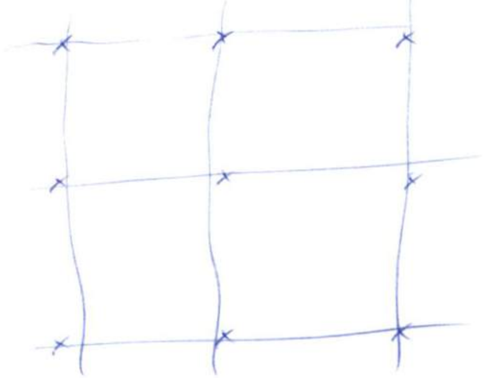
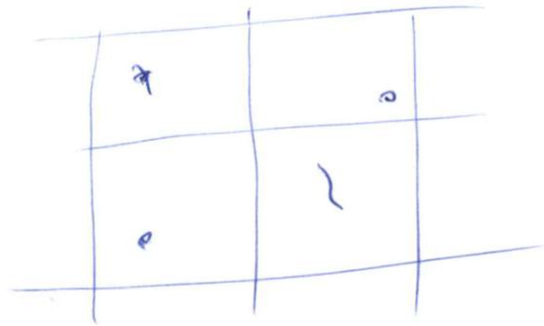
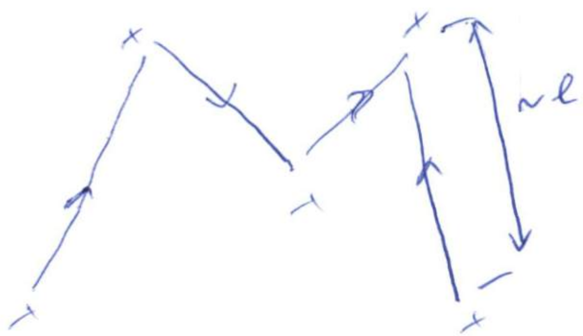
Drude (classical) model of N-Me



ideal lattice - Bloch $\bar{e}$



real - not ideal - Me -
- the conduction $\bar{e}$'s are scattered



scattering is characterized by (pheno):

l - mfp $\Rightarrow \gamma$ - scatt. rate

$\gamma^{-1} = \tau = l/v_F$ - transport time (F-velocity)

Apply weak field (the linear resp.), the $\bar{e}$-current

$\vec{J} = en\vec{V}$ mean velocity, which can be found from the classical theory

$$\dot{\vec{P}} = m\dot{\vec{V}} = e\vec{E} - \overbrace{\frac{1}{\tau}(m\vec{V})}^{\text{momentum relaxation}}$$

consider $\vec{E} = \vec{E}_0 \cdot e^{-i\omega t}$

and in the linear regime

$$\vec{p} = \vec{p}_0 e^{-i\omega t}; \quad \vec{V} = \vec{V}_0 e^{-i\omega t}$$

$$\Rightarrow -i\omega m\vec{V}_0 = e\vec{E}_0 - \tau^{-1} m\vec{V}_0; \quad \vec{V}_0 = \frac{e/m}{\tau^{-1} - i\omega}\vec{E}_0$$

and we find the current

$$\vec{j} = \underbrace{\frac{\overbrace{\frac{e^2\tau n}{m}}^{\sigma_0}}{1-i\omega\tau}}_{\sigma_D}\vec{E}$$

$$\sigma_D(\omega) = \frac{\sigma_D(\omega=0)}{1-i\omega\tau}$$ - the classical Drude σ.

Different micro-sources for τ!

defect $\to$ statical (impurities, dislocations, etc.)
defect $\to$ dynamical (phonons)

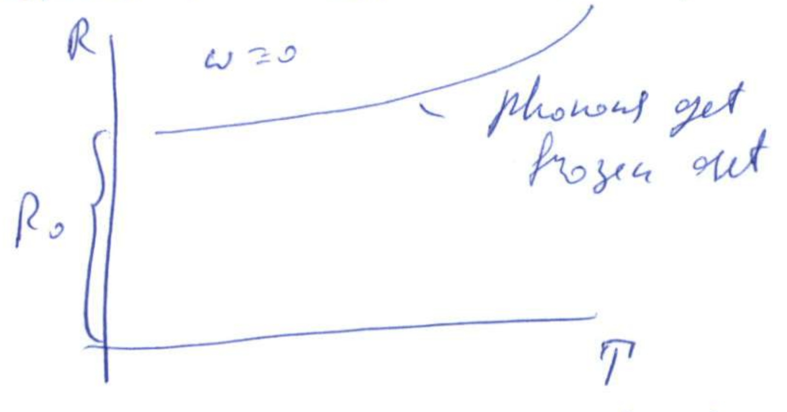
<u>The Matthiessen's rule</u> if all sources are independent $\Rightarrow$

$$\underbrace{\tau^{-1}}_{\text{total}} = \underbrace{\sum_i}_{\text{over all mechanisms}} \tau_i^{-1}{}_{\text{partial}}$$

