(p_1+p_2+p_3)\delta^4([23]\widetilde{\eta}_1 + [31]\widetilde{\eta}_2 + [12]\widetilde{\eta}_3)}{[12][23][31]}$$

$$\lambda_1 \sim \lambda_2 \sim \lambda_3$$



$$\mathcal{A}_3^{(2)} = \frac{\delta^4(p_1+p_2+p_3)\delta^8(\lambda_1\widetilde{\eta}_1 + \lambda_2\widetilde{\eta}_2 + \lambda_3\widetilde{\eta}_3)}{\langle 12\rangle\langle 23\rangle\langle 31\rangle}$$

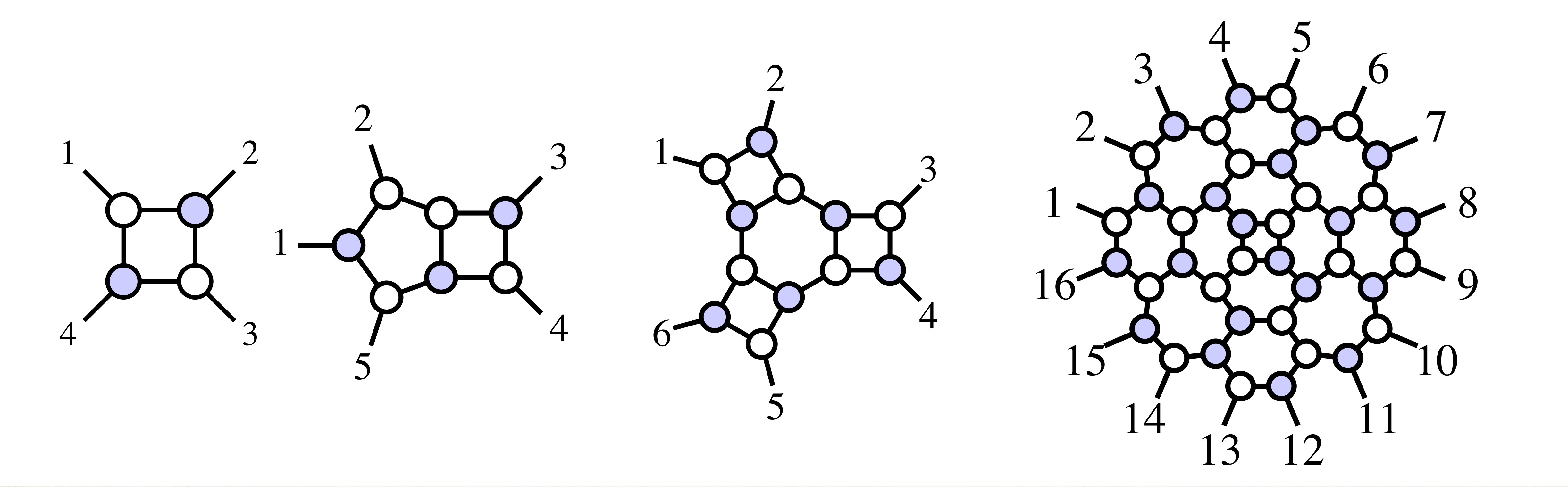
$$\widetilde{\lambda}_1 \sim \widetilde{\lambda}_2 \sim \widetilde{\lambda}_3$$

$$\Phi = G_+ + \tilde{\eta}_A\Gamma_A + \frac{1}{2}\tilde{\eta}^A\tilde{\eta}^B S_{AB} + \frac{1}{6}\epsilon_{ABCD}\tilde{\eta}^A\tilde{\eta}^B\tilde{\eta}^C\overline{\Gamma}^D + \frac{1}{24}\epsilon_{ABCD}\tilde{\eta}^A\tilde{\eta}^B\tilde{\eta}^C\tilde{\eta}^D G_-$$

On-shell diagrams

(Arkani-Hamed, Bourjaily, Cachazo, Goncharov, Postnikov, JT 2012)

- Draw planar graph with three point vertices



Cuts of loop integrands

Product of 3pt amplitudes

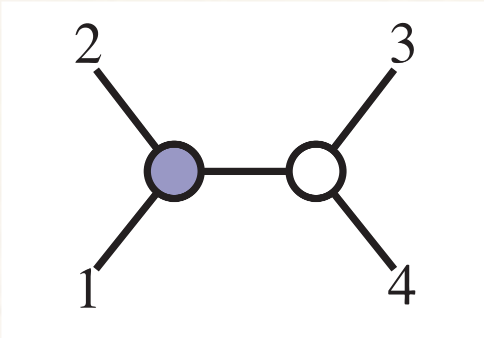
All legs are on-shell

On-shell diagrams

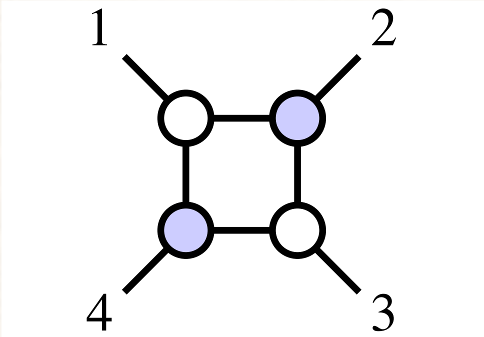
(Arkani-Hamed, Bourjaily, Cachazo, Goncharov, Postnikov, JT 2012)

- The results are functions of external kinematics

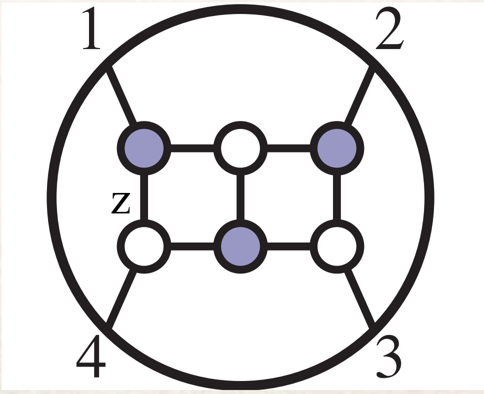
$P > 4L$ Extra delta functions



$P = 4L$ Leading singularities



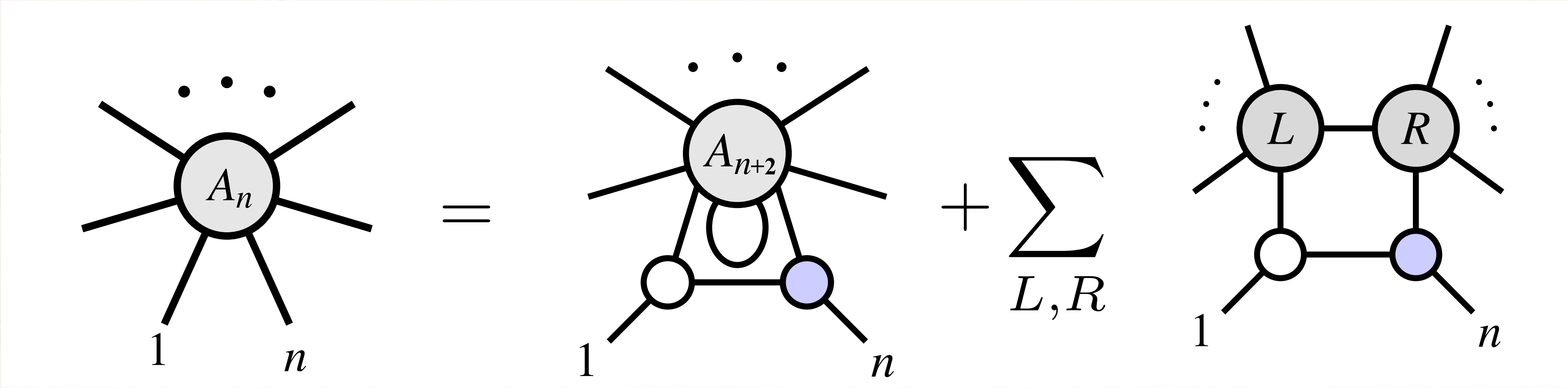
$P < 4L$ Unfixed parameters



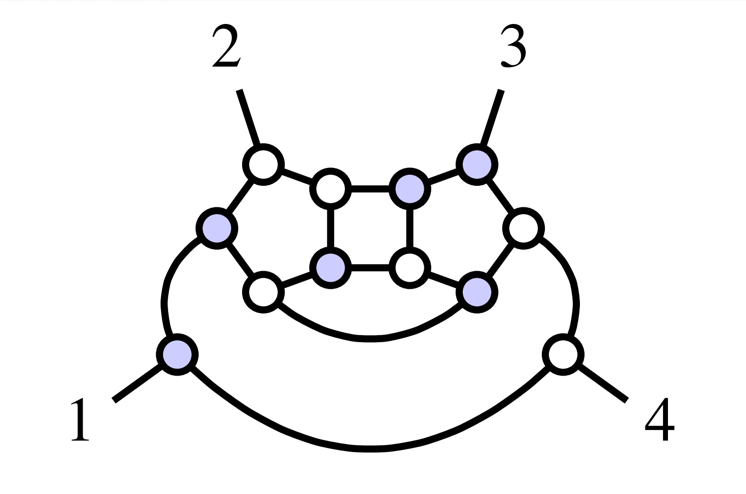
Recursion relations

(Britto, Cachazo, Feng, Witten 2005) (Arkani-Hamed, Bourjaily, Cachazo, Caron-Huot, JT 2010)

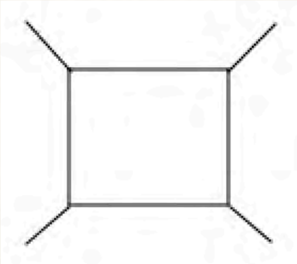
- Recursion relations for ℓ-loop integrand



- Example: 4pt 1-loop



5-loop on-shell diagram = 1-loop off-shell box



Momentum conservation

- Deep connection: on-shell diagrams vs Grassmannian
- Simple motivation: linearize momentum conservation

$$\delta(P) = \delta\left(\sum_a \lambda_a \widetilde{\lambda}_a\right)$$

- We want to write it as two linear factors

$$\delta\left(C_{ab}\widetilde{\lambda}_b\right)\,\delta\left(D_{ab}\lambda_b\right)$$

Grassmannian

(Arkani-Hamed, Bourjaily, Cachazo, Kaplan 2009)

- You get k relations, (k x n) matrix C / GL(k)

$$\delta\left(C_{ab}\widetilde{\lambda}_b\right) \quad \begin{matrix} a = 1, \ldots, k \\ b = 1, \ldots, n \end{matrix}$$

$$C \in G(k, n)$$

k-plane in n dimensions

$$\delta(D_{ab}\lambda_b) = \delta\left(C^{\perp}_{ab}\widetilde{\lambda}_b\right)$$

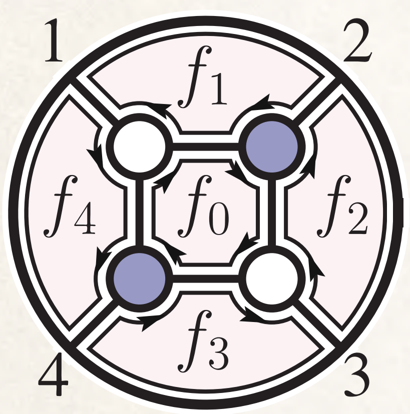
- This matrix C has many free parameters: many ways how linearize momentum conservation

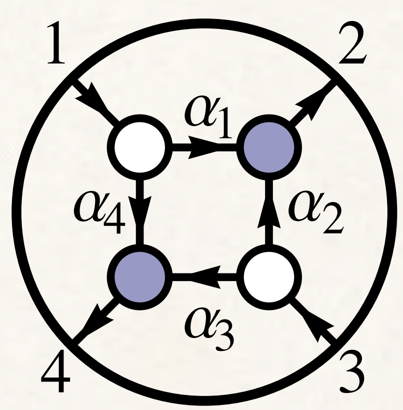
- Each on-shell diagram gives you one

Grassmannian

(Arkani-Hamed, Bourjaily, Cachazo, Goncharov, Postnikov, JT 2012)

- Building matrix: face or edge variables





$$C = \begin{pmatrix} 1 & \alpha_1 & 0 & -\alpha_4 \\ 0 & \alpha_2 & 1 & \alpha_3 \end{pmatrix}$$

- Exciting connection to mathematics

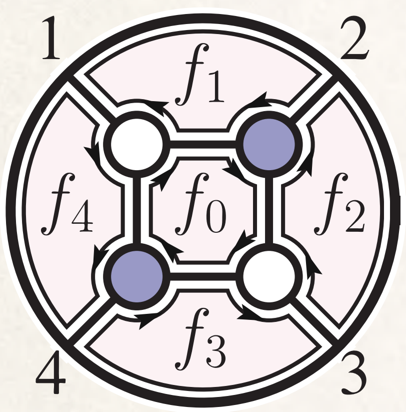
  Choose $\alpha_i > 0$: positive minors -> Positive Grassmannian

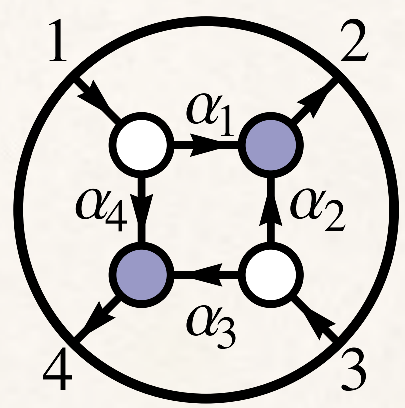
  Area of research in algebraic geometry, combinatorics

