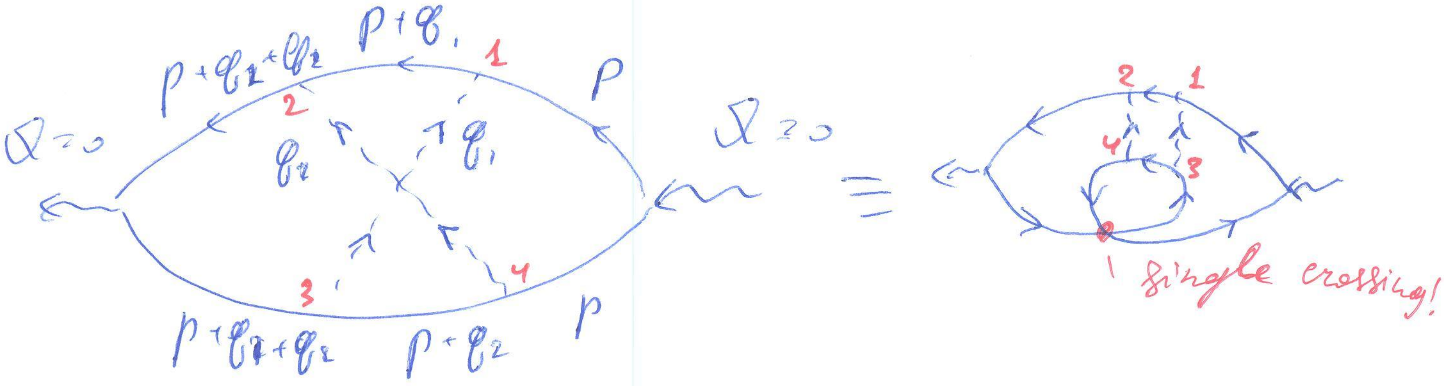
An example of max-crossed diagram (fan diagrams)



$q_{1,2}$ - two independent momentums (to be integrated over)

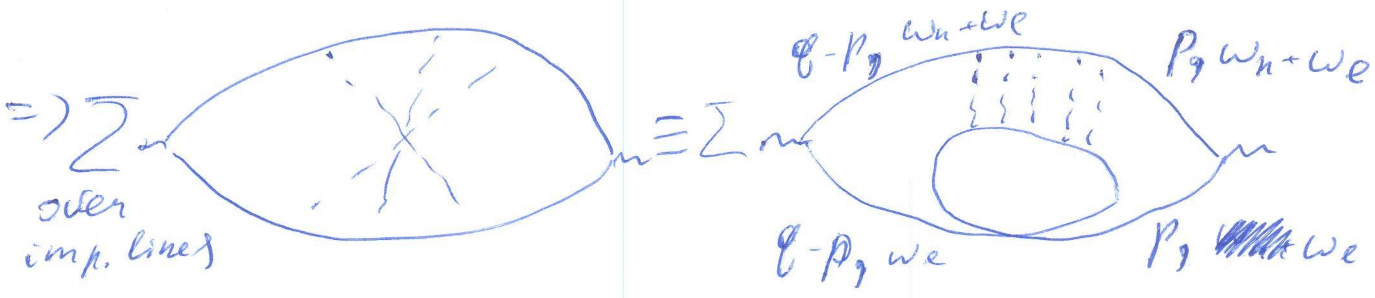
Energy of all upper/lower Green's is $i(\omega_n+\omega_e)$ and $i\omega_e$, respectively

Convenient variables

$$\vec{p}+\vec{q}_1+\vec{q}_2 = -\vec{p}+\underbrace{[2\vec{p}+\vec{q}_1+\vec{q}_2]}_{q}$$

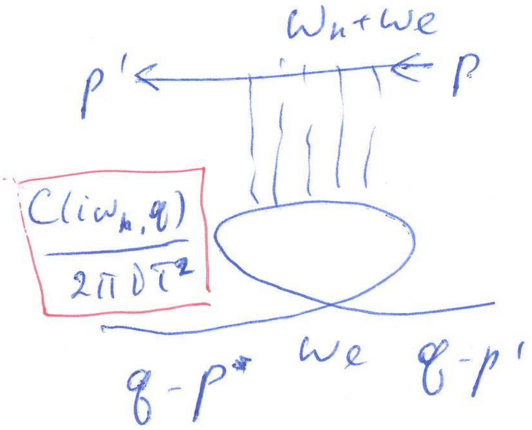
$$\vec{p}+\vec{q}_1 = \vec{p}\,'$$

$$\vec{p}+\vec{q}_2 = 2\vec{p}+\vec{q}_1+\vec{q}_2-\vec{p}-\vec{q}_1 = \vec{q}-\vec{p}\,'$$



