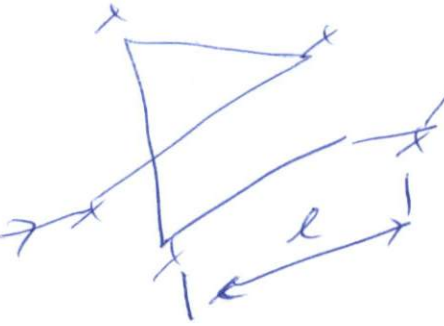
Classical conductivity of disordered Me

Reminder

the Drude model for a dirty Me



disorder is characterized by l or τ

$l = v_F \cdot \tau$ - the Fermi vel.

phenom. classical approach yields

$$\sigma_{ac}(\omega) = \frac{\sigma_{dc}}{1+i\omega\tau} \; ; \quad \sigma_{dc} = \frac{e^2 n \tau}{\underset{\text{effective}}{m^*}} \text{ - classical Drude cond.}$$

Random potential - leads to a diffusive motion, described by

$$D = \frac{v_F^2 \tau}{d} \quad d\text{ - space dimension}$$

Let's relate σ_{dc} and D.

Still semiclassical approach - the Boltzmann eq. can be used:

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \vec{\nabla}_r f + \overbrace{\left(e\vec{E} + \frac{e}{c}[\vec{v} \times \vec{H}]\right)}^{\text{the Lorenz force}} \cdot \vec{\nabla}_{\vec{p}} f = \left(\frac{\partial f}{\partial t}\right)_{\text{field}}$$

$$I[f] = \int d\vec{p}\,' \Big\{ \overbrace{f(\vec{p}\,')(1-f(\vec{p}))}^{\text{Pauli}} W(\vec{p}\,';\vec{p}) -$$

$$- \underbrace{f(\vec{p})(1-f(\vec{p}\,'))}_{\text{Pauli}} W(\vec{p};\vec{p}\,') \Big\} \text{ - collision integral}$$

Explain $f(\vec{r}, \vec{p}, t)$ & W; $\vec{\nabla}_{\vec{r}}, \vec{p}$; $\vec{E}, \vec{H}$

Kinetic equation: $\boxed{\left(\frac{\partial f}{\partial t}\right)_F = I[f]}$

In the Drude model

$\frac{1}{\tau}$ - const relaxation rate

and $I[f]$ simplifies to $-\frac{\overbrace{f-f_0}^{\equiv f_1}}{\tau}$ ↘ equilibrium

Micro-origin of τ is not yet specified.

Consider the case $\vec{E} = \text{const}$; $\vec{H}=0$ - we can neglect $\nabla_{\vec{r}}$ & $\partial/\partial t$ ⇒

$$e\vec{E}\,\vec{\nabla}_p f = -\frac{f-f_0}{\tau}$$

Let $\vec{E}$ be small - linear responce

$$e\vec{E}\,\vec{\nabla}_{\vec{p}}(f_0 + \underset{\text{subleading}}{\cancel{f_1}}) = -\frac{f_1}{\tau}$$ - linearized K.E.

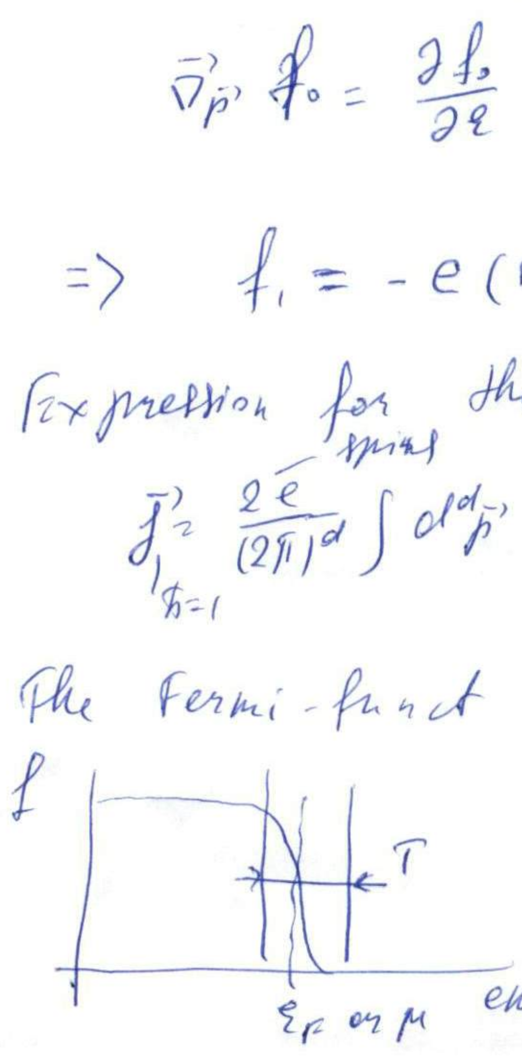
$$\vec{\nabla}_{\vec{p}} f_0 = \frac{\partial f_0}{\partial \varepsilon}\cdot\underbrace{\vec{\nabla}_{\vec{p}}\varepsilon}_{\vec{v}(\vec{p})} = \vec{v}(\vec{p})\cdot\frac{\partial f_0}{\partial \varepsilon}$$

$$\Rightarrow \quad f_1 = -e(\vec{E}\cdot\vec{v})\,\tau\,\frac{\partial f_0}{\partial \varepsilon}$$

