Galaxy clustering and spatial point processes*

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1 Introduction

An interpretation of the galaxy distribution in terms of spatial statistics has been advanced by Neyman & Scott (1958). Nowadays both analytical predictions and numerical simulations of large–scale structure are based on stochastic initial conditions (usually Gaussian random fields). So one is directly led to a statistical interpretation. Statistical methods serve as discriminatory tools between different cosmological models. However, since we live in one specific "realization", we are not restricted to this statistical interpretation, and we may gain direct physical insight, by inspecting the morphology of the galaxy distribution itself.

During the past two decades enormous progress has been made in the mapping of the distribution of galaxies in the Universe (see e.g. Coles et al. 1996). Using the redshift as a distance indicator, and knowing the angular position on the sky, we obtain a three–dimensional view of the luminous matter distribution in the Universe. The presently available redshift surveys of galaxies cover wide solid angles and have reasonable depth to permit the study of the statistical properties of the distribution of galaxies considered as a point process (Martínez 1997).

2 Second–order characteristics

Historically, the most influential analysis of the galaxy distribution were done using second–order characteristics, namely with the two–point correlation function $\xi(r)$ (related with the second order intensity function $\lambda_2(r)$ for a homogeneous and isotropic process by $\xi(r) + 1 = \lambda_2(r)/n^2$, with n being the number density), the variance of count in cells, and later with the power–spectrum and the correlation integral (directly related to Ripley's K–function). In Fig. 1 we show K and $1 + \xi$ for an observed galaxy distribution. The current results suggest that the two–point correlation function $1 + \xi$ and the correlation integral show scale invariant behaviour. The extent of this regime of "scale invariant clustering", and whether it stems from geometrical properties of the point distribution is still a controversial issue (e.g. Martínez et al. 1990, Pietronero et al. 1996, McCauley 1997, Martínez 1997). It is still not full agreement in the appropriate estimators that have to be used to obtain ξ from the redshift surveys (Martínez et al. 1997). Different procedures to correct for the effects of the edges in the estimates of ξ or the correlation integral for finite volumes, together with different weighting schemes serve to complicate the previous issue further.

Although second–order measures give important information on the spatial distribution of galaxies, they are not able to distinguish geometrical patterns like filaments, walls and clusters (Schmalzing et al. 1996). Moreover there exist several, quite natural examples of point patterns, fundamentally different by construction and by visual inspection, showing the same behaviour in second–order quantities (Baddeley & Silverman 1984 and references therein).

Other measures of clustering have been used in Cosmology, for example the void probability function $P_0(r)$ (related with the empty space function $F(r) = 1 - P_0(r)$) (Maurogordato & Lachièze–Rey 1987), the

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genus of the isodensity surfaces of the smoothed density field (Melott 1990), Minkowski functionals (Mecke et al. 1994), multifractals (Martínez et al. 1990), skewness and kurtosis (Coles & Frenk 1991), etc.

3 Minkowski functionals

Minkowski functionals are measures for characterizing the shape and the connectivity of suitably defined bodies in d–dimensional Euclidean space. Only in demanding the invariance under translations and rotations, and additivity (together with a continuity requirement), we are left with $d+1$ independent functionals (Hadwiger 1957), giving us a complete characterization of the global morphology. In three dimensions we have four functionals having a direct interpretation in terms of geometrical quantities: the volume, the surface area, the integral mean curvature, and the Euler characteristic (e.g. Schneider 1993).

For the characterization of a point distribution with Minkowski functionals one has to put flesh on the point pattern. We do this by decorating the points with balls of radius r employing the radius as a diagnostic parameter (Mecke et al. 1994). The body under consideration is now the union set of these balls. Such a model, a special case of a Boolean grain model, has been applied to investigate the clustering properties of galaxy and galaxy cluster catalogues (Kerscher et al. 1997b and Kerscher et al. 1997c). Characterizing the observed point set only, no statistical assumptions enter this analysis. In the case of the cluster distribution we compare the observed morphology with the morphology of several realizations of simulated cluster distributions, using the Minkowski functionals as a statistical tool (see also Weil 1983). As a standard of reference we use a Poisson process whose mean values of the functionals are analytically known (Mecke & Wagner 1991). In Figure 2 the Minkowski functionals for a typical galaxy distribution are shown.