Where new ladder diagram can be introduced

Cooperon



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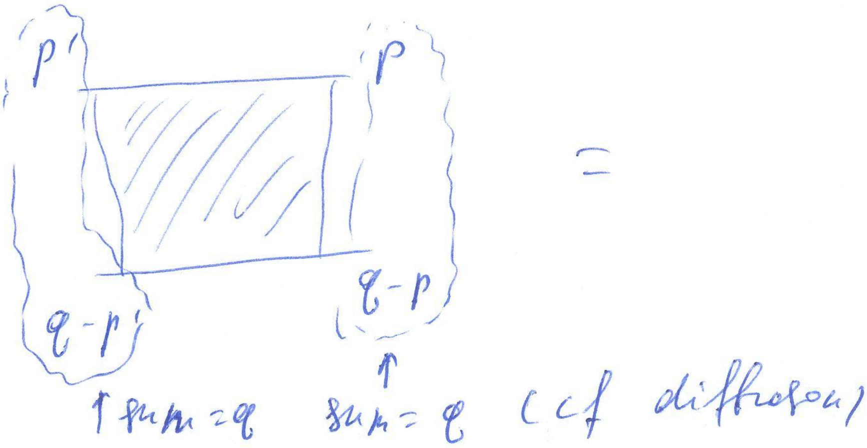

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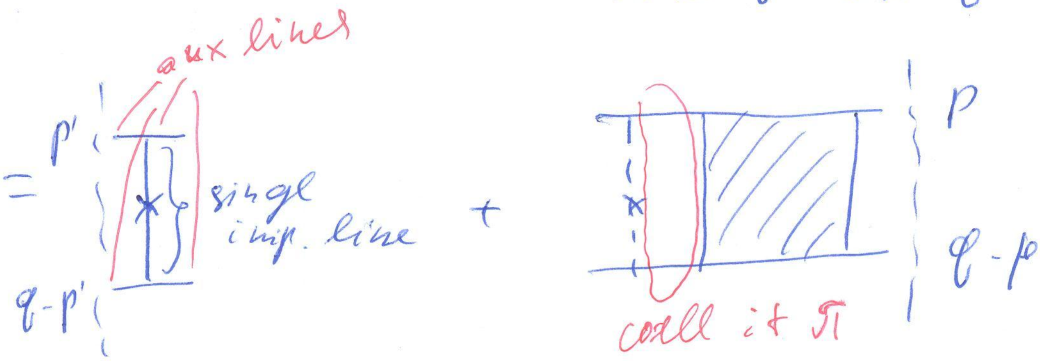


$\Rightarrow$ equation for C:

$$\frac{C(i\omega_n,q)}{2\pi\nu\tau^2} = \frac{1}{2\pi\nu\tau}\left[1+\Pi\frac{C}{2\pi\nu\tau^2}\right]$$

$$\Pi\big|_{\omega_n>0} = 2\pi\nu\tau\,\theta_1(\omega_e<0)\,\theta_2(\omega_e+\omega_n>0)\left(1-\tau[\omega_n+Dq^2]\right)$$

Solving eq. for C (omitting θs)

$$C(i\omega_n,q) = \tau + \frac{\Pi}{2\pi\nu\tau}C(i\omega_n,q)$$

$$C(i\omega_n,q) = \frac{\tau}{1-\Pi/2\pi\nu\tau} = \frac{\tau}{\omega_n\tau+Dq^2\tau}$$

$$\boxed{C(i\omega_n,q) = \frac{1}{\omega_n+Dq^2}}$$

q - sum of two momentums

$$\sigma^{\alpha\beta} = -\frac{2}{i\omega}\, \mathcal{G}_{j_\alpha j_\beta} + \text{diamagnetic term}$$

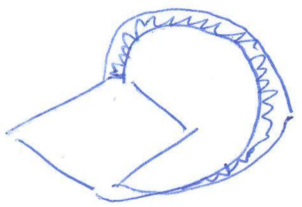
(see DIS 29-4 in notes by JvD)

