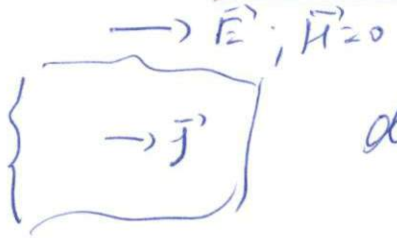
Conductivity via Green's



definition $\vec{j} = \sigma \cdot \vec{E}$

$$\hat{H}_0 = \frac{\hat{p}^2}{2m} + \underbrace{V(\vec{r})}_{\text{random}}$$

Let's choose the gauge of the vector potential:

$$(\vec{A}, \varphi=0)\ ,\ \text{such that}\ \begin{cases} \vec{E} = -\dfrac{\partial \vec{A}}{\partial t} & // c=1 // \\ \vec{H} = \vec{\nabla}\times\vec{A} = 0 \end{cases}$$

choose $\vec{A} = \frac{\vec{E}}{i\omega}$ // single harmonics //

$$\Rightarrow \quad \hat{H} = \frac{(\hat{\vec{p}} + e\vec{A})^2}{2m} + V(\vec{r})$$

The current density is given by

$$\vec{j} = \mathrm{Tr}(\hat{\rho}\,\hat{j})$$

$\hat{j}$ – current operator

$\hat{\rho}$ – density matrix

where $\hat{\rho}$ obeys the evolution equation

$$i\hbar \frac{\partial \hat{\rho}}{\partial t} = [\hat{H}, \hat{\rho}]$$

Decompose

$$\rho = \rho_0 + \delta\rho$$

$\delta\rho$ – caused by $\vec{E}$

ρ_0 – equilibrium at $E=0$

and regularize the long time evolution by

adding $-i\eta(\rho - \rho_{eq})\big|_{\eta\to+0}$ – relaxation to ρ_0 new equilibrium

Assume that E is infenitesimal small – we need only leading terms in $\vec{A}$. In particular

$$\hat{H} \simeq \hat{H}_0 + \underbrace{\frac{e}{2m}[\hat{\vec{p}}\vec{A} + \vec{A}\cdot\hat{\vec{p}}]}_{H_1\ \text{minimal coupling}}$$

Introducing $\delta\rho_{eq} = \underbrace{\rho_{eq} - \rho_0}_{\sim A}$

and selecting in the evolution equation only terms of order A:

$$i\hbar \frac{\partial \delta\rho}{\partial t} = [\hat H_0, \delta\rho] + [\hat H_1, \rho_0] - i\eta(\delta\rho - \delta\rho_{eq})$$

The current operator is also changed

$$\hat{\vec{j}} = -\frac{e}{2}\Big(\hat n \underbrace{\frac{\hat{\vec{p}}}{m}}_{\hat{\vec{v}}} + \hat{\vec{v}}\,\hat n\Big) \to \hat{j}_0 + \hat{j}_1$$

$$\Big|\ \hat{\vec{p}} \to \hat{\vec{p}} + e\vec{A}$$

with

$$\hat{\vec{j}}_0 = -\frac{e}{2}\Big(\hat n \frac{\hat{\vec{p}}}{m} + \frac{\hat{\vec{p}}}{m}\hat n\Big)$$

and

$$\hat{\vec{j}}_1 = -\frac{e^2}{2m}\Big(\hat n \vec{A} + \vec{A}\hat n\Big)$$ – will be canceled in N-Me due to the gauge invariance, $\vec{j}$ cannot depend of $\vec{A}$ (only on $\vec{B}$)

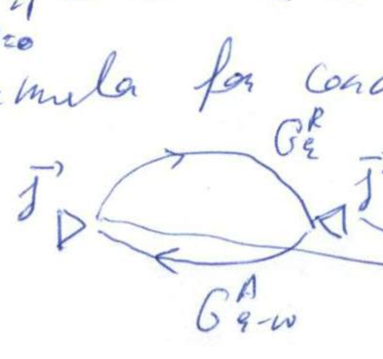
Simplifying the current in the case of the weak $\vec{E}$

$$\vec{J} = Tr(\hat\rho\,\hat{\vec{j}}) \simeq Tr(\hat\rho_0\,\hat{\vec{j}}_1) + Tr(\delta\rho\,\hat{j}_0)$$