Connection to amplitudes

(Arkani-Hamed, Bourjaily, Cachazo, Goncharov, Postnikov, JT 2012)

- Building matrix: face or edge variables





$$C = \begin{pmatrix} 1 & \alpha_1 & 0 & -\alpha_4 \\ 0 & \alpha_2 & 1 & \alpha_3 \end{pmatrix}$$

- Same function as a product of 3pt amplitudes equal to

$$\boxed{\Omega = \frac{d\alpha_1}{\alpha_1}\frac{d\alpha_2}{\alpha_2}\frac{d\alpha_3}{\alpha_3}\frac{d\alpha_4}{\alpha_4}\delta(C \cdot Z)}$$

Solves for α_i
in terms of $\lambda_i, \widetilde{\lambda}_i$
and gives $\delta(P)\delta(Q)$

$$\delta(C \cdot Z) = \delta(C \cdot \widetilde{\lambda})\delta(C^{\perp} \cdot \lambda)\delta(C \cdot \widetilde{\eta})$$

Hidden symmetries

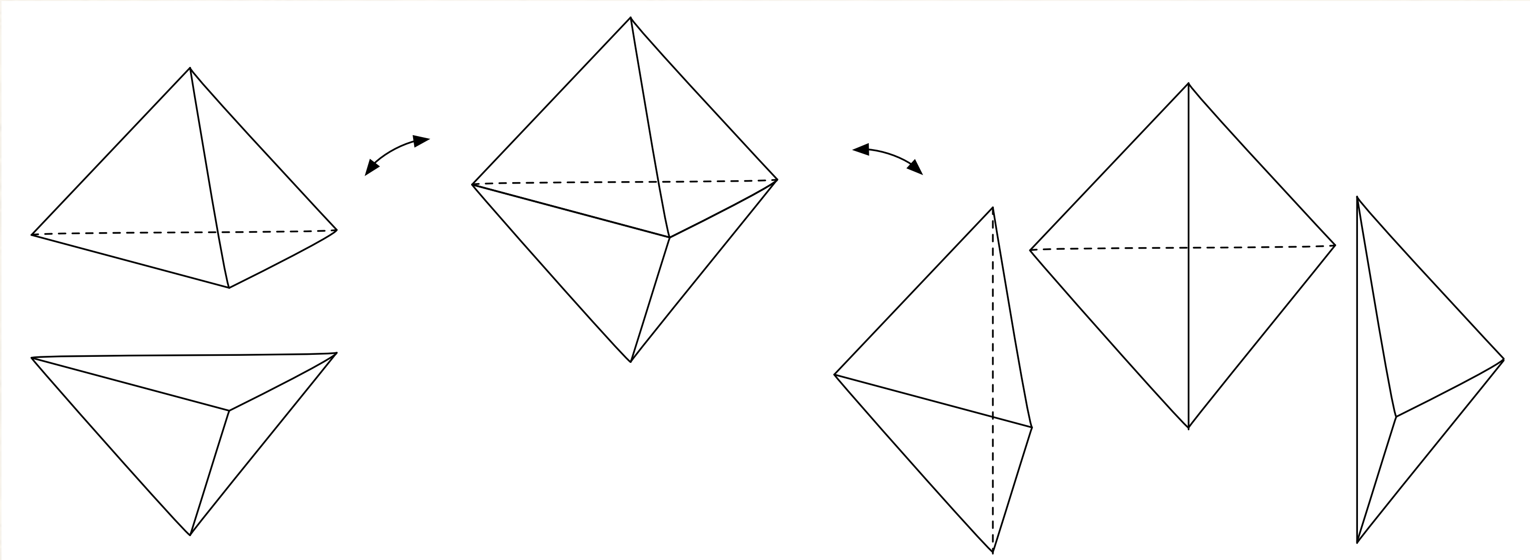
- Each on-shell diagram:
  - Dual conformal symmetry, Yangian
  - Logarithmic singularities $\frac{dx}{x}$
- Recursion relations: true for amplitudes
- Geometric formulation using Grassmannian
  - All dependence on kinematics: delta function
- Role of recursion relations, complete geometry picture?

Amplituhedron

(Arkani-Hamed, JT 2013)

- Motivation: Grassmannian + polytope picture

(Hodge 2009)



Amplitudes are volumes!

- Definition of the space $Y = C \cdot Z$

- Loop integrand = volume form $$\Omega = \frac{dx_1}{x_1}\frac{dx_2}{x_2}\ldots\frac{dx_m}{x_m}$$

Amplituhedron

(Arkani-Hamed, JT 2013)

- The definition of the space if known to all loops
- Main challenge: find the form
    - Triangulate the space into “simplices”
    - Find the form from the definition
- Our goal is non-planar: use of diagrams, unitarity
- Formulate the Amplituhedron in this language

Properties of Amplituhedron

- Assumptions:
  - Dual conformal symmetry: momentum twistors
  - Logarithmic singularities: definition of the form
- Implications:
  - All-loop order definition of the integrand
  - Proof: reproduces the same singularity structure
  - Dual Amplituhedron: volume of some region

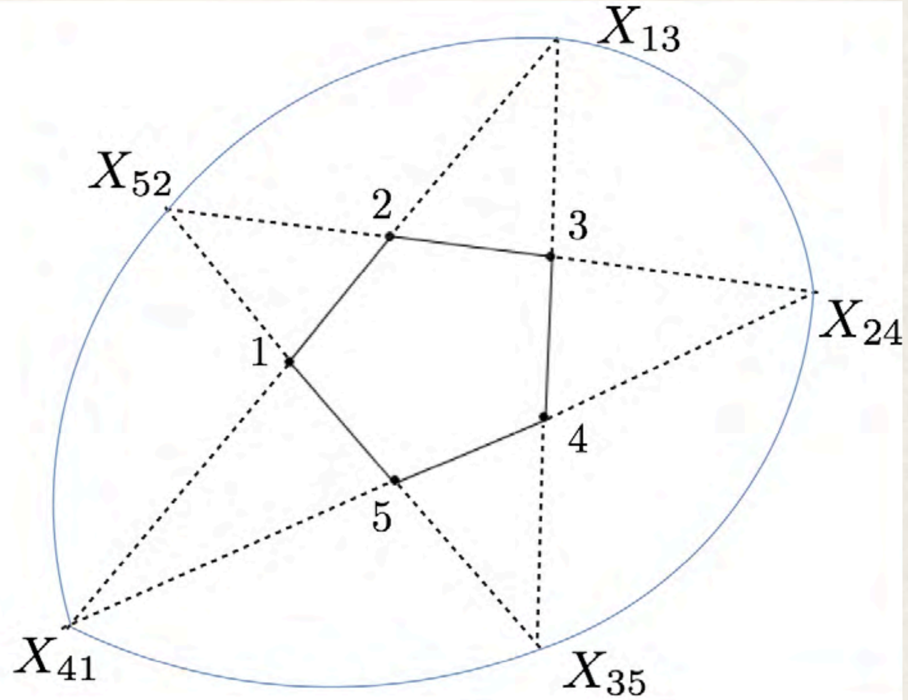
Positivity of amplitudes

(Arkani-Hamed, Hodges, JT, 2014)

- Volume form of the Amplituhedron

$$I = \frac{(\text{Numerator})}{(\text{all poles})}$$

- Numerator fixed by **zeroes**
  - Points outside Amplituhedron
  - Canceling higher poles



Surface outside Amplituhedron

- Amplitude **positive** (for points inside): volume interpretation

- **Vanishing** on a conic of illegal points

Implications in unitarity methods

- Expansion of the amplitude

$$\mathcal{A} = \sum_j a_j \int d\mathcal{I}_j$$

Yangian invariant coefficients fixed by vanishing cuts

Special basis of integrals:
- Dual conformal symmetry
- Logarithmic singularities

- Step 1: construct the basis of special integrals

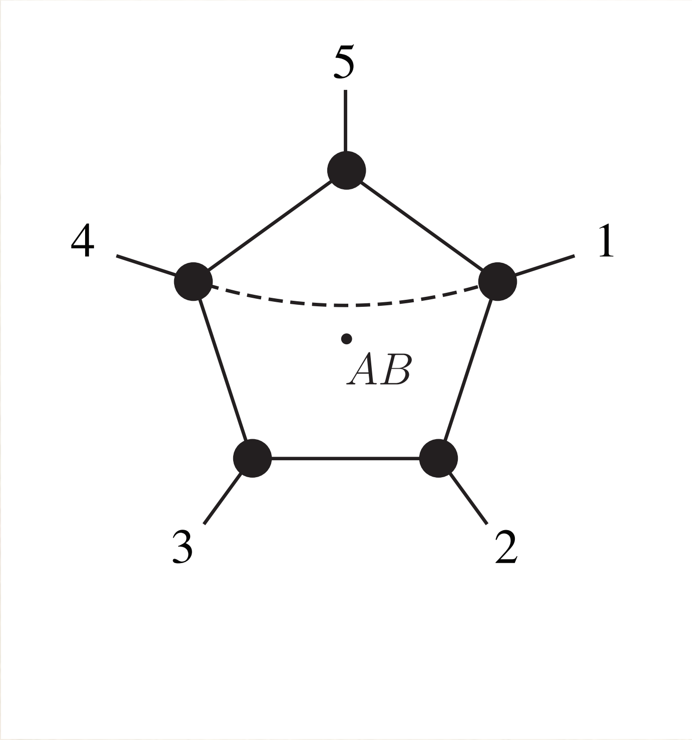
- Step 2: fix the coefficients by checking vanishing cuts

Dual conformal symmetry

- All integrals in the basis: dual conformal invariant
- Simple rule: function of momentum twistors
- How to see dual conformal symmetry in the cut structure of individual integrals?

DCI in action

Unit leading singularities



On all 4L-cut the residue is 1

Chiral vs scalar pentagon

For n>6 can be cross ratio

$$\frac{\langle ABd^2A\rangle\langle ABd^2B\rangle\langle AB13\rangle\langle 2345\rangle\langle 4512\rangle}{\langle AB12\rangle\langle AB23\rangle\langle AB34\rangle\langle AB45\rangle\langle AB51\rangle}$$

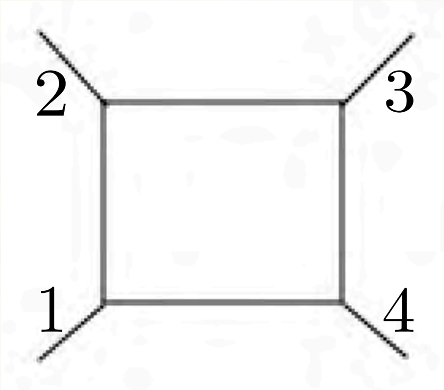
Unit

$$\frac{\langle ABd^2A\rangle\langle ABd^2B\rangle\langle ABI\rangle}{\langle AB12\rangle\langle AB23\rangle\langle AB34\rangle\langle AB45\rangle\langle AB51\rangle}$$

Non-unit

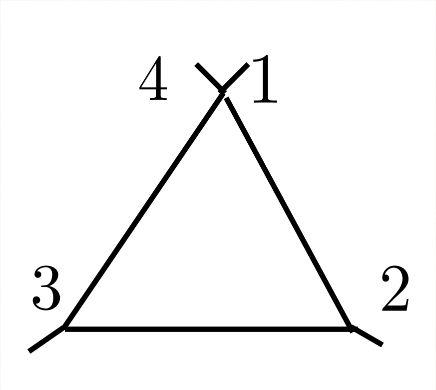
DCI in action

No poles at infinity $\ell \to \infty$



$$\frac{\langle ABd^2A\rangle\langle ABd^2B\rangle\langle 1234\rangle^2}{\langle AB12\rangle\langle AB23\rangle\langle AB34\rangle\langle AB41\rangle}$$

No pole



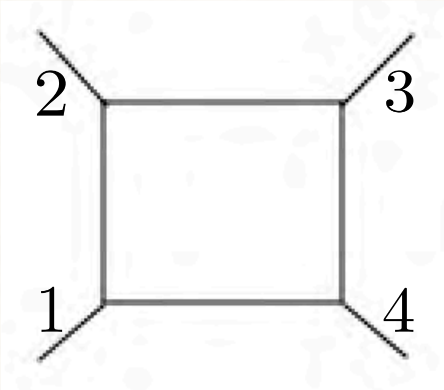
$$\frac{\langle ABd^2A\rangle\langle ABd^2B\rangle\langle 1234\rangle\langle 23I\rangle}{\langle AB12\rangle\langle AB23\rangle\langle AB34\rangle\langle ABI\rangle}$$

Pole

Cut this propagator

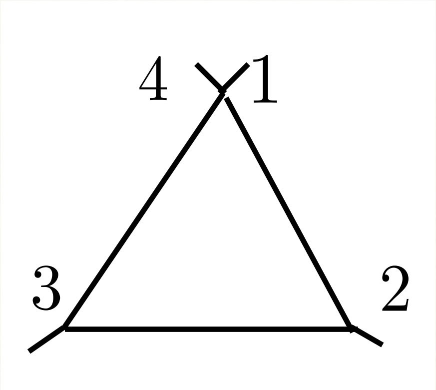
DCI in action

No poles at infinity $\ell \to \infty$



$$\frac{\langle ABd^2A\rangle\langle ABd^2B\rangle\langle 1234\rangle^2}{\langle AB12\rangle\langle AB23\rangle\langle AB34\rangle\langle AB41\rangle}$$

