1 Magnetic susceptibility

1.1 Susceptibility of a free spin

Consider a free spin $s = 1/2$. Find its susceptibility χ with respect to a weak constant magnetic field. Show that χ obeys the Curie law

$$\chi = \beta\mu^2\,, \quad \beta \equiv 1/T\,; \tag{1}$$

where μ is the magnetic moment of the spin.

Hints:

1. write the Hamiltonian of the system and equation for the Matsubara susceptibility χ^M of the spin;
2. to derive χ^M, calculate expectation value of the product of the spin operators $\hat{s}_z(\tau)\hat{s}_z(0)$ with respect to the unperturbed Hamiltonian (without the magnetic field);
3. write down an answer for χ^M in a form which is convenient for the analytic continuation and perform the analytic continuation;
4. take the static limit $\omega \to 0$.

1.2 Response of a free spin in crossed magnetic fields

Consider a free spin at finite temperature in a transverse field B_x. Find its static magnetic susceptibility $\chi(B_x, T)$ in response to an additionally applied small field B'_z:

$$\chi = \frac{\partial}{\partial B'_z}\langle G|S_z|G\rangle\Big|_{B'_z=o}\,. \tag{2}$$

χ should be calculated in two ways:

1. by using a perturbation theory to find perturbed $|G\rangle$, and using Eq.(2)
2. via the Kubo formula by evaluating the response function explicitly.

Note that both approaches must yield the same answer (check this!). Analyze different limits: i. $T \to 0$ and $T \to \infty$; ii. $B_x \to 0$ (compare this case with the answer of the previous problem) and $B_x \to \infty$.