Backreaction and the evolution of cosmological parameters

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The standard picture of the evolution of the Universe relates typical observables to global scalar cosmological parameters such as the mean matter density and the global expansion rate. These parameters are derived from solutions of Einstein's or Newton's laws of gravitationally interacting *homogeneous and isotropic* systems. The key argument of applicability of the homogeneous–isotropic models is the conjecture that *on average* the Universe may still follow the evolution of these simple models.

For a finite domain $\mathcal{D}$ one defines the scale factor via the volume $V(t) = |\mathcal{D}(t)|$ of the domain: $a_\mathcal{D}(t) = (V(t)/V(t_\mathbf{i}))^{\frac{1}{3}}$. Here, $V(t_\mathbf{i})$ is the volume at initial time. For a homogeneous and isotropic matter distribution the volume evolves as $a_\mathcal{D}^3 = a^3$, where $a(t)$ is a solution of the Friedmann equation. However in the presence of inhomogeneities the domain shows deformations and $a_\mathcal{D} \neq a$. Pictures illustrating both cases are shown in Fig. 1.

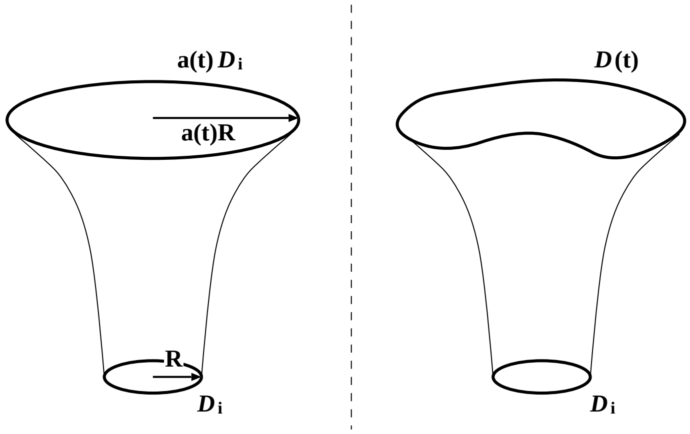

Figure 1: The evolution of an initial domain within the Hubble flow with global scale factor $a(t)$ (left) and for a general inhomogeneous setting (right)

The basic equation for the description of the average evolution of a finite domain in the Newtonian approximation [1] and also in the full relativistic case [2] is the generalized Friedmann equation for the scale factor $a_\mathcal{D}(t)$:

$$3\frac{\ddot{a}_\mathcal{D}}{a_\mathcal{D}} + 4\pi G \left\langle \varrho \right\rangle_\mathcal{D} - \Lambda = Q_\mathcal{D}, \tag{1}$$

with the average mass density $\left\langle \varrho \right\rangle_\mathcal{D} = M/V(t)$, Newton's gravitational constant G, the cosmological constant Λ, and the "backreaction term" $Q_\mathcal{D}$ depending on fluctuations of the velocity field inside the domain. $Q_{\mathcal{D}}$ may be written in terms of kinematical scalars, the expansion rate θ, the rate of shear σ, and the rate of vorticity ω:

$$Q_{\mathcal{D}} = \frac{2}{3}\left(\left\langle\theta^2\right\rangle_{\mathcal{D}} - \langle\theta\rangle^2_{\mathcal{D}}\right) + 2\left\langle\omega^2 - \sigma^2\right\rangle_{\mathcal{D}} . \tag{2}$$

In Eq. (1) there are two sources for the evolution of the scale factor $a_{\mathcal{D}}$, the average density *and* the backreaction term: the evolution of a spatial domain is triggered by an over/under–density *and* velocity fluctuations.

For domains, small compared to the horizon, deviations from the behavior of the scale factor of the standard Friedmann model may be quantified using the Newtonian approximation. We calculate the backreaction term (2) using the the "Zel'dovich approximation" [4], insert it into the generalized expansion law (1), and solve numerically for $a_{\mathcal{D}}(t)$ starting at redshift 200. Initial conditions are chosen according to an Einstein–de–Sitter background. The fluctuations of the initial displacement field are calculated from the standard cold dark matter power spectrum, also depending on the size of the initial domain. Details may be found in [3].

In an initially spherical domain with a radius of $R = 0.5$Mpc (a scaled radius of $a(t)R =$ 100Mpc, compare Fig. 1) significant deviations from the Friedmann evolution are possible even for one–σ fluctuations in the initial conditions. For three–σ the scale factor may show a deviation larger than 10% from the Friedmann evolution. A spherical domain with scaled radius of 100Mpc corresponds to a cube with sidelength 160Mpc, comparable with the size of periodic cubes typically used in cosmological N–body simulations. However, simulations with periodic boundary conditions do not take this effect into account [1].

Knowing the time evolution of $a_{\mathcal{D}}$ for given initial conditions we can calculate the time evolution of the domain dependent density parameters

$$\Omega^{\mathcal{D}}_m(t) = \frac{H^2(t_{\mathbf{i}})\ (1 + \langle\delta_{\mathrm{i}}\rangle_{\mathcal{D}_{\mathbf{i}}})}{\dot{a}_{\mathcal{D}}(t)^2 a_{\mathcal{D}}(t)}, \quad \Omega^{\mathcal{D}}_k(t) = -\frac{k_{\mathcal{D}}}{\dot{a}_{\mathcal{D}}(t)^2}, \quad \Omega^{\mathcal{D}}_Q(t) = \frac{1}{3\ a^2_{\mathcal{D}} H^2_{\mathcal{D}}}\int_{t_{\mathrm{i}}}^{t} \mathrm{d}s\ Q_{\mathcal{D}}\ 2\dot{a}_{\mathcal{D}} a_{\mathcal{D}}, \tag{3}$$