No pole

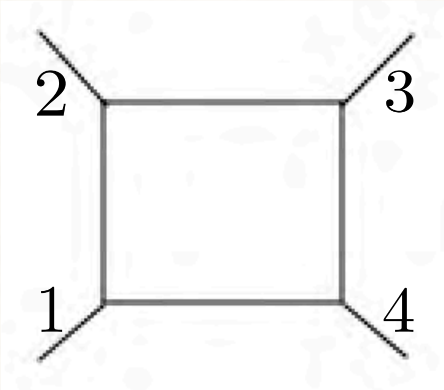


$$\frac{d^4\ell}{\ell^2(\ell+k_2)^2(\ell+k_2+k_3)^2}$$

Pole

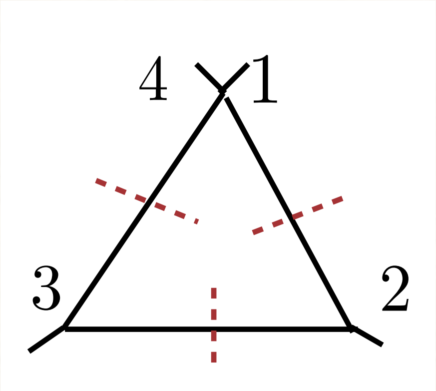
DCI in action

No poles at infinity $\ell \to \infty$



$$\frac{\langle ABd^2A\rangle\langle ABd^2B\rangle\langle 1234\rangle^2}{\langle AB12\rangle\langle AB23\rangle\langle AB34\rangle\langle AB41\rangle}$$

No pole

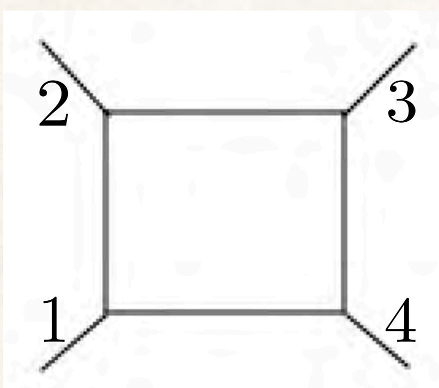


$$\frac{d^4\ell}{\underset{\downarrow\atop 0}{\ell^2}\,\underset{\downarrow\atop 0}{(\ell+k_2)^2}\,\underset{\downarrow\atop 0}{(\ell+k_2+k_3)^2}}$$

Pole

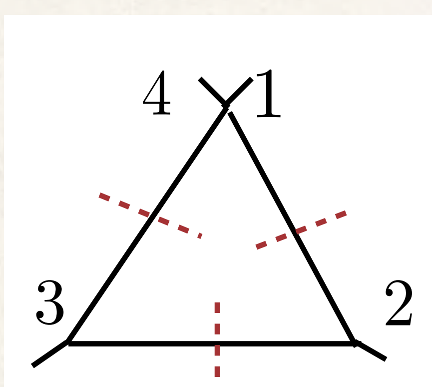
DCI in action

No poles at infinity $\ell \to \infty$



$$\frac{\langle ABd^2A\rangle\langle ABd^2B\rangle\langle 1234\rangle^2}{\langle AB12\rangle\langle AB23\rangle\langle AB34\rangle\langle AB41\rangle}$$

No pole



Pole

$$\frac{d\alpha}{\alpha} \qquad \ell + k_2 = \alpha\lambda_2\tilde{\lambda}_3$$

$\alpha \to \infty$ $\ell \to \infty$

Logarithmic singularities

- **Logarithmic singularities** $\dfrac{dx}{x}$
  - Statement about types of poles in the cut structure
  - Link to the uniform transcendentality
- More than single poles $\dfrac{dx\,dy}{xy(x+y)} \xrightarrow{x=0} \dfrac{dy}{y^2}$
- Certain integrals also have this property

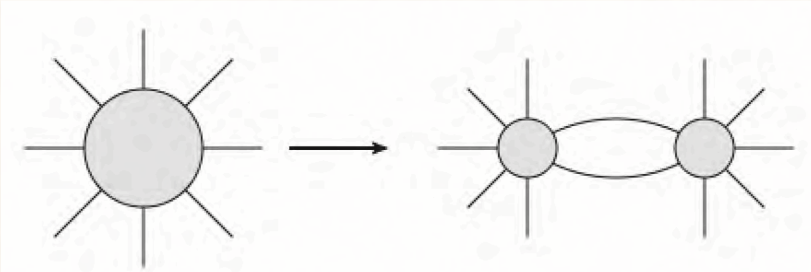
$$dI_4 = \frac{d^4\ell\, st}{\ell^2(\ell+k_1)^2(\ell+k_1+k_2)^2(\ell-k_4)^2} = \frac{df_1}{f_1}\frac{df_2}{f_2}\frac{df_3}{f_3}\frac{df_4}{f_4}$$

$$f_1 = \frac{\ell^2}{(\ell-\ell^*)^2}$$

Homogeneous constraints

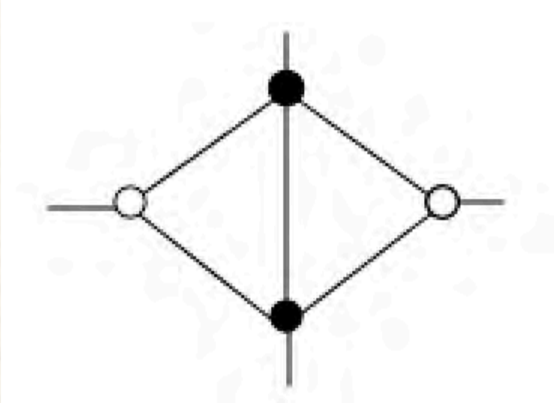
(Arkani-Hamed, Hodges, JT, 2014) (Bern, Herrmann, Litsey, Stankowicz, JT 2015)

- Basis is constructed: fix the coefficients using cuts
- Standard unitarity methods



- (Dual) Amplituhedron: only vanishing cuts enough

Example:
Illegal cut of the
2-loop 4pt amplitude



Fixes the relative coefficient
of two planar double boxes

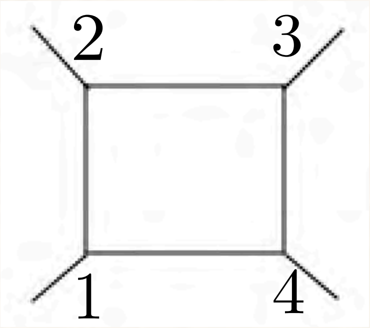
Non-planar amplitudes in N=4 SYM

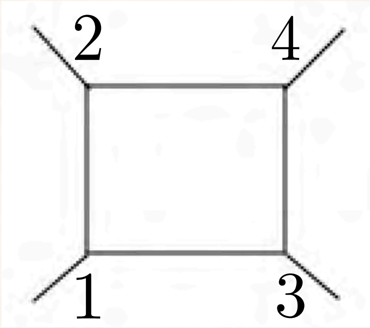
(Arkani-Hamed, Bourjaily, Cachazo, JT, 2014)

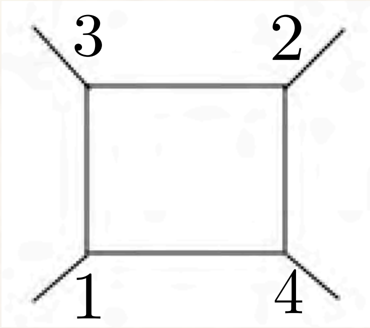
(Bern, Herrmann, Litsey, Stankowicz, JT 2014, 2015)

Non-planar problems

- No unique integrand, labeling problem



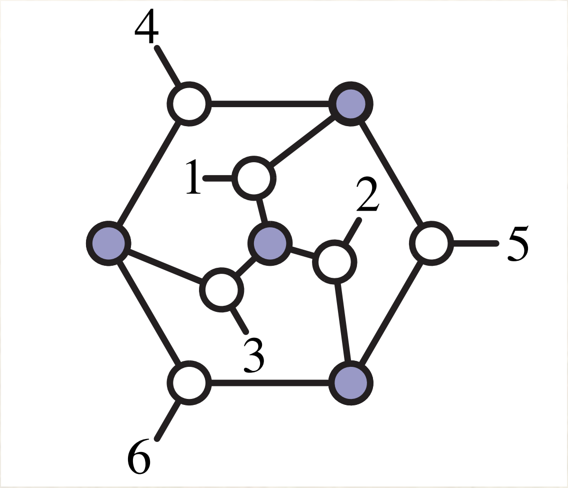




What is ℓ ?

- No momentum twistors, no known symmetries

- On-shell diagrams for singularities



No recursion relations

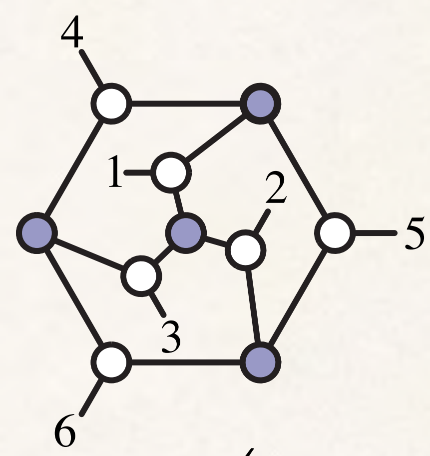
Non-planar on-shell diagrams

- Non-planar diagrams

(Arkani-Hamed, Bourjaily, Cachazo, Postnikov, JT, 2014)

(Franco, Galloni, Penante, Wen 2015)

(Bourjaily, Franco, Galloni, Wen 2016)



see talks by
Jake and Daniele

Same logarithmic form

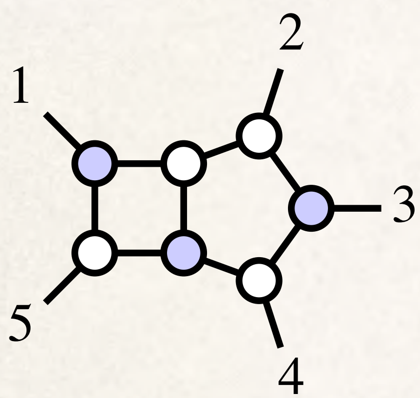
$$C = \begin{pmatrix} * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \end{pmatrix} \in G(3,6)$$

$$\Omega = \frac{d\alpha_1}{\alpha_1}\frac{d\alpha_2}{\alpha_2}\frac{d\alpha_3}{\alpha_3}\frac{d\alpha_4}{\alpha_4}\frac{d\alpha_5}{\alpha_5}\frac{d\alpha_6}{\alpha_6}\delta(C\cdot Z)$$

- **Conjecture**: logarithmic singularities of the amplitude

MHV on-shell diagrams

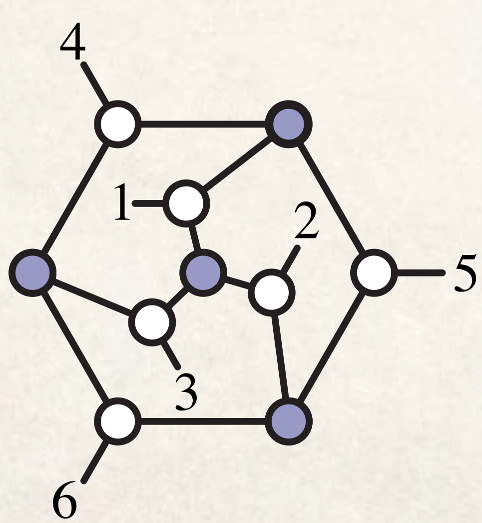
- Planar sector: all are Parke-Taylor factors



$$= \frac{1}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 51\rangle} = PT(12345)$$

required by superconformal symmetry

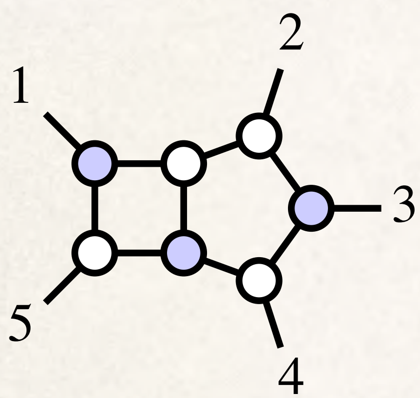
- Non-planar diagrams: holomorphic functions



$$= \frac{(\langle 34\rangle\langle 51\rangle\langle 62\rangle + \langle 14\rangle\langle 25\rangle\langle 63\rangle)^2}{\langle 12\rangle\langle 23\rangle\langle 31\rangle\langle 25\rangle\langle 56\rangle\langle 62\rangle\langle 34\rangle\langle 46\rangle\langle 63\rangle\langle 45\rangle\langle 51\rangle\langle 14\rangle}$$

MHV on-shell diagrams

- Planar sector: all are Parke-Taylor factors



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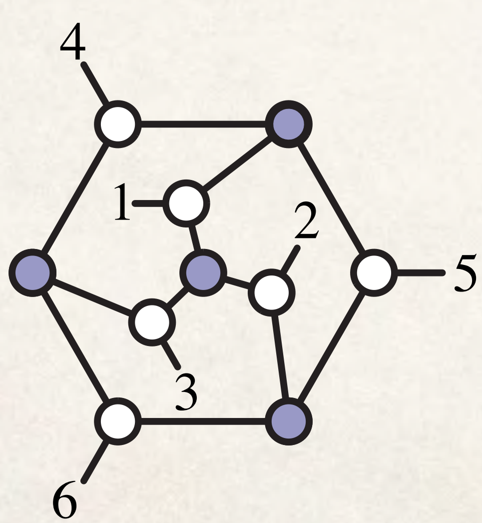


$$= PT(123456) + PT(124563) + PT(142563) + PT(145623)$$
$$+PT(146235) + PT(146253) + PT(162345)$$

Parke-Taylor factors: similar to planar

No poles at infinity

Non-planar amplitudes

- No unique integrand, no recursion relations
- On-shell diagrams: cuts of amplitudes
- Conjecture: amplitude has the same properties
  - Logarithmic singularities
  - No poles at infinity

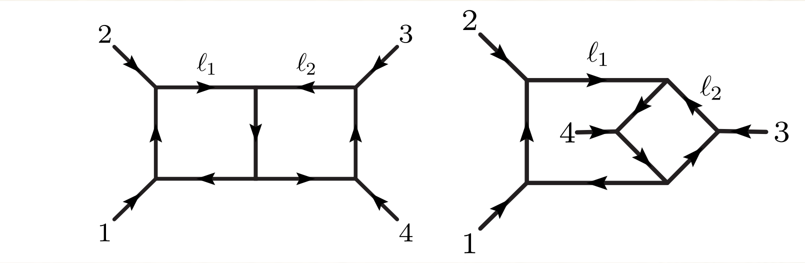
Geometric construction?

