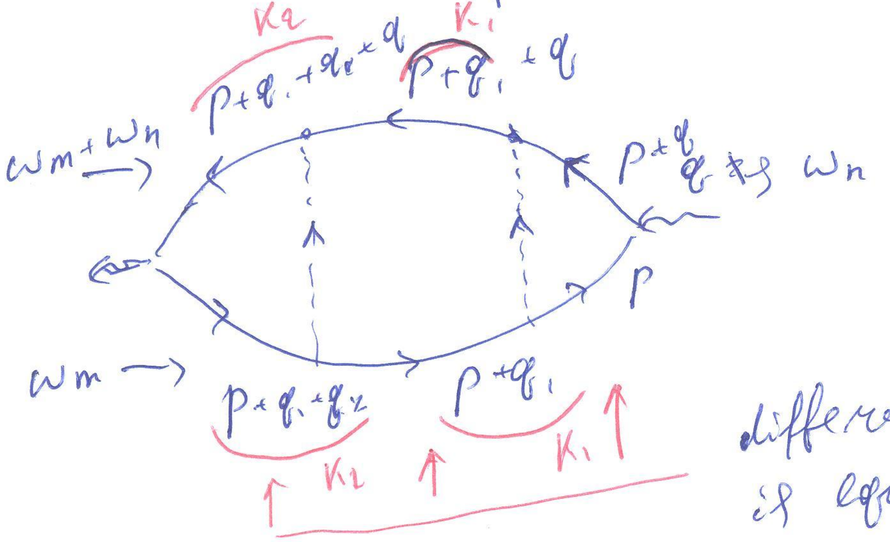
Disorder averaged bubble

$$K = \langle \chi \rangle = T \sum_{\omega_m} \int \frac{d^3k}{(2\pi)^3} \langle G(i\omega_m, k)\, G(i(\omega_m+\omega_n), k) \rangle$$

(ω_m)	$(\omega_m+\omega_n)$	
>0	>0	$\langle G^R G^R \rangle$
<0	<0	$\langle G^A G^A \rangle$
>0	<0	$\langle G^R G^A \rangle$
<0	>0	$\langle G^A G^R \rangle$



difference of momentums is equal to the external q

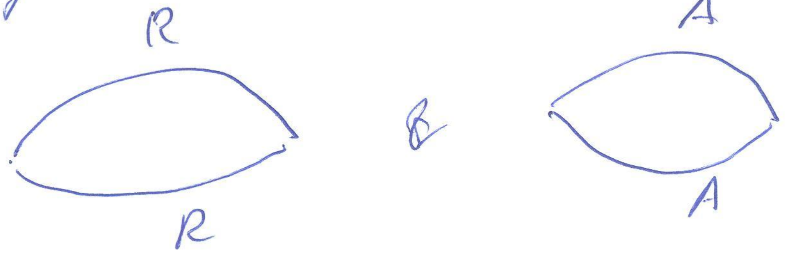
Limits for ω_m, assume $\omega_n > 0$

- RR: $\omega_m > 0$ – just all positive frequencies
- AR: $-\omega_n < \omega_m < 0$ – small window

slow convergence, the pole theorem (over q) cannot be applied, large q are important

ω_m - sum is limited, the pole theorem (over q) can be used and the answer is governed by small q (small vicinity of the F-surface).

Leading $\langle RR \rangle$ and $\langle AA \rangle$ contributions



(vertices are not dressed since this will yield $\frac{1}{k_F l}$ - corrections)

Explanation of scales:

$$G^{R/A}(r)\, G^{R/A}(-r) \sim e^{2ip_F|r| - |r|/l}$$

fast oscillations

which will kill integral if $p_F|r| > 1$

thus typical momentum

$$p_{typ} \sim \frac{1}{|r|} \sim p_F \gg q$$ – small ω_n & q

can be neglected in these diagrams

$$\Rightarrow K_0 \simeq T \sum_{\omega_m} \int \frac{d^3k}{(2\pi)^3} \left[G^{(m)}(i\omega_m, k)\right]^2 =$$

$$= -\frac{\partial}{\partial\mu} \underbrace{\sum_{\omega_m} \int \frac{d^3k}{(2\pi)^3} G^{(m)}(i\omega_m, k)}_{\bar{n} \text{ - mean density}} = -\frac{\partial}{\partial\mu}\bar{n} = -\nu$$