with $k_{\mathcal{D}} = (\Omega^{\mathcal{D}}_m(t_{\mathbf{i}}) - 1)\dot{a}_{\mathcal{D}}(t_{\mathbf{i}})^2$, and the average initial density contrast $\langle\delta_{\mathrm{i}}\rangle_{\mathcal{D}_{\mathbf{i}}}$. In Fig. 3 the time evolution of these $\Omega^{\mathcal{D}}$ parameters for the three–σ initial conditions of a domain with scaled radius of 100Mpc are shown (compare with Fig. 2). In the over–dense case the domain dependent matter density parameter $\Omega^{\mathcal{D}}_m$ may be overestimated by more than 100%. In this case $\Omega^{\mathcal{D}}_k$ and $\Omega^{\mathcal{D}}_Q$ give a significant contribution, although we started with small $\Omega^{\mathcal{D}}_k$ and zero $\Omega^{\mathcal{D}}_Q$. For an under–dense domain we underestimate the matter density by nearly 40%. Here the $\Omega^{\mathcal{D}}_Q$ is small throughout the whole evolution, but it triggers the strong evolution of both $\Omega^{\mathcal{D}}_m$ and $\Omega^{\mathcal{D}}_k$.

References

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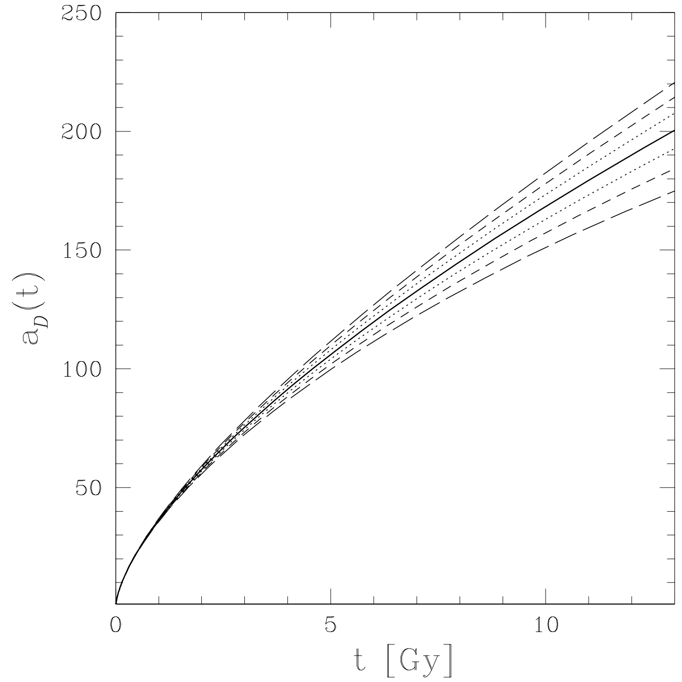

Figure 2: The evolution of the scale factor $a_{\mathcal{D}}(t)$ for spherical domains with a scaled radius of 100Mpc. We considered one–σ (dotted line), two–σ (short dashed line), and three–σ (long dashed line) fluctuations in the initial conditions. The homogeneous background solution is given by the solid line.

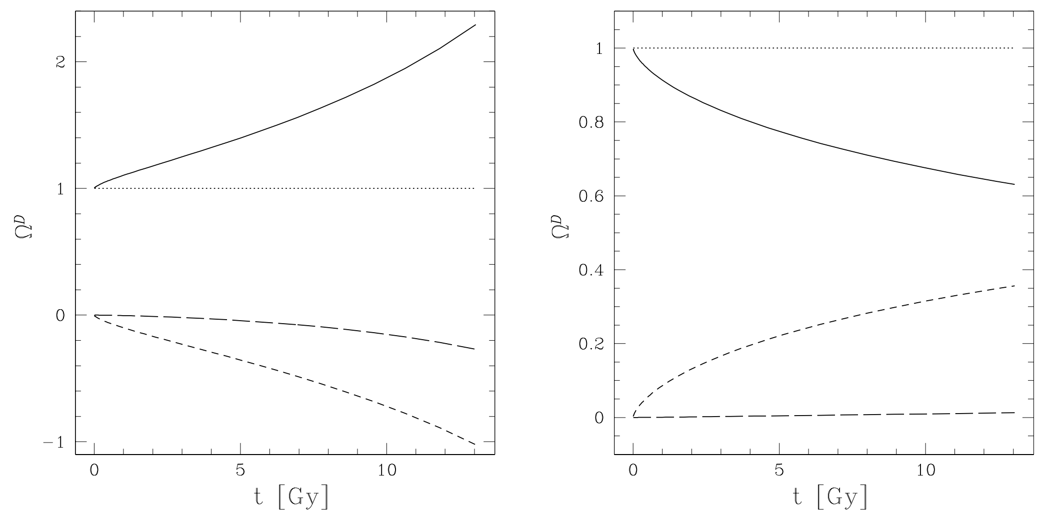

Figure 3: In the right plot the evolution of the "cosmological parameters" $\Omega_m^{\mathcal{D}}$ (solid line), $\Omega_k^{\mathcal{D}}$ (short dashed), and $\Omega_Q^{\mathcal{D}}$ (long dashed) is shown for an over–dense domain with initial radius of 0.5Mpc (a scaled radius of 100Mpc). The dotted line is marking $\Omega_m^{\mathcal{D}} + \Omega_k^{\mathcal{D}} + \Omega_Q^{\mathcal{D}}$. The evolution in an under–dense domain is shown in the right plot.