New symmetries?

Non-planar amplitudes

- Conservative approach: sum of integrals

$$A = \sum_i a_i \cdot C_i \cdot I_i$$

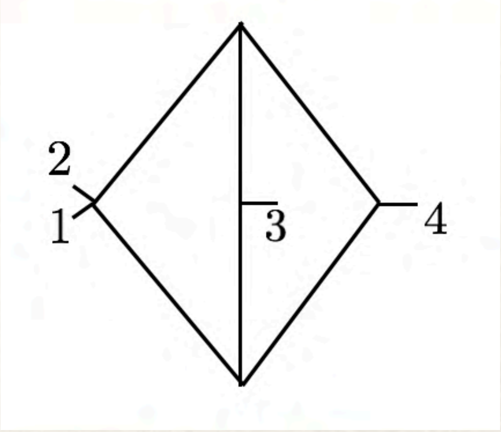


$$f^{1ab} f^{bcd} \dots f^{4ef}$$

- Conditions imposed term-by-term: special numerators

Logarithmic unit leading singularities and no poles at infinity

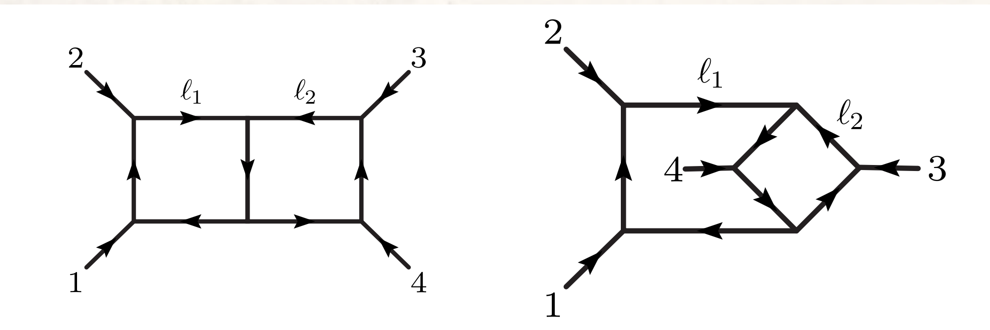
- Some diagrams forbidden

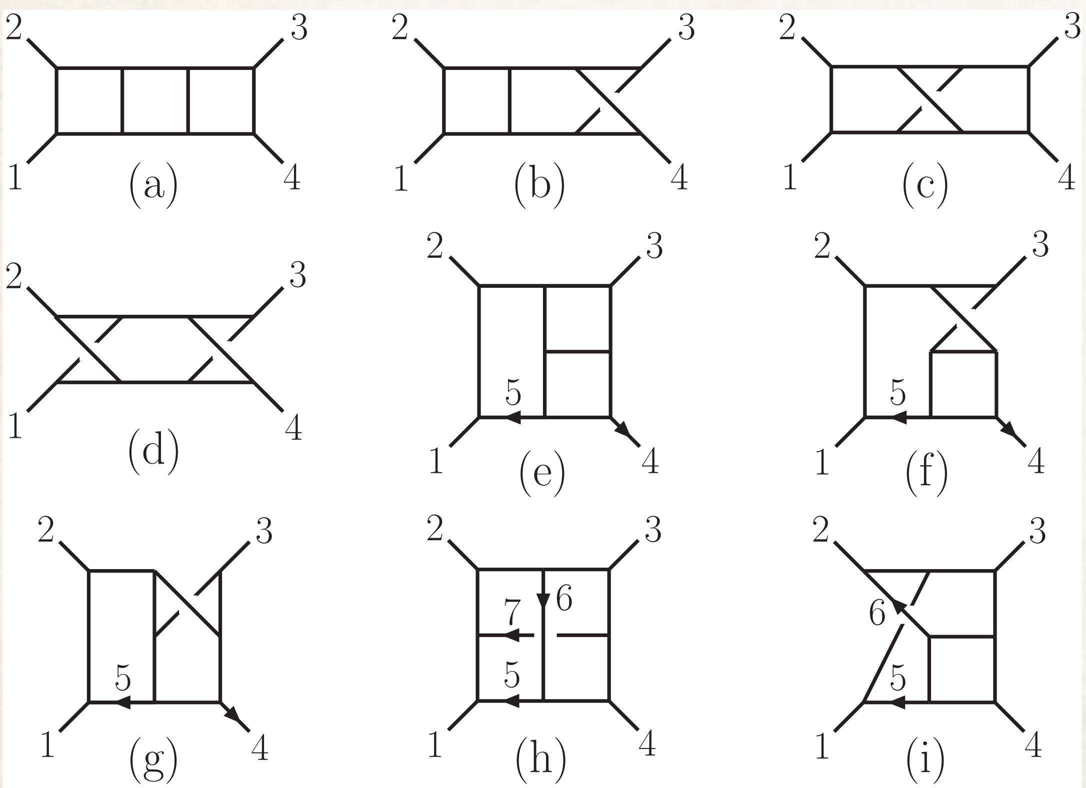


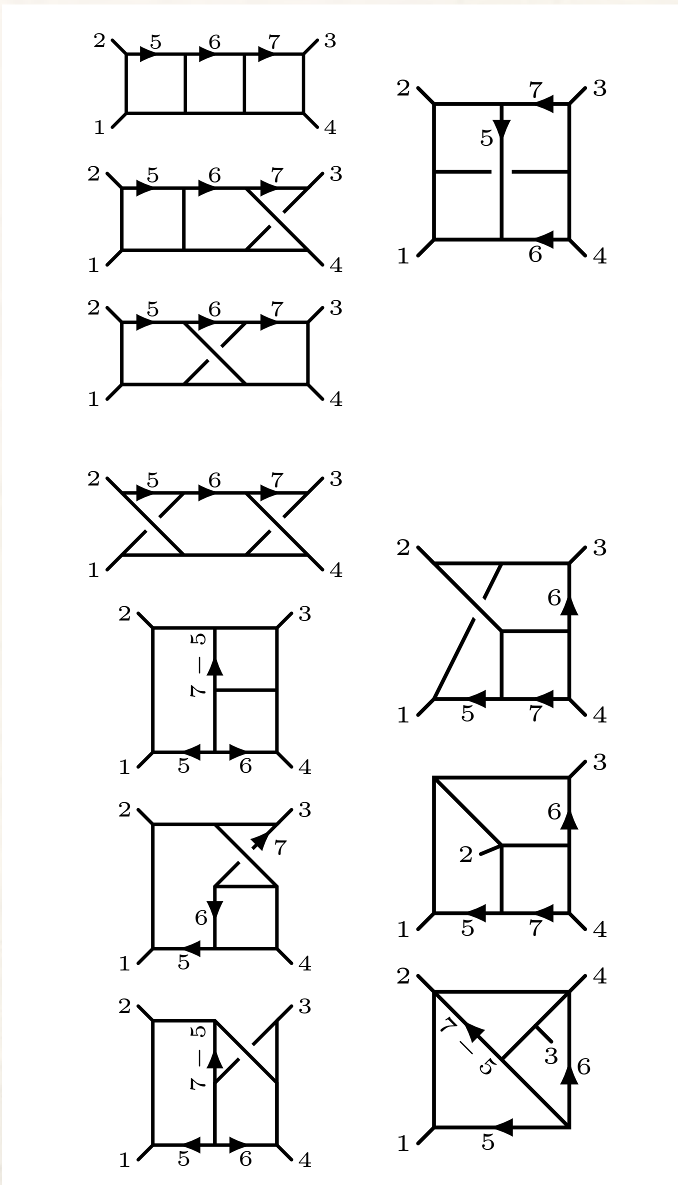
Explicit checks

(Arkani-Hamed, Bourjaily, Cachazo, JT, 2014) (Bern, Herrmann, Litsey, Stankowicz, JT 2014, 2015)

- Standard / BCJ basis for 4pt at 2-loop and 3-loop, 5pt 2-loop

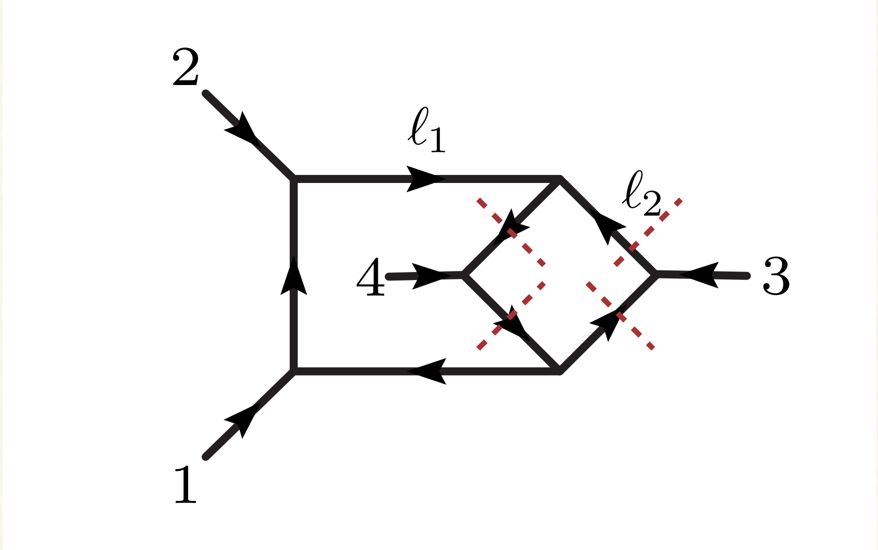






Individual terms do not satisfy our constraints

Non-planar double box

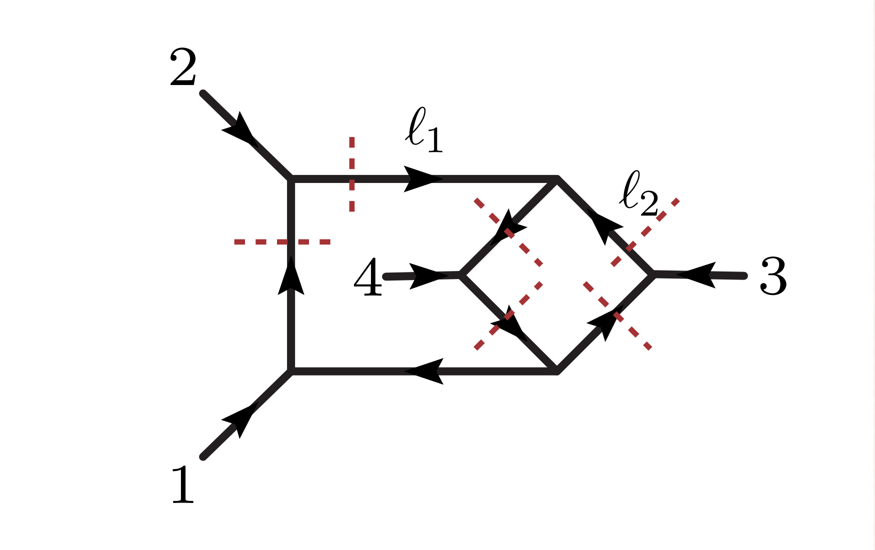


$$dI = \frac{d^4\ell_1\, d^4\ell_2\ (p_1+p_2)^2}{\ell_1^2(\ell_1-k_2)^2(\ell_1-k_1-k_2)^2\ell_2^2(\ell_2-k_3)^2(\ell_1+\ell_2)^2(\ell_1+\ell_2+k_4)^2}$$

Perform cuts $\quad \ell_2^2 = (\ell_2-k_3)^2 = (\ell_1+\ell_2)^2 = (\ell_1+\ell_2+k_4)^2 = 0$

Localize ℓ_2 completely

Non-planar double box

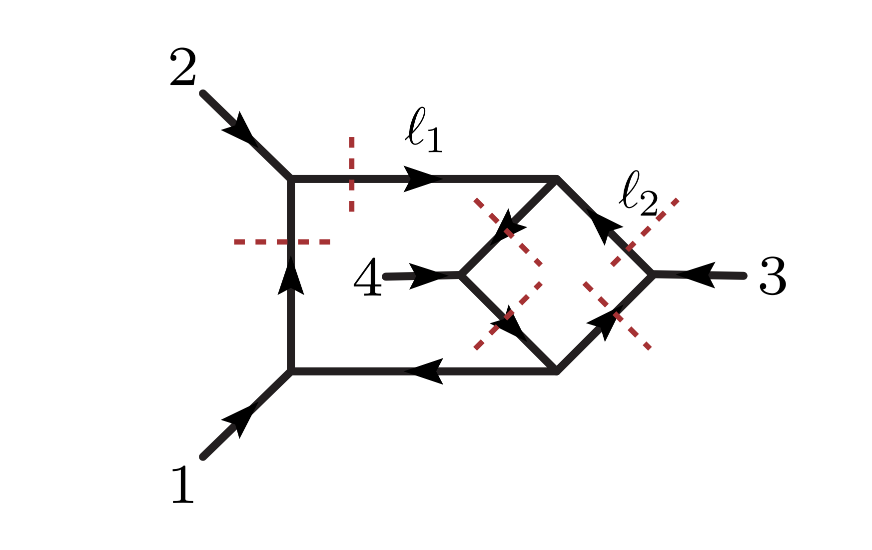


$$\mathrm{Cut}_1\, dI = \frac{d^4\ell_1}{\ell_1^2(\ell_1-k_2)^2(\ell_1-k_1-k_2)^2[(\ell_1+k_3)^2(\ell_1+k_4)^2-\ell_1^2(\ell_1+k_3+k_4)^2]}$$