$$K_0 \simeq -\nu + \cancel{O\left(\tfrac{1}{k_F l}\right)}$$

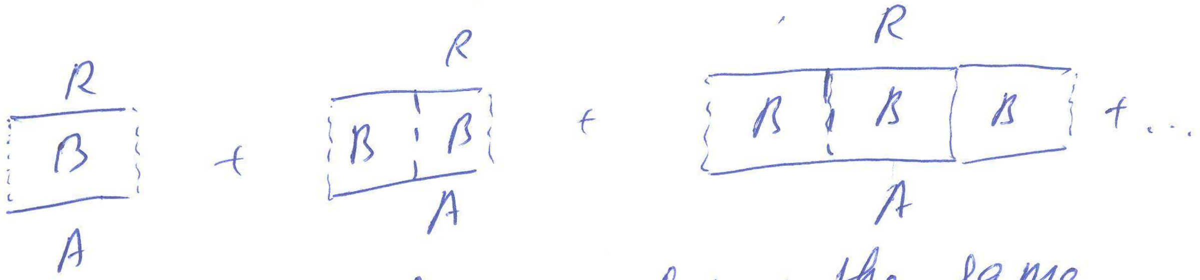
Block $B \equiv \pi \overset{(2\pi\nu\tau)}{=} \left(1 - \tau(|\omega_n| + Dq^2)\right)$ – see sep. calculs

which does not depend on ω_m

Σ over ω_m always yields

$$T\sum_{\omega_m}\left(\begin{matrix}\theta(\omega_m>0)\,\theta(\omega_m+\omega_n<0)\\ \text{or}\\ \theta(\omega_m<0)\,\theta(\omega_m+\omega_n>0)\end{matrix}\right) = T|n| = \frac{|\omega_n|}{2\pi}$$

Blocks in different power



Note: a.) Blocks are always the same

b.) $= \dfrac{1}{2\pi\nu\tau}$

$$\Rightarrow K_1 = \underbrace{2}_{\text{spins}}\ \underbrace{\frac{|\omega_n|}{2\pi}}_{M\text{-sum}}\ 2\pi\nu\tau \sum_{k=1}^{\infty}\left(\frac{B}{2\pi\nu\tau}\right)^k = \frac{|\omega_n|}{\pi}\,\frac{B}{1-B/2\pi\nu\tau}$$

$$K_1 = \frac{1}{\cancel{\pi}}\,\frac{|\omega_n|\,2\pi\nu\tau\,(1-\overbrace{\tau(|\omega_n|+Dq^2)}^{\text{small}})}{X-[X-\tau(|\omega_n|+Dq^2)]} = \frac{\overset{\text{full}}{\nu\,|\omega_n|}}{|\omega_n|+Dq^2}$$

$$K = K_0 + K_1 \simeq +\nu\left(-1+\frac{|\omega_n|}{|\omega_n|+Dq^2}\right) = -\nu\,\frac{Dq^2}{|\omega_n|+Dq^2}$$

Building block Π

Π-1

$$\Pi(i\omega_n, q) = \int \frac{d^3k}{(2\pi)^3} \, G(i[\omega_n+\omega_e], \vec{k}+\vec{q})\, G(i\omega_e, \vec{k})$$

$$\simeq \nu \int d\xi_k \int \frac{d\Omega_k}{4\pi} \frac{1}{i[\omega_e+\omega_n] - \xi_{k+q} + \frac{i}{2\tau}\,\mathrm{sign}(\omega_n+\omega_e)} \times$$

$$\times \frac{1}{i\omega_e - \xi_k + \frac{i}{2\tau}\,\mathrm{sign}(\omega_e)}$$

Let $\omega_n > 0 \Rightarrow \dfrac{\times\ \omega_e+\omega_n}{\times\ \omega_e} >$ pole structure

$$\Rightarrow \Pi = \nu\, 2\pi i\; \Theta(\omega_e<0)\, \Theta(\omega_e+\omega_n>0) \int \frac{d\Omega_k}{4\pi} \times$$

$$\times \frac{1}{i\omega_n + i/\tau - \underbrace{[\xi_{k+q} - \xi_k]}_{\to\, -\vec{v}_k\cdot\vec{q} \,=\, -v_F \cos\theta\, q}}$$

Diffusion approximation

$$\omega\tau \ll 1, \quad q\,l \ll 1$$

$$\Rightarrow \Pi = \nu\tau\, \frac{2\pi \cancel{i}}{\cancel{i}}\, \Theta_1 \Theta_2 \int \frac{d\Omega_k}{4\pi} \frac{1}{1+\omega_n\tau + i v_F q \cos\theta}$$

$$\simeq 2\pi\nu\tau\, \Theta_1 \Theta_2 \int \frac{d\Omega_k}{4\pi} \left(1 - \omega_n\tau - i v_F q\cos\theta - (v_F q \cos\theta)^2\right)$$

$$\Pi = 2\pi\nu\tau\, \Theta_1 \Theta_2 \left(1 - [\omega_n\tau + \tau D q^2]\right)$$

where $D = \dfrac{v_F^2 \tau}{3}$ - diffusion const.

Note: $\omega_n < 0$ will result in $-\omega_n \to \omega_n \Rightarrow |\omega_n|$