Next - lengthy but trivial algebra of operators and computing the matrix elements (note that Tr is base-invariant). This yields //M-book, pp. 296-301// for the stationary regime (cf. Levitov (5.12)) at $\vec{E} = (E_x, 0, 0)$

$$\sigma_{xx}\Big|_{\substack{\hbar=1\\ T=0}} = \frac{\overset{\text{from spin}}{(2)}}{2\pi\, Vol}\left\langle Tr\left[\hat{j}_x \hat{G}^R_\varepsilon \hat{j}_x \hat{G}_{\varepsilon-\omega}\right]\right\rangle_{dis}$$

– the Kubo formula for conductivity in the b-invariant form

Pictorially: 

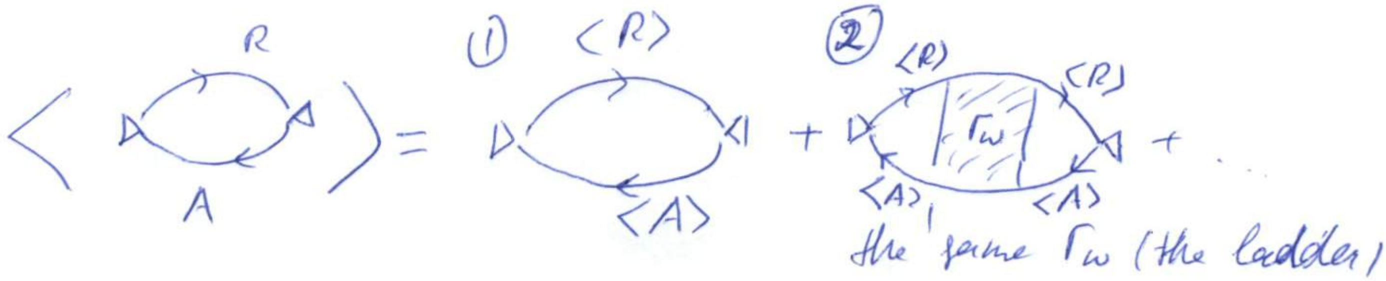
the vector vertices

The most convenient is the momentum repr.

$\vec{j} \to \frac{e\vec{k}}{m}$ + consider homogeneous $\vec{E}$!

$$\Rightarrow \sigma_{xx}(q=0;\omega) = \frac{2\, e^2/m^2}{2\pi \mathrm{Vol}} \sum_{\vec{k},\vec{k}'} \left\langle \frac{\cancel{e}k_x}{\cancel{m}}\, G^R_\xi(\vec{k},\vec{k}')\, \frac{\cancel{e}k'_x}{\cancel{m}}\, G^A_{\xi-\omega}(\vec{k}',\vec{k}) \right\rangle_{dis}$$

Now the disorder averaging: let's start with the same approximations as we used for P:



① $\langle R/A\rangle$ is translationally invariant, i.e., diagonal in $k \Rightarrow$

$$\sigma^{(1)}_{xx}(q=0,\omega) = \frac{2e^2}{2\pi m^2 \mathrm{Vol}} \sum_{\vec{k}} k_x^2 \langle G^R_\xi(k)\rangle \langle G^A_{\xi-\omega}(k)\rangle$$

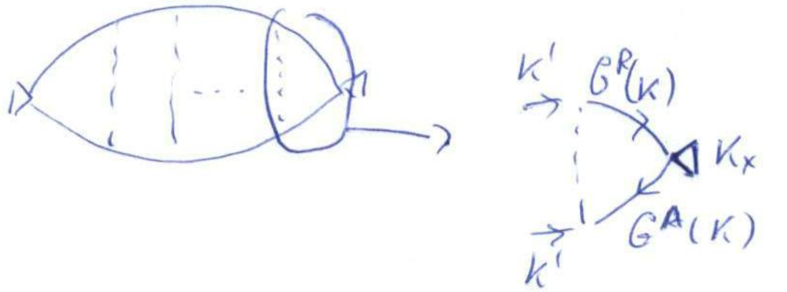
$$\to \frac{2e^2}{2\pi m^2 \cancel{\mathrm{Vol}}} \int d\Omega_d \int d\xi\, \underbrace{\nu(\xi)}_{\nu_0 = \nu(\xi_F)}\, \overbrace{k^2}^{k_F^2}\, \hat{n}_x^2 \langle G^R_\xi(\xi)\rangle \langle G^A_{\xi-\omega}(\xi)\rangle$$

$$= \frac{2e^2 v_F^2 \nu_0}{\cancel{2\pi}} \underbrace{\int \frac{d\xi}{2\pi} \langle G^R_\xi(\xi)\rangle \langle G^A_{\xi-\omega}(\xi)\rangle}_{\text{pole integral} = P_0 \text{ up to const}} \underbrace{\int d\Omega_d\, \hat{n}_x^2}_{1/d}$$