Localize $\ell_1 = \alpha k_2$ by cutting $\ell_1^2 = (\ell_1 - k_2)^2 = 0$

and the Jacobian

Non-planar double box

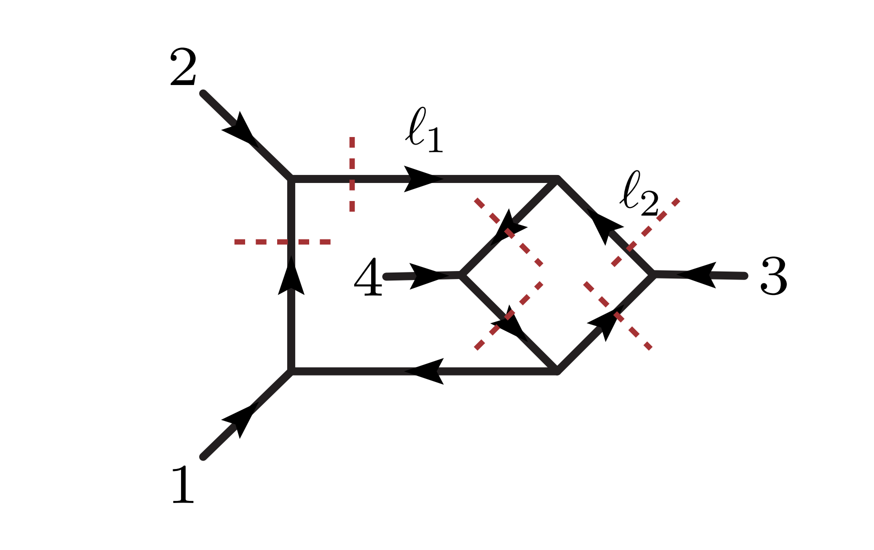


$$\mathrm{Cut}_{1,2}\, dI = \frac{d\alpha}{(\alpha+1)\alpha^2 tu}$$

$\boxed{\text{Double pole for } \alpha = 0}$

- There is also pole at infinity
- We want to find a numerator which cancels all that

Non-planar double box



$$\mathrm{Cut}_{1,2}\, dI = \frac{d\alpha}{(\alpha+1)\alpha^2 tu}$$

$\boxed{\text{Double pole for } \alpha = 0}$

<u>New numerator</u>

$$\boxed{N = (\ell_1 + k_3)^2 + (\ell_1 + k_4)^2}$$

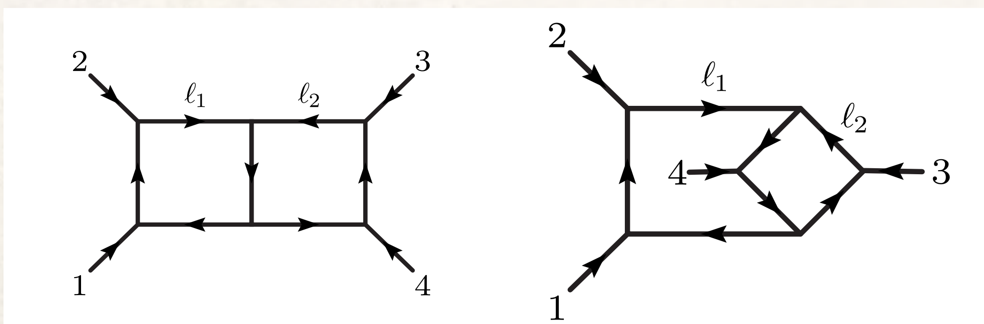
Cancels double pole

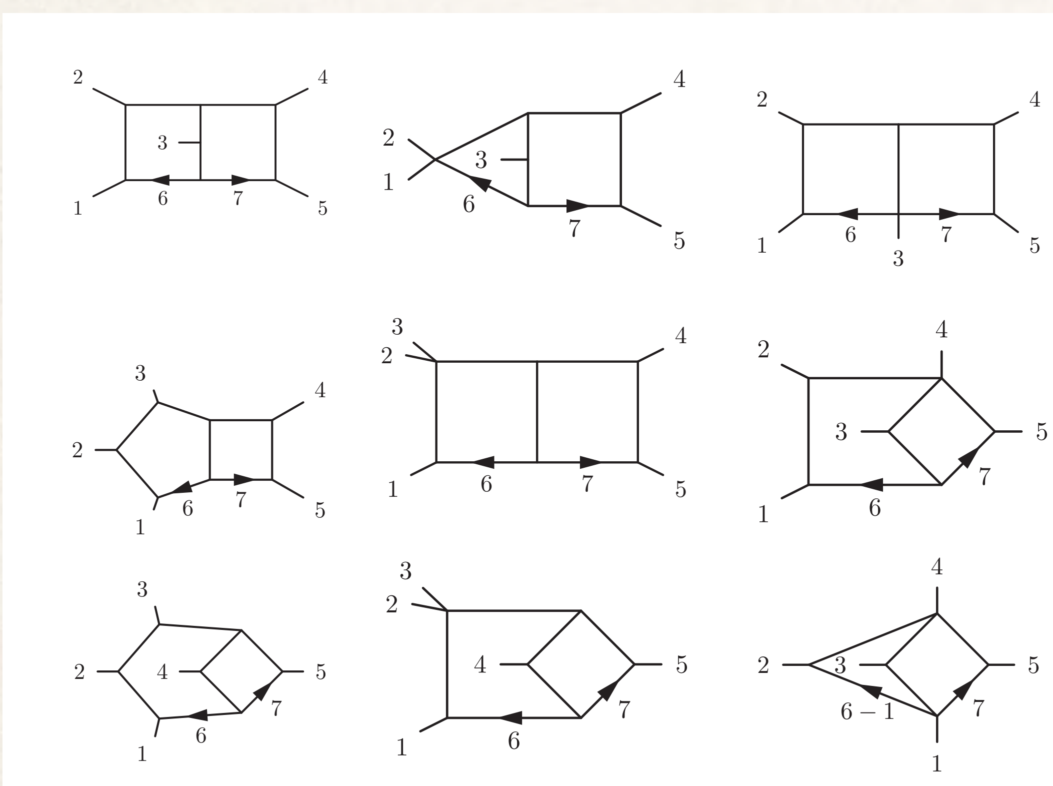
$$N \to \alpha s$$

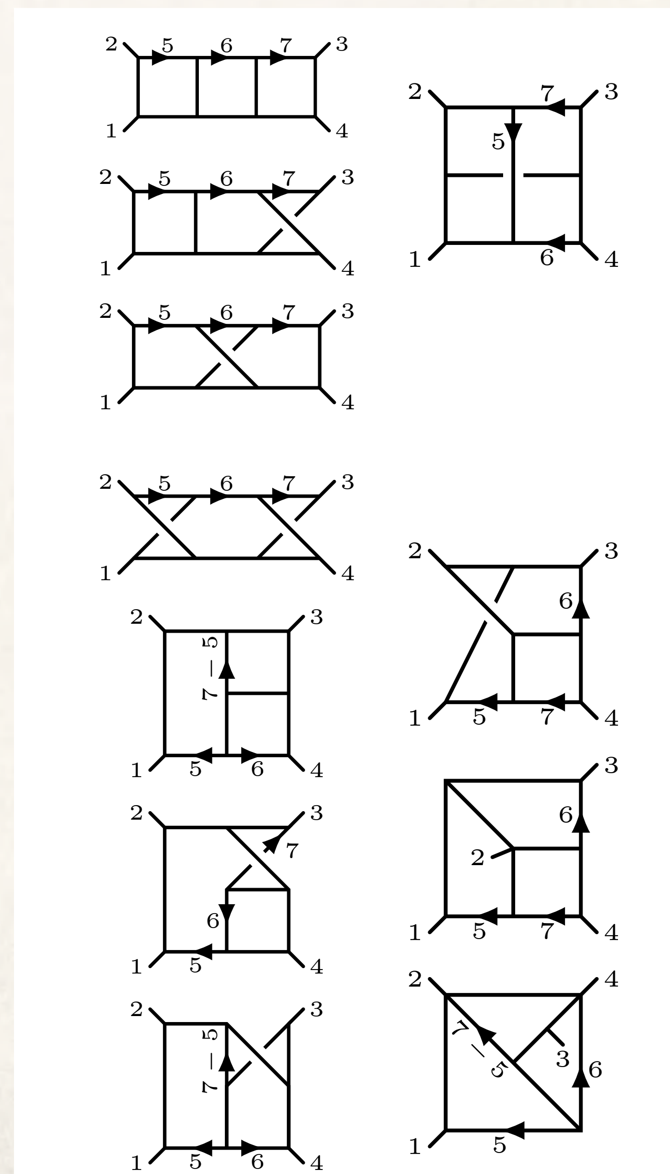
Explicit checks

(Arkani-Hamed, Bourjaily, Cachazo, JT, 2014) (Bern, Herrmann, Litsey, Stankowicz, JT 2014, 2015)

- Construct basis for 4pt at 2-loop and 3-loop, 5pt 2-loop





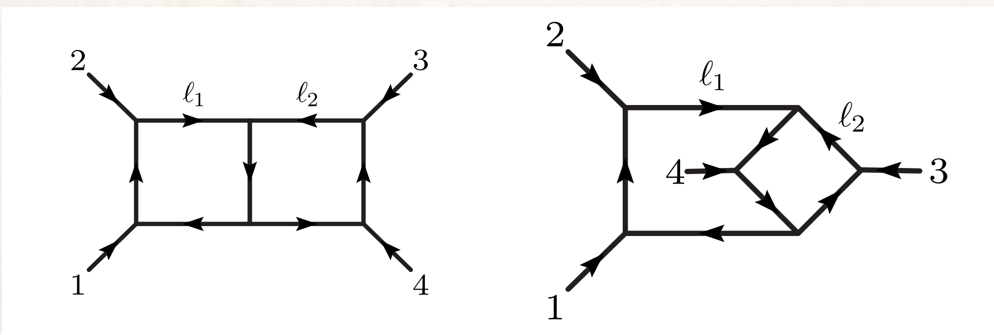


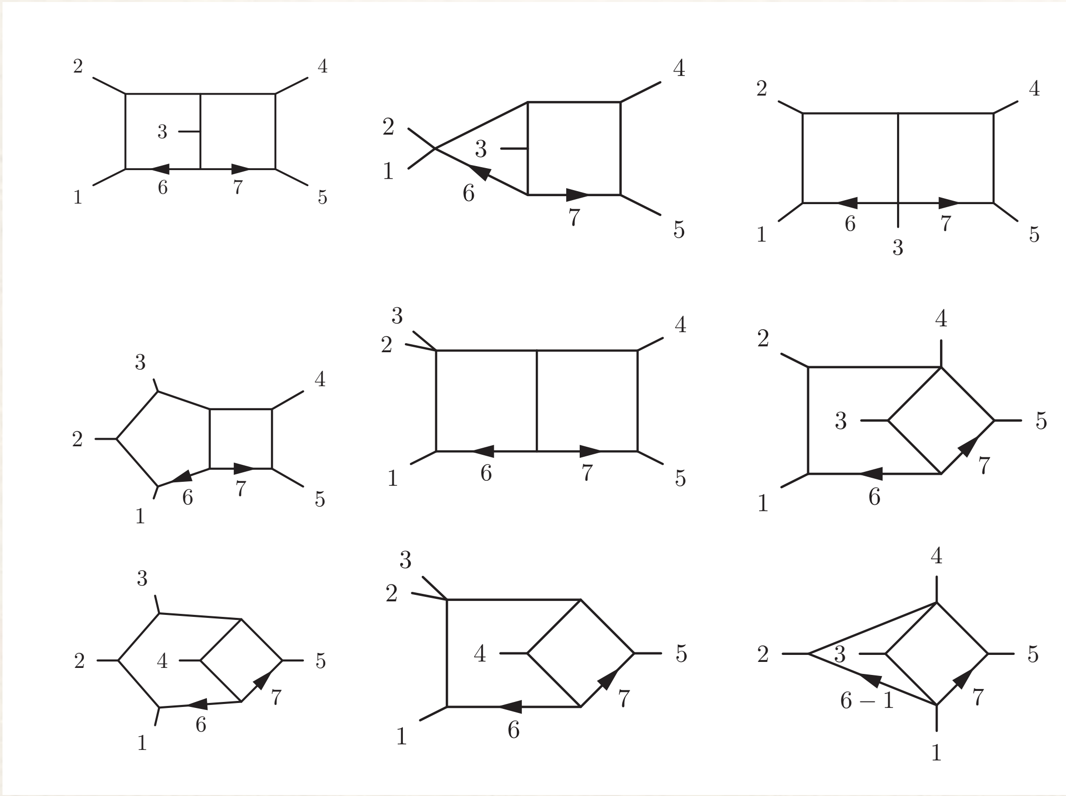
Expand the amplitude:

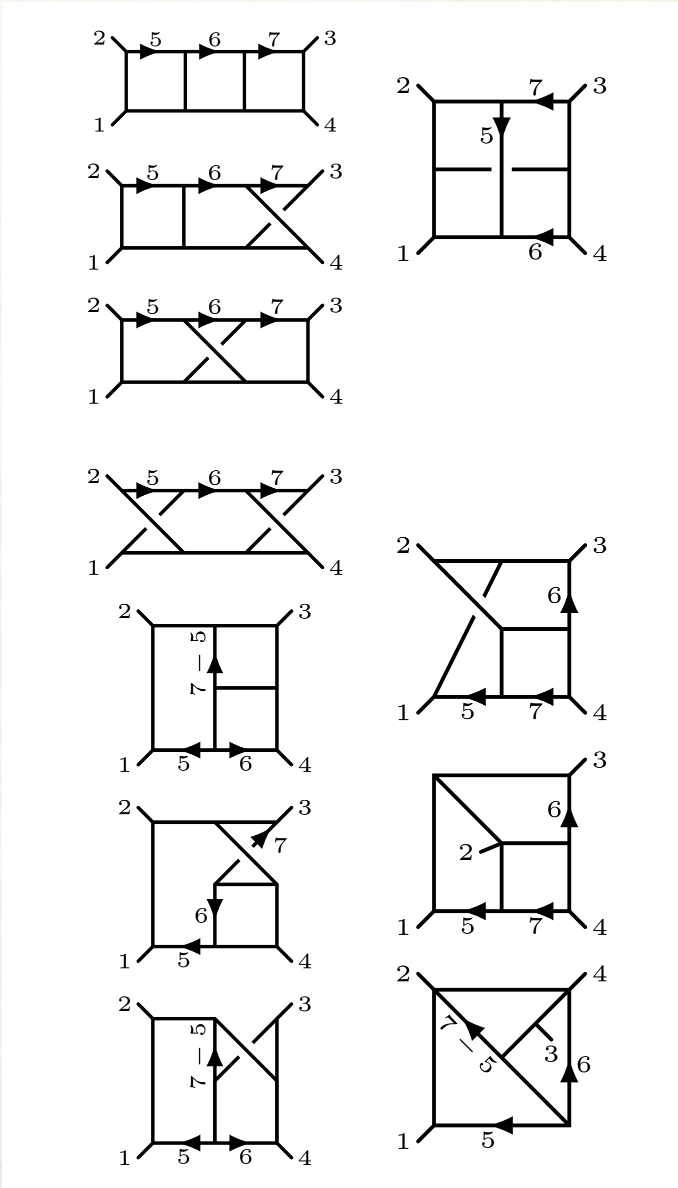
Explicit checks

(Arkani-Hamed, Bourjaily, Cachazo, JT, 2014) (Bern, Herrmann, Litsey, Stankowicz, JT 2014, 2015)

- Construct basis for 4pt at 2-loop and 3-loop, 5pt 2-loop







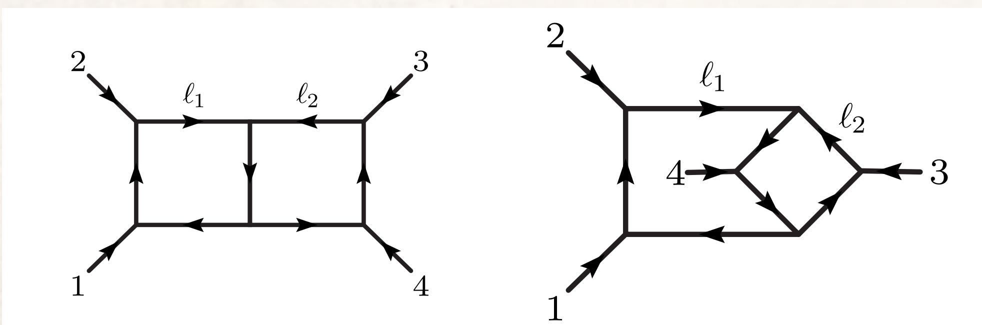
Expand the amplitude: ✓

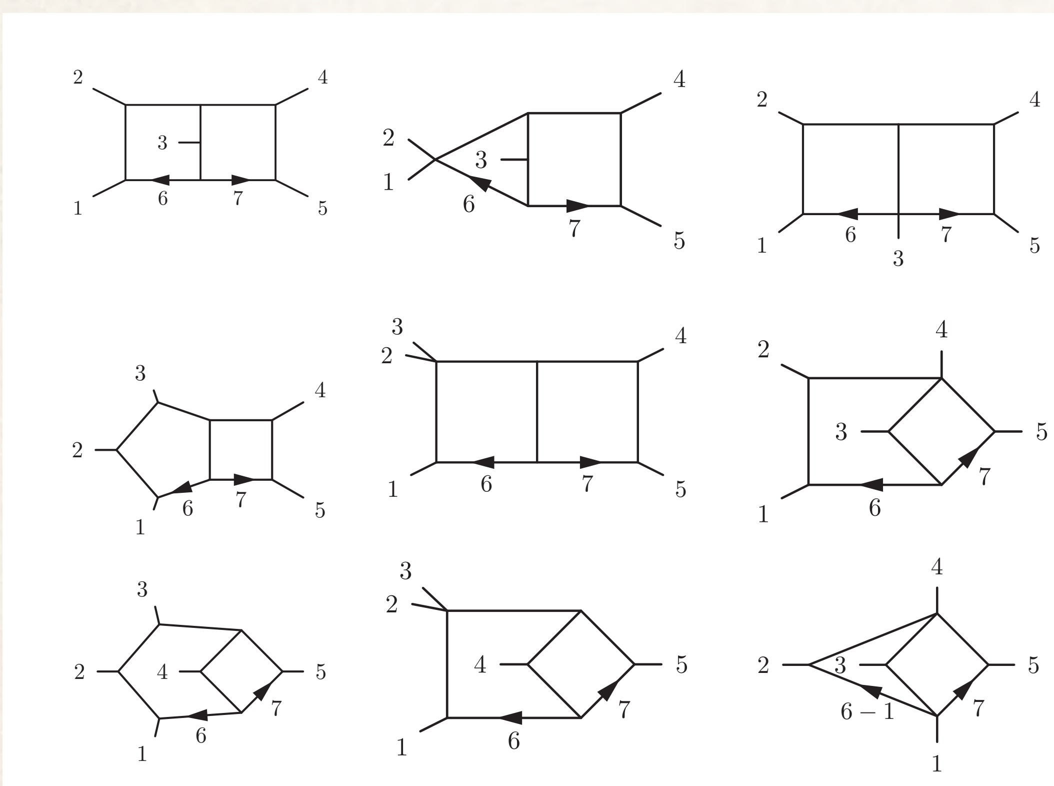
Coefficient fixed by standard unitarity methods

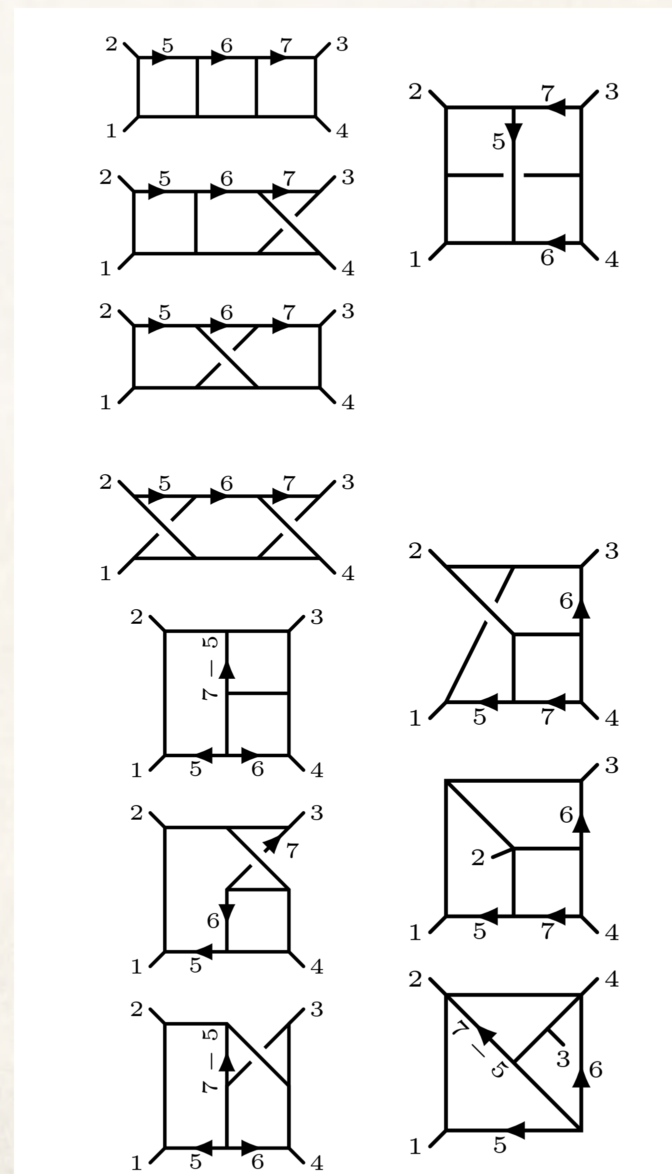
Explicit checks

(Bern, Herrmann, Litsey, Stankowicz, JT, 2015)

- Construct basis for 4pt at 2-loop and 3-loop, 5pt 2-loop







Expand the amplitude:

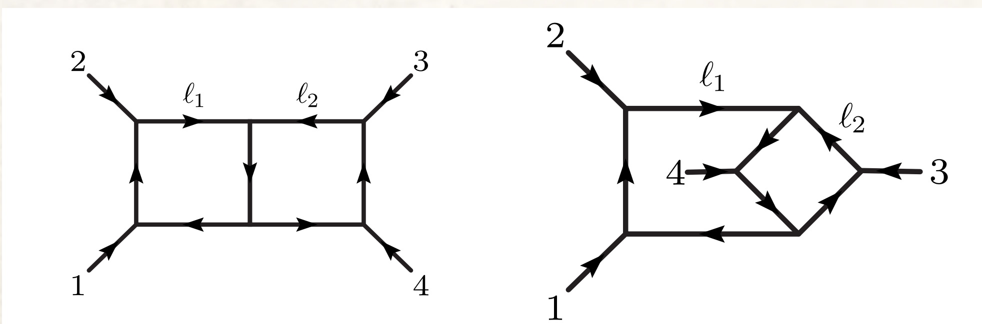
?