Expression for the current insert

$$\underset{\hbar=1}{\vec{j}} = \frac{\overset{\text{spins}}{2}e}{(2\pi)^d}\int d^d\vec{p}\;\vec{v}\cdot f_1 = -e^2\tau\int d\varepsilon\;\overset{\text{DoS}}{\nu(\varepsilon)}\int\frac{d\Omega}{4\pi}\,\vec{v}(\vec{E}\cdot\vec{v})\times\frac{\partial f_0}{\partial \varepsilon}$$

The Fermi-funct



$\Rightarrow \frac{\partial f_0}{\partial \varepsilon} \neq 0$ only in

$\mu - T < \varepsilon < \mu + T$ with $T \ll \mu$

Good approximation

$$\partial_\varepsilon f_0 \simeq -\delta(\varepsilon-\varepsilon_\mu) + O\left(\cancel{\left(\frac{T}{\mu}\right)^2}\right)$$ – Sommerfeld corrections

$$\Rightarrow \vec{j} \simeq (-1)^2 e^2 \tau \int \frac{d\Omega}{4\pi} \int_{-\infty}^{+\infty} d\varepsilon\, \nu(\varepsilon)\, \vec{v}\,(\vec{E}\cdot\vec{v})\, \delta(\varepsilon-\varepsilon_F)$$

$\delta(\varepsilon-\varepsilon_F)$ yields

$$\nu \to \nu(\varepsilon_F)$$

$$\vec{v} \to V_F \cdot \vec{n}_v \quad \text{with } |\vec{n}_v| = 1$$

Assume, for example, $\vec{E} = (0, 0, E_z)$

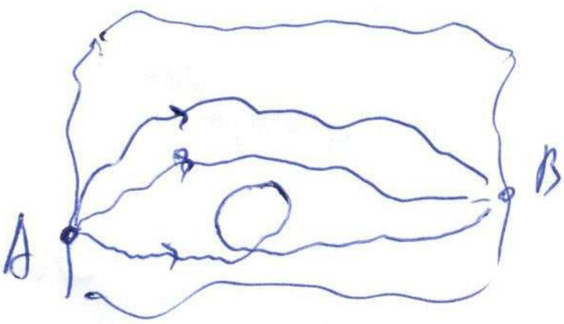
$$\Rightarrow \vec{j} = e^2 \tau V_F^2\, \nu(\varepsilon_F)\, E_z \underbrace{\int \frac{d\Omega}{4}\, \vec{n}_v \cdot n_z}_{\substack{1/d,\ \text{check yourselves only for } \vec{n}_z \\ 0 \text{ otherwise}}}$$

$$\Rightarrow \vec{j} = e^2 \frac{\tau V_F^2}{d}\, \nu(\varepsilon_F)\, \vec{E} \equiv e^2 D\, \nu(\varepsilon_F)\, \vec{E}$$

or $\boxed{\sigma_{dc} = e^2 D\, \nu(\varepsilon_F)}$ – the Einstein relation.

What is expected beyond the classical picture?

WL (interference) correction



A & B connected by different trajectories

To study transport, we need a probability

$$A \to B.$$

which can be found by summing all amplitudes:

$$W_{A\to B} = \Big|\sum_{i} A_i\Big|^2$$

i – over all trajectories (recall the Feynman q-mechanics)

Separate out diagonal / off-diagonal terms

$$W = \underbrace{\sum_i |A_i|^2}_{\text{no phase-classical contribution}} + \overbrace{\sum_{i\neq j} A_i A_j^*}$$

– phase-dependent, accounts for the interference

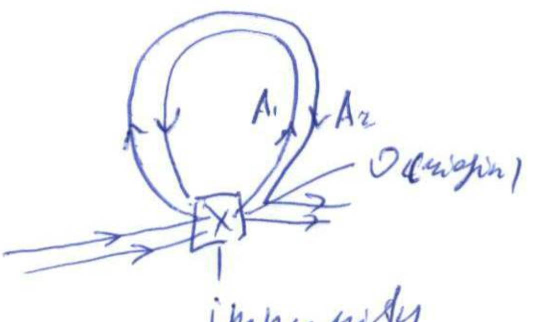
The phase of a given trajectory

$$\varphi_j = \frac{1}{\hbar}\int_A^B (\vec{p}\cdot d\vec{l})_j$$

j – trajectory number.

Typically $L_i \neq L_j \Rightarrow \varphi_i - \varphi_j$ is large – the second term oscillates quickly and disappears after calculating $\sum_{i,j}$.

But there are special contributions by loops.