$$= 2e^2 \frac{v_F^2}{d} \nu_0 \frac{\cancel{2\pi\nu}}{\cancel{2\pi}} \frac{1}{\cancel{\nu}/\tau_e + (-i)\omega} = \frac{2e^2 \overbrace{\left(\frac{v_F^2 \tau_e}{d}\right)}^{D} \nu_0}{1 - i\omega\tau} =$$

$$= \frac{2e^2 D \nu_0}{1 - i\omega\tau} \Rightarrow \boxed{\begin{cases} \sigma^{(1)}_{xx}(q=0,\omega) = \frac{\sigma_0}{1-i\omega\tau} & \text{– the ac-Drude} \\ \sigma_0 = 2e^2 D\nu_0 & \text{– the Einstein relation} \end{cases}}$$

(2) - vanishes because



- all momentums are independent

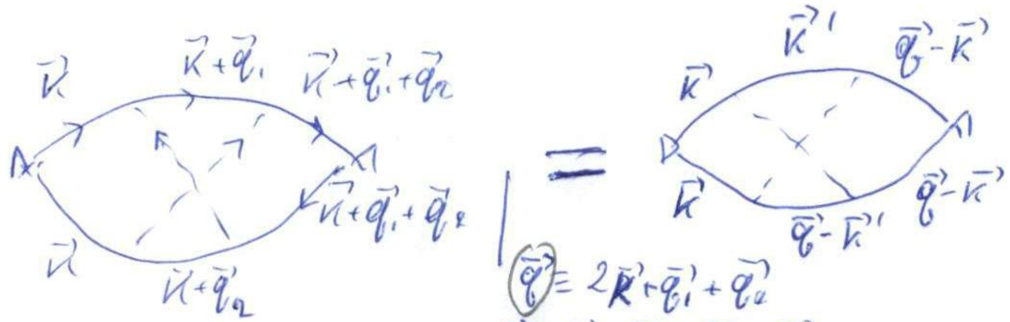
$$\Rightarrow \sigma_{xx}^{(2)} \propto \sum_{\vec{k}} k_x \langle G^R_\varepsilon(|\vec{k}|)\rangle \langle G^A_{\varepsilon-\omega}(|\vec{k}|)\rangle = 0 -$$

due to the angle averaging.

How to find QM-corrections?

Let's find the leading in $\frac{1}{\varepsilon}$ (weak disorder!) diags, which is beyond the classical one.

(Ivy)



$\vec{q} = 2\vec{k}+\vec{q}_1+\vec{q}_2$

$\vec{k}+\vec{q}_1+\vec{q}_2 = \vec{q}-\vec{k}$

$\vec{k}+\vec{q}_1 = \vec{k}'$

$\vec{k}+\vec{q}_2 = \vec{q}-\vec{k}'$

Note that vertices here are $\vec{k}$ & $\vec{q}-\vec{k}$, i.e., they are <u>not</u> independent and this does not kill this diag.

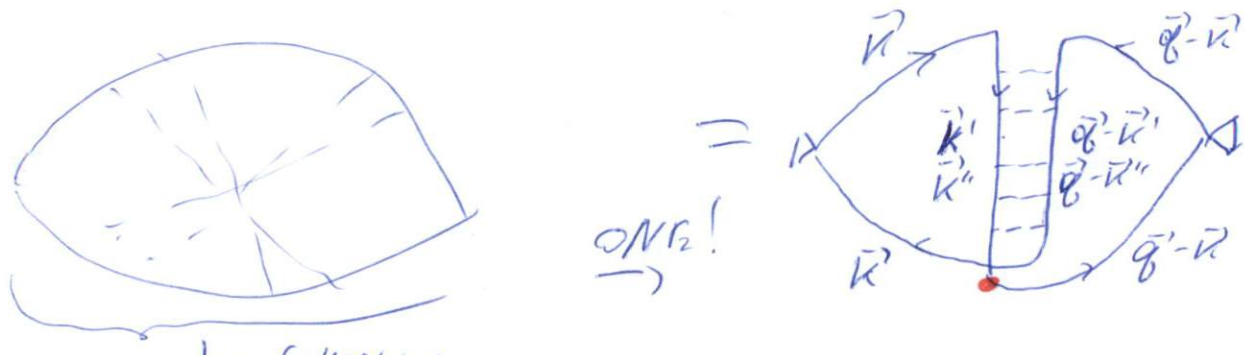
Rough power counting

$\propto 1/\varepsilon^2$ must be scaled by some energy

In the simplest case it's scaled by $\omega \Rightarrow \frac{1}{(\omega\tau)^2}$ - ~~can be~~ is large in the diffusive limit