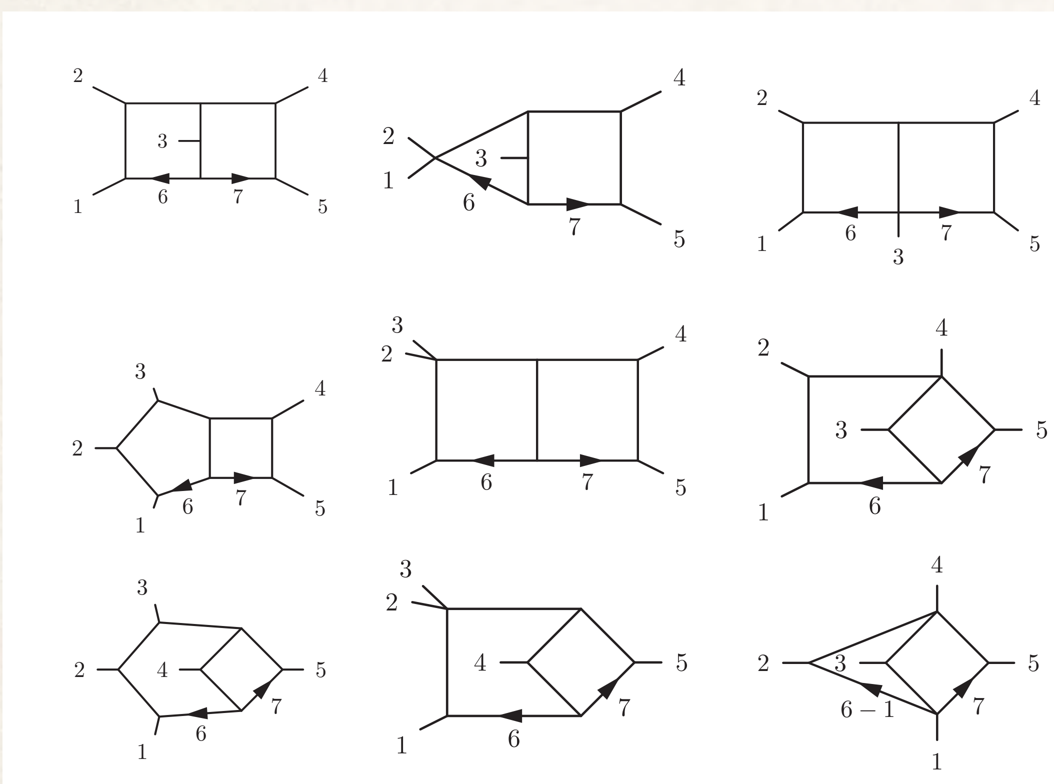
Using only **vanishing** cuts

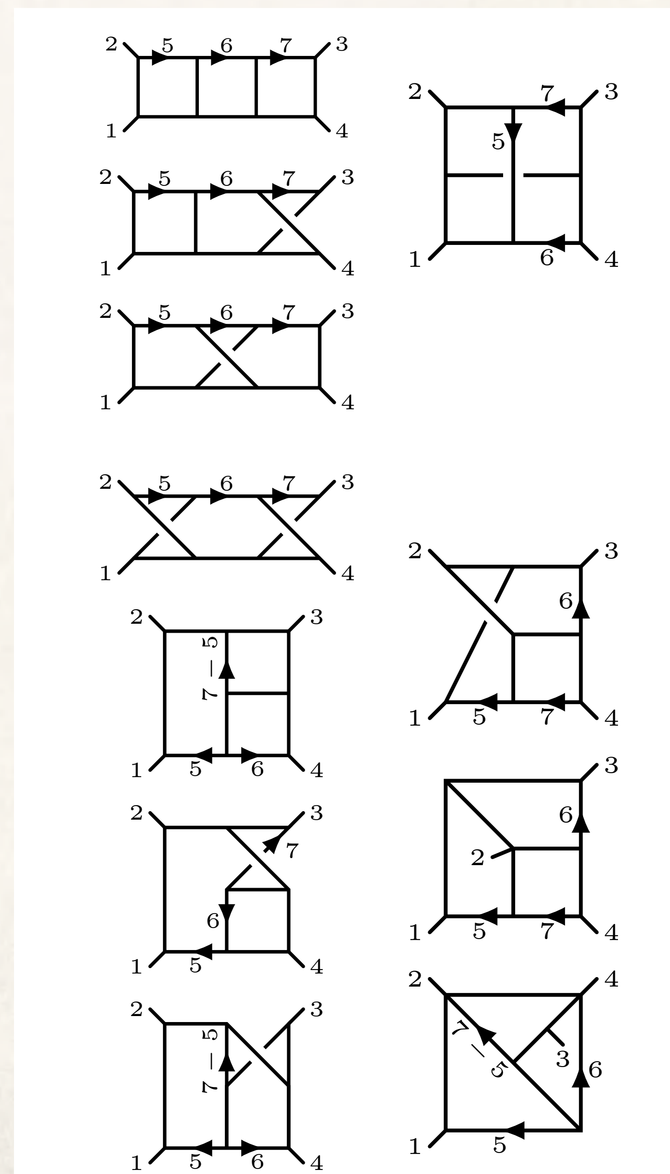
Explicit checks

(Bern, Herrmann, Litsey, Stankowicz, JT, 2015)

- Construct basis for 4pt at 2-loop and 3-loop, 5pt 2-loop







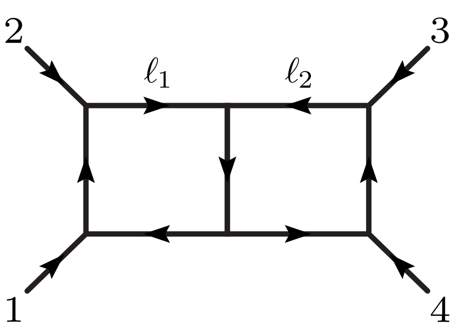
Expand the amplitude: ✓

Using only **vanishing** cuts

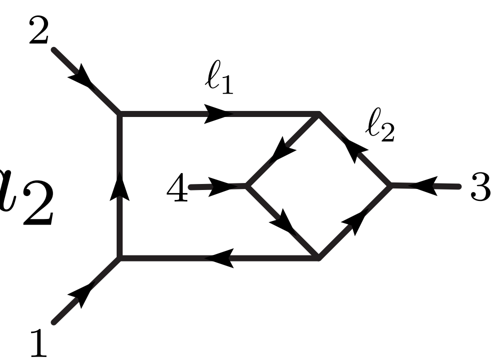
Example of zero condition

- Expansion of the amplitude

$$M_2 = \sum_{\sigma} \quad a_1$$



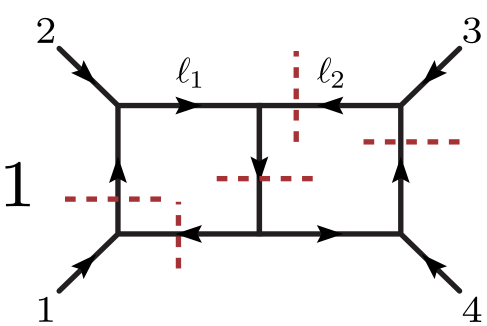
$$+ \quad a_2$$



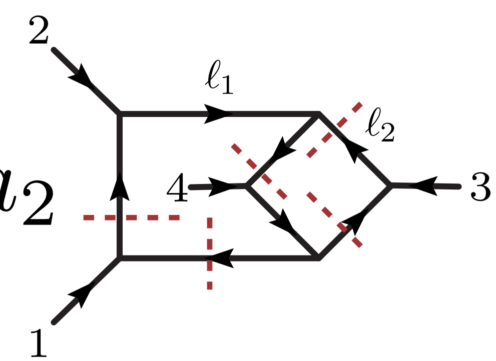
Example of zero condition

- Expansion of the amplitude

$$\text{Cut } M_2 = \sum_{\sigma} \quad a_1$$

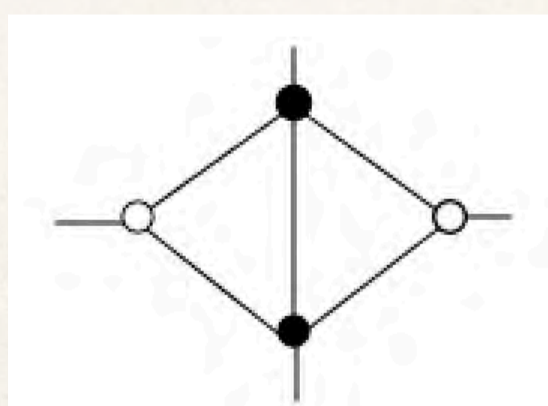


$$+ \quad a_2$$



$$= 0$$

Illegal 5-cut



$k = 1$

Fixes relative coefficient

$$a_1 = a_2$$

Non-planar conclusion

- Same properties: planar and non-planar
- In planar: hidden symmetries + Amplituhedron
- Non-planar: see implications of something new
    - Do not follow from known symmetries
    - Problem of labels: formulate as symmetry / geometry

Non-planar conclusion

- The complete N=4 SYM is the simplest QFT!
- Open questions: new symmetries, role of color factor, complete geometric formulation…..
- Relation to final amplitudes: uniform transcendentality, structure in cross-ratios, etc.

Comments on N=8 amplitudes

New hope for gravity

- One step forward: N=8 amplitudes
- Reasons to hope there is a new approach:
    - BCJ between N=4 and N=8
    - Magic properties of tree-level amplitudes
    - Possible special behavior in UV
- No planar sector of gravity, no natural conjectures

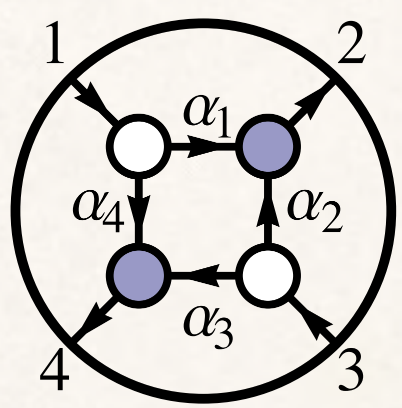
Gravity on-shell diagrams

(Herrmann, JT 2016) (Heslop, Lipstein 2016)

- Natural start: gravity on-shell diagrams
- Well-defined in any QFT: cuts of the loop amplitudes
- Each diagram can be written in Grassmannian
- Question: Is there a universal form (like dlog form)?
- What do we learn about loop amplitudes?

Grassmannian formula

- Edge variables for each edge



$$C = \begin{pmatrix} 1 & \alpha_1 & 0 & -\alpha_4 \\ 0 & \alpha_2 & 1 & \alpha_3 \end{pmatrix}$$

- The value of the diagram in Yang-Mills

$$\boxed{\Omega = \frac{d\alpha_1}{\alpha_1}\frac{d\alpha_2}{\alpha_2}\dots\frac{d\alpha_m}{\alpha_m}\cdot\delta(C\cdot Z)}$$

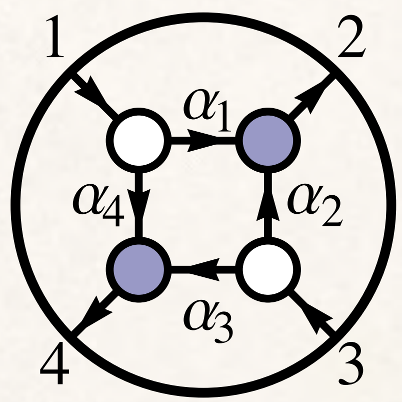
$$\delta(C\cdot Z) = \delta(C\cdot\widetilde{\lambda})\delta(C^\perp\cdot\lambda)\delta(C\cdot\widetilde{\eta})$$

Solves for α_i
in terms of $\lambda_i, \widetilde{\lambda}_i$
and gives $\delta(P)\delta(Q)$

Grassmannian formula

(Herrmann, JT 2016) similar representation (Heslop, Lipstein 2016)

- Edge variables for each edge



$$C = \begin{pmatrix} 1 & \alpha_1 & 0 & -\alpha_4 \\ 0 & \alpha_2 & 1 & \alpha_3 \end{pmatrix}$$

- The value of the diagram in gravity

$$\boxed{\Omega = \frac{d\alpha_1}{\alpha_1^3}\frac{d\alpha_2}{\alpha_2^3}\dots\frac{d\alpha_m}{\alpha_m^3}\prod_v \Delta_v \cdot \delta(C\cdot Z)}$$

Special numerator: factor in each vertex

$$\delta(C\cdot Z) = \delta(C\cdot\widetilde{\lambda})\delta(C^\perp\cdot\lambda)\delta(C\cdot\widetilde{\eta})$$

Properties of the formula

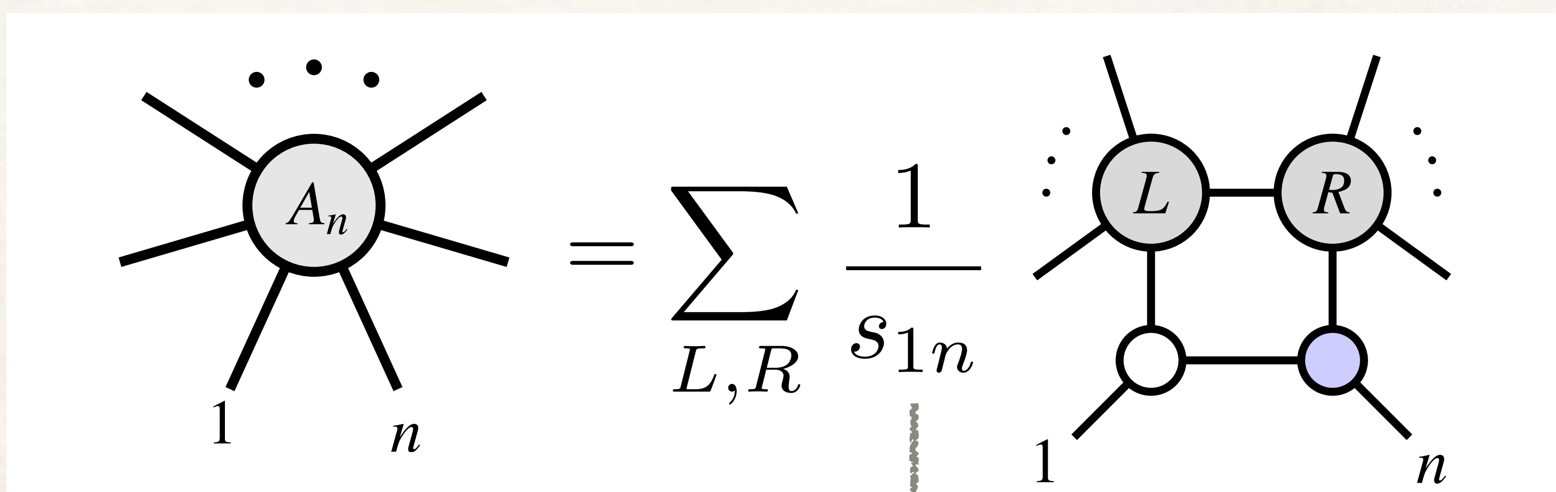
(Herrmann, JT 2016)

- Analyzing: $$\Omega = \frac{d\alpha_1}{\alpha_1^3}\frac{d\alpha_2}{\alpha_2^3}\dots\frac{d\alpha_m}{\alpha_m^3}\prod_v \Delta_v \cdot \delta(C\cdot Z)$$
  - Higher poles present
  - Reduces to single poles if erasing edge (pole for finite momentum)
  - Diagram vanishes if in any vertex three momenta are collinear
- Similar formula for N<8 SUGRA

Tree-level recursion relations

(Heslop, Lipstein 2016)

- Gravity on-shell diagrams in the context of BCFW recursion relations
- Gravity tree amplitudes not just sum of on-shell diagrams



Extra kinematical factors

see Arthur's talk

Conjectures for the loop amplitude

- No recursion for amplitude using on-shell diagrams
  - Absence of variables, generic non-planar problem
  - Dimensionality: extra kinematical factors
- Implications for amplitude: optimistic conjectures
  - Collinearity conditions
  - Finite poles
  - Poles at infinity

$\left.\begin{array}{l}\text{Collinearity conditions}\\ \text{Finite poles}\end{array}\right\} \longrightarrow \text{IR}$

$\text{Poles at infinity} \longrightarrow \text{UV}$

Finite poles

- On-shell diagrams: all finite cuts logarithmic

$$\text{Integral} \to \text{Cut} \to \text{Cut} \to \frac{d\alpha}{\alpha^2}$$

never happens

- Strong hint from BCJ relations

$$A^{(YM)} = \sum_i \frac{n_i^{(BCJ)} c_i}{s_i} = \sum_i \frac{n_i^{(dlog)} c_i}{s_i}$$