Correction to $\mathcal{G}_{j_\alpha j_\beta}$ from the Cooperon:

$$\Delta \mathcal{G}_{\alpha\beta} = T \sum_\ell \int \frac{d^3\vec q}{(2\pi)^3} \int \frac{d^3 p}{(2\pi)^3}\, e^2 \underbrace{\left(\frac{2\vec p}{2m}\right)_\alpha}_{\text{right. vert.}} \underbrace{\left(\frac{\vec q - p}{m}\right)_\beta}_{\text{left vert}}$$

$$G(i[\omega_n+\omega_\ell], q-p)\; G(i[\omega_n+\omega_\ell], p) \times$$

$$\times\, G(i[\omega_n\ \cancel{\ldots}], p)\; G(i\omega_\ell, q-p)\; \frac{C(i\omega_n, q)}{2\pi\nu\tau^2}$$



(cf. p. 1)

Note that ω_n - small external frequency

$C(i\omega_n, q)$ diverges at $\omega_n, q \to 0$ – this is the main contribution. Let's expand in small q

// scale separation $q \ll p \sim p_F$ //

$\int d\varphi = 2\pi$ $\quad\int \cos\theta \cos\theta \, d\cos\theta = \frac{1}{3}$

$$\Rightarrow \Delta g_{\alpha\beta} \simeq T\left(\frac{e}{m}\right)^2 \sum_\ell \underbrace{\int \frac{d\Omega_p}{4\pi}\, \vec p_\alpha (\vec q - \vec p)_\beta}_{\simeq -p_F^2\, \delta_{\alpha\beta} \cdot \frac{2}{2\cdot 3}} \;\underbrace{\int \frac{d^3 q}{(2\pi)^3} \frac{C(i\omega_n, q)}{2\pi\nu\tau^2}}_{}$$

$\Rightarrow -\frac{e^2 n}{m\nu}\delta_{\alpha\beta}$ (check $\frac{4}{4}$)

$$\underbrace{\nu \int d\xi_p \frac{1}{\left[i[\underbrace{\omega_n+\omega_\ell}_{\omega_n > \omega_n+\omega_\ell > 0}] - \xi_p + i/2\tau\right]^2 \left[i\omega_\ell - \xi_p - i/2\tau\right]^2}}_{\text{Pole integral}}$$

$\omega_\ell < 0$

$$+2\pi i \frac{-2i\tau^3}{(1+\omega_n\tau)^3} = 4\pi \frac{\tau^3}{(1+\cancel{\omega_n\tau})^3}$$

small

Matsubara sum

$$T \sum_{l} \theta(\omega_l < 0)\, \theta(\omega_l + \omega_n > 0) = nT = \frac{\omega_n}{2\pi}$$

Inserting these results into eq. for $\Delta\sigma$:
and doing anal. cont.

$$\Delta\sigma^{\alpha\beta} = \cancel{-}\frac{2}{\cancel{i\omega}}\left[-\frac{e^2 n}{m v}\delta_{\alpha\beta}\right] v\, \cancel{4\pi\tau^3}^{\,2\tau}\, \frac{\cancel{(-i\omega)}}{2\pi}\, \frac{1}{\cancel{2\pi v \tau^2}}$$

$$\times \int \frac{d^3 q}{(2\pi)^3} C^R(\omega, q)$$

$$\boxed{\Delta\sigma^{\alpha\beta} = -\underbrace{\frac{2 e^2 n \tau}{m}\delta_{\alpha\beta}}_{\sigma_{\alpha\beta}^{\text{Drude}}} \frac{1}{\pi v} \int \frac{d^3 q}{(2\pi)^3} C^R(\omega, q)}$$

3d

$$n = (2)\frac{2}{3}\frac{p_F^3}{(2\pi)^2\hbar^3}\,;\quad v = \frac{m}{\pi\hbar^2}\frac{\sqrt{2m\varepsilon_F}}{\pi\hbar} = \frac{p_F m}{\pi^2\hbar^3}$$

$$n/v = \frac{4}{3}\frac{p_F^3}{(2\pi)^2}\frac{\pi^2}{p_F m} = \frac{4}{3}\frac{1}{4}\frac{p_F^2}{m} = \frac{1}{3}\frac{p_F^2}{m}$$