How to obtain meaningfull answer? – let's try to collect all similar diagr. again with minimal number of crossings Cond-5

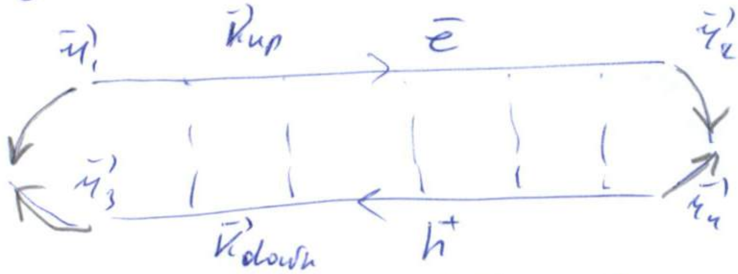
Consider



how many crossing are there?

Let's compare ladders

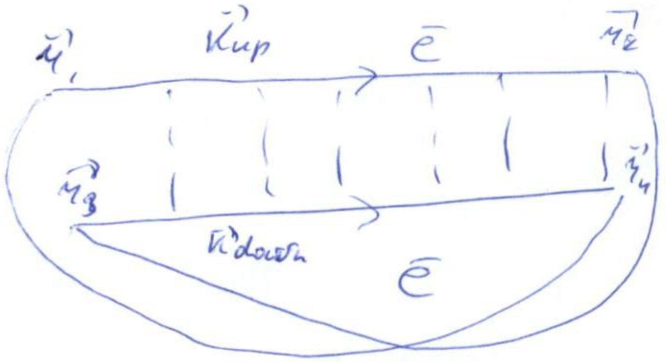
(D)



$\rightarrow \dfrac{1}{Dq^2 - i\omega}$ – small $Dq^2 \sim \omega$ yield the main contr.

$\bar{e}-h^+$ channel, $\begin{cases} \vec{r}_1 \simeq \vec{r}_3 \\ r_2 \simeq r_4 \end{cases}$ in real space OR $\underline{\vec{K}_{up} \ominus \vec{K}_{down} = \vec{q}}$

(C)



$\rightarrow$ again $\dfrac{1}{Dq^2 - i\omega}$

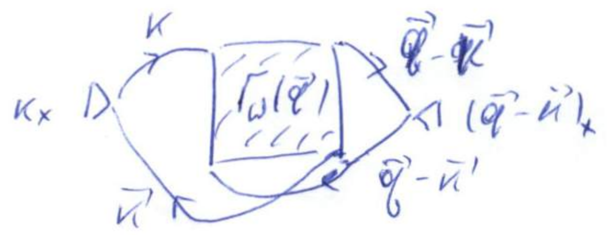
but the meaning of q is <u>different</u>

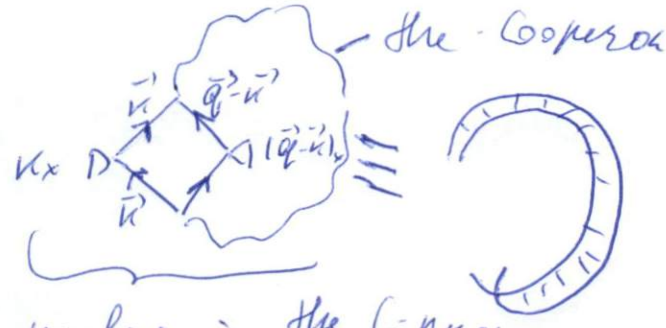
$\underbrace{\bar{e}-\bar{e}\text{-channel}}$, $\begin{cases} \vec{r}_1 \simeq \vec{r}_4 \\ \vec{r}_2 \simeq \vec{r}_3 \end{cases}$. OR $\vec{K}_{up} \oplus \vec{K}_{down} = \vec{q}$

therefore is called the Cooperon

If all $\vec{r}_{1,2,3,4}$ are close to each other – it describes QM return probability

Now use either the vertex funct. or C - propagator (Cond - 6)





Analytical expression

$$\Delta\sigma = \frac{2e^2}{2\pi m^2 V_0 d} \sum_{\vec{k},\vec{q}} \underbrace{k_x (\vec{q}-\vec{k})_x}_{\text{vertices}} \underbrace{2\pi \rho_0 P_0(k,\omega)}_{\text{left angle}} \times$$

$$\times \underbrace{\frac{C(\vec{q},\omega)}{2\pi V_0 \tau_e^2}}_{\Gamma_\omega(\vec{q})} \underbrace{2\pi \rho_0 P_0(\vec{q}-\vec{k},\omega)}_{\text{right angle}}$$