The use of Minkowski functionals is not restricted to the analysis of a Boolean grain model but they may be also employed to give a complete morphological characterization of isodensity surfaces of random fields (see Weil 1983 and Schmalzing & Buchert 1997). This method unifies and extends a widely applied statistics in cosmology which measures the genus of isodensity surfaces of smoothed cosmological fields (Melott 1990).

4 Next neighbour distributions

The next neighbour distribution functions F and G are quite common tools in the analysis of spatial point patterns (see e.g. Stoyan & Stoyan 1994 or Ripley 1988), where $G(r)$ is the distribution function of the distance r of a point of the process to the nearest other point of the process, and $F(r)$ is the distribution function of the distance r of an arbitrary point to the nearest point of the process. In cosmology $P_0 = 1-F$ is known as the void probability function (White 1979); similarly F is nothing but the volume density of the first Minkowski functional of the Boolean grain model.

Recently van Lieshout & Baddeley (1997) suggested to use the quotient $J := (1-G)/(1-F)$ as a morphological measure, distinguishing clustering and regular patterns (in a statistical sense). In Kerscher et al. (1997a) we apply J to a Matern process (see e.g. Stoyan & Stoyan 1994) mixed with a Poisson process as a simple model of galaxy clustering. There we show that the J descriptor, applied to galaxy catalogues gives information on the strength of clustering i.e. the fraction of galaxies residing in clusters. In Figure 3 we show these measures for a distribution of galaxies.

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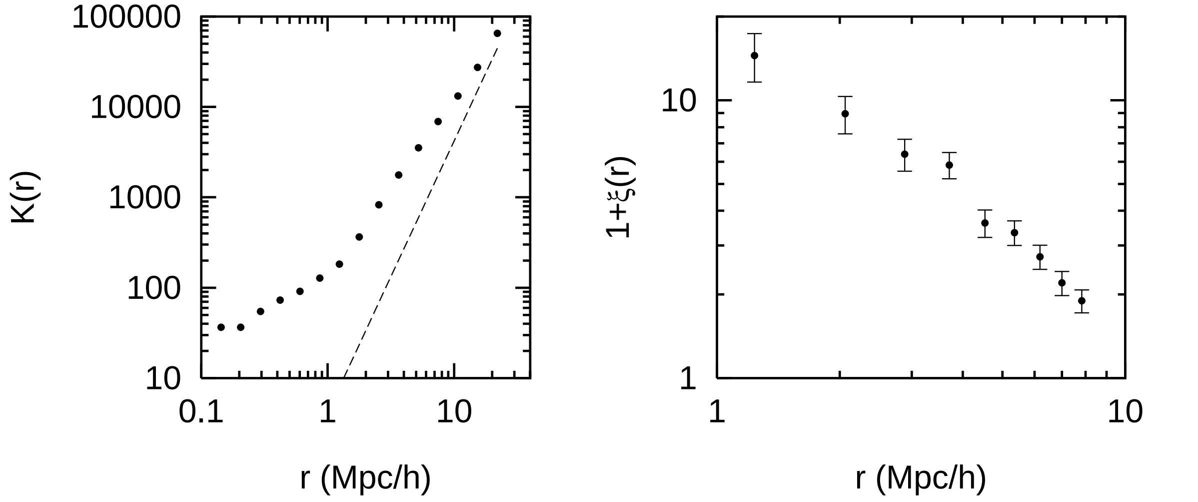

Figure 1: The left plot shows Ripley's K–function (the dashed line for a Poisson distribution) and the right plot the correlation function $1+\xi$ with Poissonian errors for a volume limited sample of the CfA1 galaxy catalogue with $80h^{-1}$Mpc depth.