The probability to return to the origing

$$W_{ret} = |A_1 + A_2| =$$

$$|A_1|^2 + |A_2|^2 + A_1 A_2^* + A_2 A_1^*$$

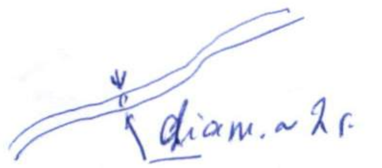
Yet $L_{A_1} = L_{A_2}$, they are related only by the transformation $\vec{p} \to -\vec{p}$ & $d\vec{l} \to -d\vec{l}$ (two time-reversed paths) $\Rightarrow \varphi_{A_1} = \varphi_{A_2}$, $|A_1| = |A_2|$

$$\Rightarrow W_{ret} = 2|A_1|^2 + 2|A_1|^2 = 4|A_1|^2 = 2 W_{ret}^{(cl)}$$

– the return prob. is increased due to the interference.

W_{tot} is increased $\Rightarrow$ $W_{A\to B}$ is decreased (DIS-5)
and we may expect a negative q-correction to σ_{dc} - called WL correction - caused by the self-intersecting trajectories.

Estimate $\Delta\sigma_{WL}$ We need a probability of the self-intersection. QM trajectory must be considered as a tube



Elementary volume of the tube

$$dV \sim \underbrace{\lambda_F^2}_{\text{diameter}} \times \underbrace{v_F\,dt}_{\text{elementary length}}$$

Then $P \sim \int \frac{dV}{V_{tot}} = \int \frac{\lambda_F^2 \cdot v_F\,dt}{V_{tot}(t)}$ total volume covered by diffusing trajectory

$X_{typ} \sim \sqrt{Dt} \Rightarrow V_{tot} \sim X_{typ}^{d}\big| = (Dt)^{3/2}$

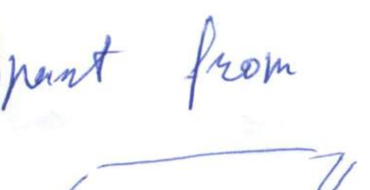
Assuming that $\Delta\sigma_{WL}/\sigma_{dc} \sim -P \Rightarrow$

$$\boxed{\frac{\Delta\sigma_{WL}}{\sigma_{dc}} \sim \underbrace{-}_{\text{must be negative}} \int_{t_{min}}^{t_{max}} \frac{v_F \lambda_F^2\,dt}{(Dt)^{3/2}}}$$

Limits: $t_{min} = \tau$ since there is no diffusion for $t < \tau$. t_{max} (called τ_φ) sets the IR cut-off for coherent contributions from loops.

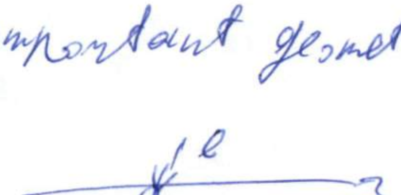
Apart from $d=3$ - two other important geometries

2d



$\lambda_F \ll \ell \ll L_x, L_y$

quasi-1d



In both cases $\ell \lesssim l$

Restoring units in a generic case

$$\boxed{\frac{\Delta\sigma_{WL}}{\sigma_{dc}} \sim -\int_{\tau}^{\tau_\varphi} \frac{v_F\lambda_F^2\,dt}{(Dt)^{d/2}\,b^{3-d}}}$$

Integrate in 3d

$$\frac{\Delta\sigma_{WL}}{\sigma_{dc}} \sim -\frac{v_F\lambda_F^2}{D^{\cancel{3/2}}}\left(\frac{1}{\sqrt{\tau D}}-\frac{1}{\sqrt{\tau_\varphi D}}\right)$$

up to coefficients

$D \sim v_F^2\tau = l v_F$; $D\tau \sim l v_F \tau = l^2$

$D\tau_\varphi \equiv L_\varphi^2$ – coherence length (explain it)

$$\Rightarrow \left.\frac{\Delta\sigma_{WL}}{\sigma_{dc}}\right|_{3d} \sim -\frac{\cancel{v_F}\lambda_F^2}{\cancel{v_F}\,l}\left(\frac{1}{l}-\frac{1}{L_\varphi}\right)$$

l is related to ellastic scattering, L_φ – to inellastic.

$\Rightarrow$ typically $L_\varphi \gg l$, $\frac{1}{l} \gg \frac{1}{L_\varphi}$

$$\left.\frac{\Delta\sigma_{WL}}{\sigma_{dc}}\right|_{3d} \sim -\left(\frac{\lambda_F}{l}\right)^2 \sim -\left(\frac{1}{k_F l}\right)^2 \ll 1$$

Parameter $1/k_F l$ controlls the smallness of QM-corr.

Integrate in 2d

$$\frac{\Delta\sigma_{WL}}{\sigma_{dc}} \sim -\frac{v_F\lambda_F^2}{Db}\log\left(\tau_\varphi/\tau\right) \sim -\frac{\lambda_F^2}{lb}\log\left(\frac{\tau_\varphi}{\tau}\right)$$

– may become large if $\log(\tau_\varphi/\tau)$ is large.

QM corrections are especially important in l-dim.

τ_φ itself – to be discussed later.