Let's now separate scales

$k \sim k_F$, as usually, while the main contribution is governed by the pole of C with $Dq^2 \sim \omega \ll \varepsilon_F$

$$\Rightarrow k \ll q$$

and one can factorize integrals:

$$\Delta\sigma = \frac{2e^2}{2\pi m^2 V_0 d} \underbrace{\sum_{\vec{k}} (\overset{\text{negative!}}{-k_x^2}) \left(2\pi \rho_0 P_0(\vec{k},\omega)\right)^2}_{\text{simplify as previously}} \times \frac{1}{2\pi V_0 \tau_e^2} \sum_{\vec{q}} C(\vec{q},\omega)$$

$$\to -2e^2 v_F^2/d\, V_0 \int \underbrace{\frac{d\xi}{2\pi} \left(2\pi \rho_0 P_0(\xi,\omega)\right)^2}_{\text{poles: } 2\tau_e^3} \cdot \frac{1}{2\pi V_0 \tau_e^2} \sum_{\vec{q}} C(\vec{q},\omega)$$

can be neglected since it's slow

$$= -\overbrace{\left(2e^2 \frac{v_F^2 \tau_e}{d} V_0\right)}^{\sigma_0} \cdot \frac{1}{\pi V_0} \sum_{\vec{q}} C(\vec{q},\omega) = -\frac{\sigma_0}{\pi V_0} \sum_{\vec{q}} C(\vec{q},\omega)$$

Properties

$$\frac{\Delta\sigma}{\sigma_0} = -\frac{1}{\pi\nu_0}\sum_{q}\frac{1}{Dq^2 - i\omega} \to \begin{cases} UV \text{ in } 3d \\ \log \text{ in } 2d \\ IR \text{ in } 1d \end{cases}$$

in a full agreement with the naive estimate. Let's look at 1d & 2d, where $\Delta\sigma$ can be important.

Transport $\Rightarrow$ $q \neq 0$, $|q|_{min} \sim 1/L$; $Dq^2_{min} = \Gamma_{Th} =$

$1/\tau_{Th}$ - the time it takes for particle to traverse the system

If $L \to \infty$, $\Gamma_{Th} \to 0 \Rightarrow$ IR cut-off is ω. Rewrite

$$\frac{\Delta\sigma}{\sigma_0} = -\frac{1}{\pi\nu_0}\int dt\, e^{+i\omega t} \underbrace{C(t, r=0)}_{\text{QM return probability}}$$

If $\omega \to 0$ IR is needed in 1d, 2d $\Rightarrow$ substitute

$$C(t, r=0) \to C_0(t, r=0)\exp\left(-t/\tau_H - t/\tau_{dw} - t/\tau_\varphi\right)$$

H-field dwelling dephasing

τ_H & τ_φ influence real measurements (wait for the seminar)

Analize smallness of the correction in IR at finite L

$$\left.\frac{\Delta\sigma}{\sigma_0}\right|_{dc} \sim -\frac{1}{\nu_0}\sum_q \frac{1}{Dq^2 - \cancel{i\omega}} \to \frac{1}{\rho_0}\int \frac{Q^{d-1}dQ}{DQ^2}\Big|_{Q\to\frac{1}{L}}$$

$$\to \frac{(1/L)^{d-2}}{\rho_0 D} = \frac{L^2/D}{\rho_0 Vol} = \frac{1}{\nu_0 \Gamma_{Th}}$$

Def.: $\frac{1}{\nu_0} \equiv \Delta$ - mean distance between energy levels

$\Rightarrow \frac{\Delta\sigma}{\sigma_0} \sim \frac{\Delta}{\Gamma_{Th}}$. Def: $g_0 = \frac{\Gamma_{Th}}{\Delta}$ - conductance (Thouless eq.)

Cond-8

<u>Conclusion</u> WL correction can be small only iff $g_0 \gg 1$ (good Me).

Non-perturbative approach is needed otherwise.

How to generate subleading corrections?

G_0

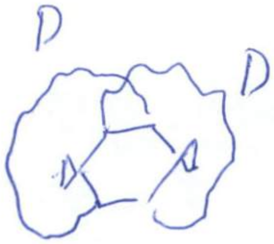
C

$O(G_0/g_0)$

one loop in the ladder.

<u>Rule</u> each loop in ladders is able to add $1/g_0$

Example of two loop diagrams:



$- \; O(G_0/g_0)$ - do not forget combinatorics

This is called: „The loop-expansion in D & C"

Objects ◇, ⬡, ... are called Hikami boxes - they represent short-ranged part of diagrams. (can be very unpleasant in real calculus)