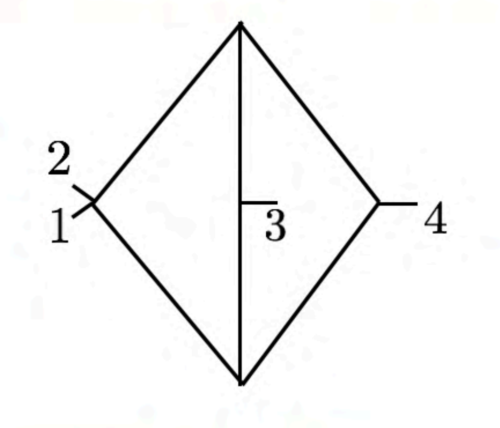
Double poles canceled by Yang-Mills numerator

$$\downarrow$$

$$A^{(GR)} = \sum_i \frac{n_i^{(dlog)} n_i^{(BCJ)}}{s_i}$$

Finite poles

- Almost certainly correct statement
- Interesting implications: some diagrams absent



Double poles in the cut structure

This diagram and his higher loop friends: divergent in

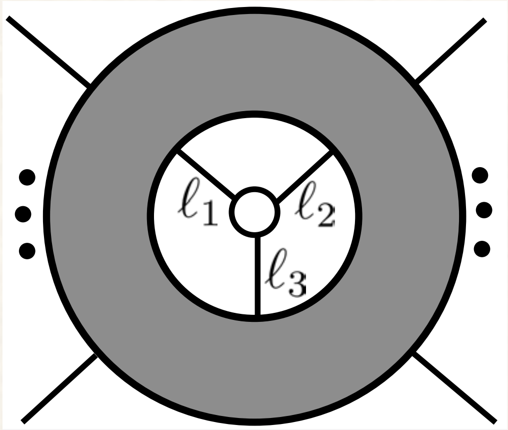
$$D = 4 + \frac{4}{L}$$

vs

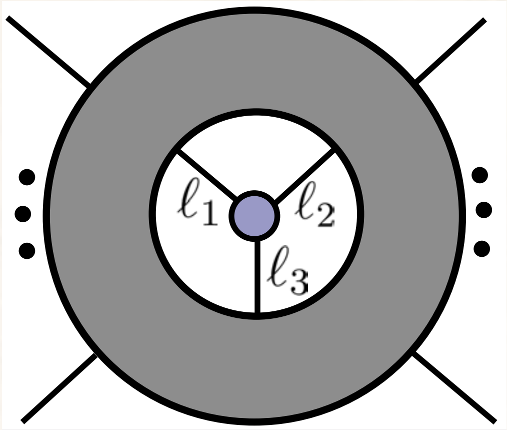
$$D = 4 + \frac{6}{L}$$

Collinearity conditions

- On-shell diagrams: any cut of the form



$\sim [\ell_1\,\ell_2]$



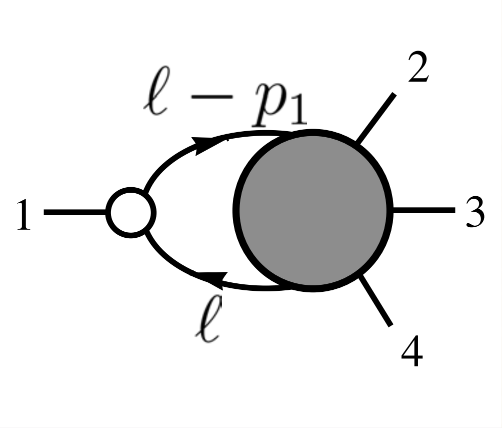
$\sim \langle \ell_1\,\ell_2 \rangle$

- In special case of external legs: collinear limit

$$A \sim \frac{[12]}{\langle 12 \rangle} \cdot (\dots)$$

Collinearity conditions

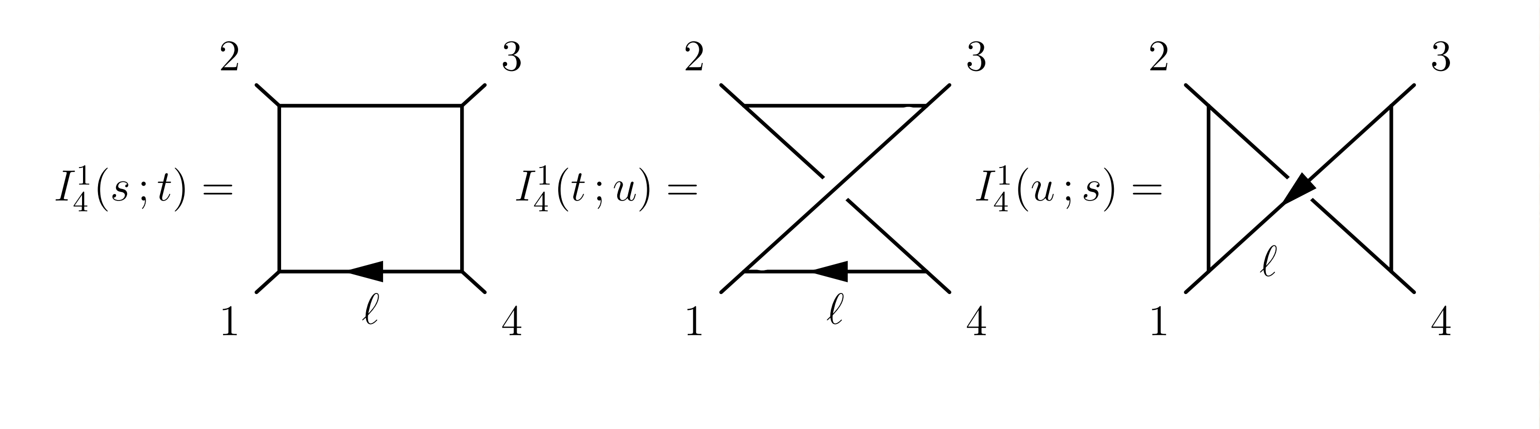
- Double cut of the amplitude:

$\ell = \lambda_1 \tilde{\lambda}_\ell$ 

$\sim [\ell\, 1]$

Can not be implemented term-by-term in the amplitude

- Expansion of the 4pt 1-loop amplitude: three boxes

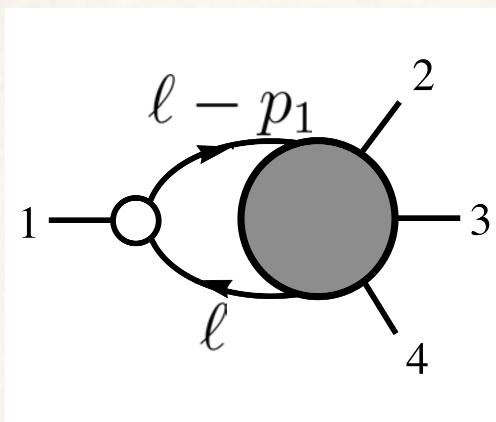


Each scales $\sim \dfrac{1}{[\ell\, 1]}$

Collinearity conditions

- Double cut of the amplitude:

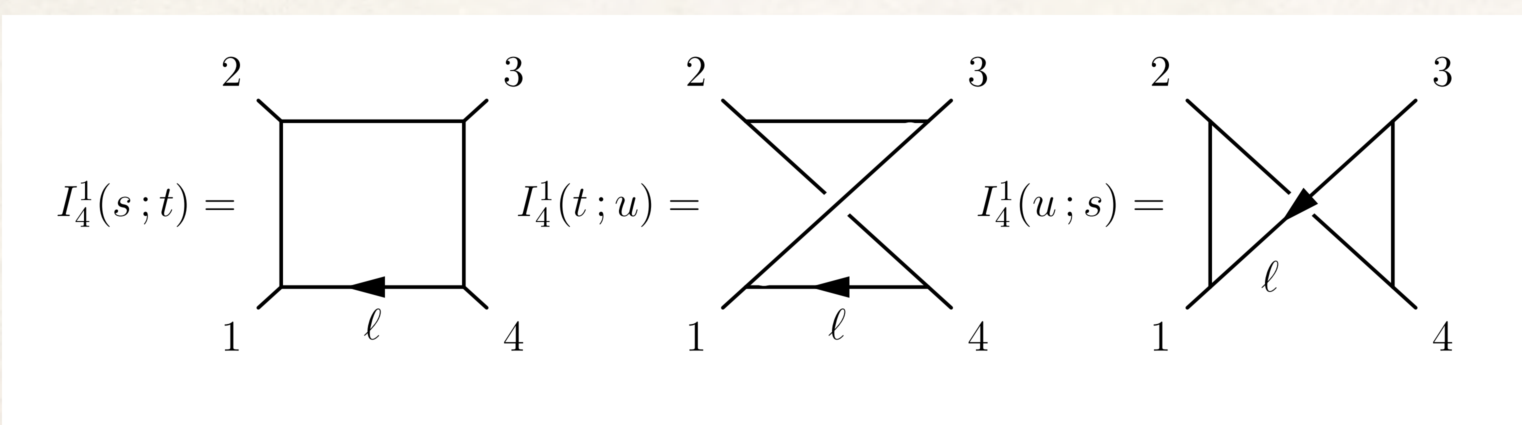
$\ell = \lambda_1 \widetilde{\lambda}_\ell$



$\sim [\ell\, 1]$

Very non-trivial statement even for 4pt 1-loop

- Expansion of the 4pt 1-loop amplitude: three boxes

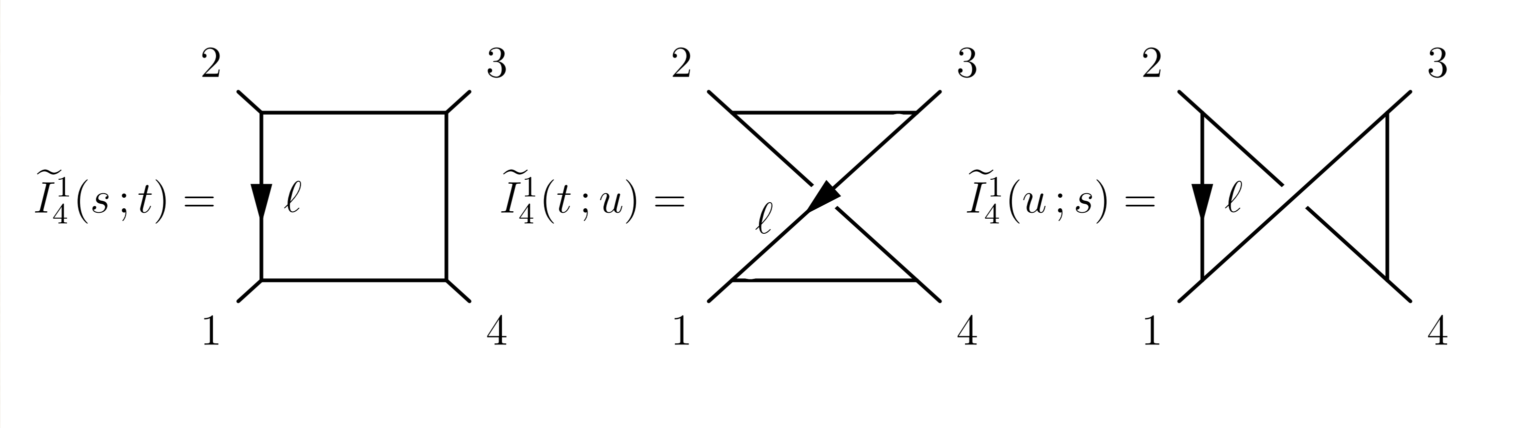


Sum

~ 1

Collinearity conditions

- We made a choice in labeling diagrams
- Consider another labels, and sum over both options



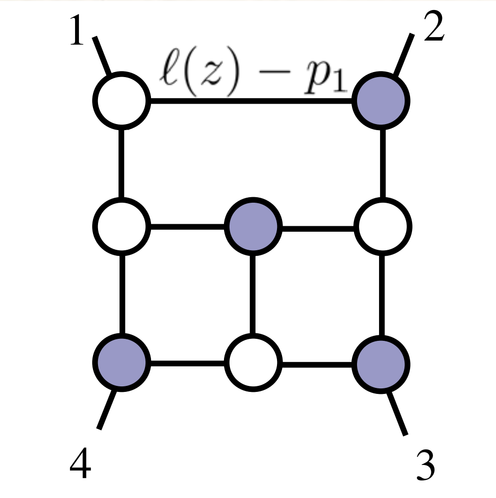
Total sum

$\sim [\ell\, 1]$

- Two loop checks even more impressive, symmetrization!
- Close connection to IR singularities of gravity

Poles at infinity

- They are present starting at 3-loops



$$\sim \frac{dz}{z}$$

Pole at

$$z \to \infty$$
$$\ell(z) \to \infty$$

- Higher poles at higher loops

- Generically everything complex, the detailed description of the space of poles at infinity needed

Gravituhedron?

- Before dreaming about the geometric formulation for N=8 SUGRA: describe in details singularity structure
- In N=4 SYM: logarithmic singularities
- In N=8 SUGRA: more complicated
  - Logarithmic singularities at finite momenta
  - Multiple poles at infinity
- Precise description of poles at infinity needed

Conclusion

Conclusion

- Amplitudes in the complete N=4 SYM
  - Logarithmic singularities
  - No poles at infinity
  - Fixed by vanishing cuts

Evidence for non-planar geometric construction

- Amplitudes in N=8 SUGRA
  - Grassmannian formula for on-shell diagrams
  - Simple IR properties, complicated structure of poles at infinity

Thank you for your attention