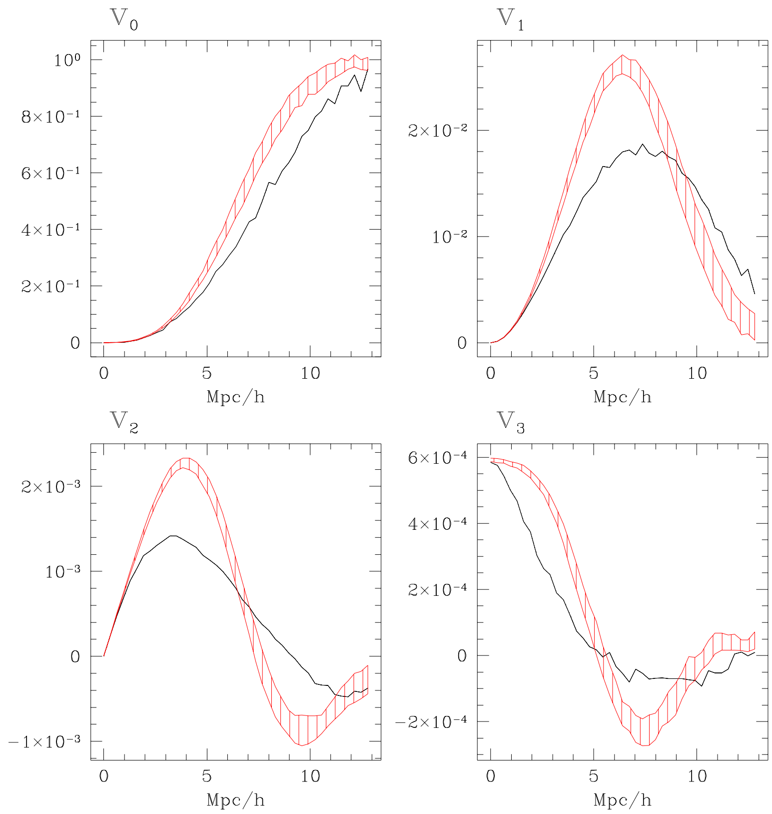

Figure 2: The volume densities of the four Minkowski functionals for a volume limited sample of the CfA1 galaxy catalogue with $80h^{-1}$Mpc depth. The shaded area depicts the 1σ error range of a Poisson distribution with the same number of galaxies, estimated from 15 realizations.

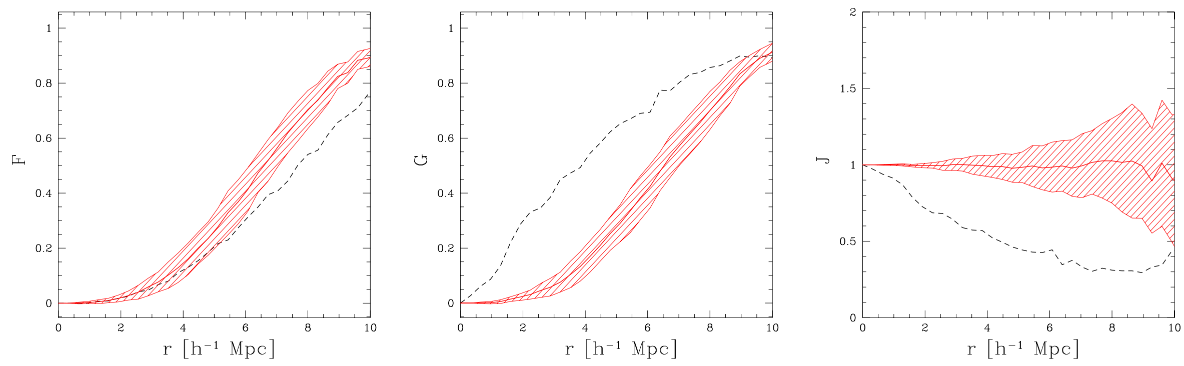

Figure 3: From left to right we show the empty space function F, the next neighbour distribution G, and the J descriptor for a volume limited sample of the CfA1 galaxy catalogue with $80h^{-1}$Mpc depth. The shaded area depicts the 1σ error range of a Poisson distribution with the same number of galaxies, estimated from 15 realizations.

Summary

In this talk we present a brief overview on statistical methods currently used in the analysis of the spatial galaxy distribution. We discuss the need for methods going beyond second–order characteristics and propose measures like Minkowski functionals and next neighbour distributions as advanced tools for the description of the morphology of the galaxy distribution.

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