The simplest case

$$\frac{1}{\tau} = \frac{1}{\tau_{imp}} + \frac{1}{\tau_{ph}}, \quad \text{where } \tau_{ph}^{-1} \propto T^{\alpha}, \ \alpha > 0$$

Consider the resistivity of N-Me, $R = \sigma^{-1}$



The residual resistivity is due to impurities

$$R_0^{-1} = \frac{e^2\tau_{imp} n}{m} = \text{const}$$

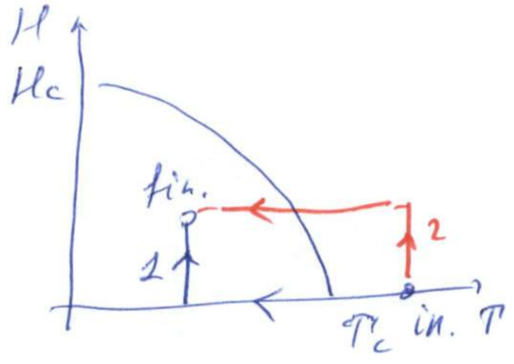
BUT $\to$ go to slides

Basic properties of type-I SC at $\begin{cases} T < T_c \\ H < H_c \end{cases}$ 3

$$\boxed{R = 0, \quad \vec{B} = 0} \quad (1)$$

Is it a N-Me, where all impurities disappeared?

Consider two different paths



① $T_i > T_c$, $H_i = 0$

- $T \searrow$ below T_c
- $H \nearrow$ but $H < H_c(T)$

$$\vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}, \quad \text{but } \vec{E} = \vec{j} \cdot \underbrace{R}_{=0} = 0$$

$$\Rightarrow \frac{\partial \vec{B}}{\partial t} = 0, \quad \text{i.e. } B = \text{const} = B_i = 0$$

② the same initial state but

- $H \nearrow$ at first
- $T \searrow$ at second

$\Rightarrow B \neq 0$ inside the sample

① ≠ ② for „ideal M"

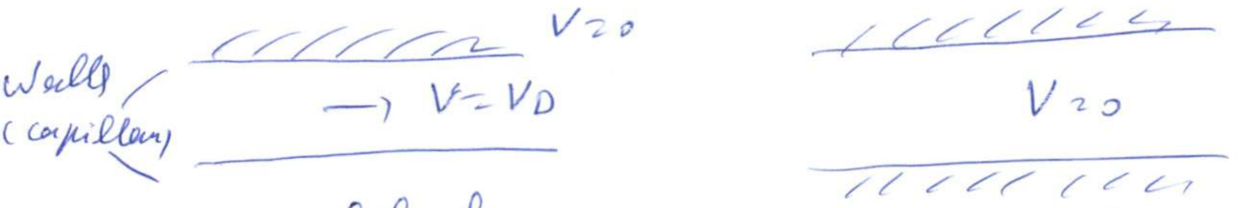
BUT ① ≡ ② for Sup Cond, (1) always holds true - we expect that Sup Con is different state with completely rearranged |GS⟩ & excitatio

How to explain the "absence" of impurities?

11 (K-O) → 57 (BCS), 58 (Bogolubov) ⇒ 46 years of theoretical effort to explain the micro-origin of SC.

First step was stipulated by the discovery of superfluidity ⟨ → Kapitsa (38), → Landau (41)

He^4 at $T < T_\lambda$ is superfluid, no viscosity at $V < V_c$



lab. frame   He - frame

He - frame: $E_{kin} = 0$, $\vec{P} = 0$

Viscosity would mean that qu-particles (excitations) can appear. Note V_D is small so we do not expect a motion of all He^4.

QP: $\varepsilon(\vec{p})$, $\vec{p}$ lead to finite E, P

Back to lab-frame (Galilean transformation)

$$\vec{P}' = \vec{P} + \underset{\text{mass of } He^4}{M} V_D \quad ; \quad E' = E + \vec{P}\cdot\vec{V}_D + \frac{M V_D^2}{2}$$

~~Energy conservation~~ where $\varepsilon \to E$ and $\vec{p} \to \vec{P}$

$$\Rightarrow E' = \varepsilon(\vec{p}) + \vec{p}\cdot\vec{V}_D + \frac{M V_D^2}{2}$$

This process is energetically favorable IFF $E < 0$, i.e. $\varepsilon(\vec{p}) + \vec{p}\cdot\vec{V}_D < 0$

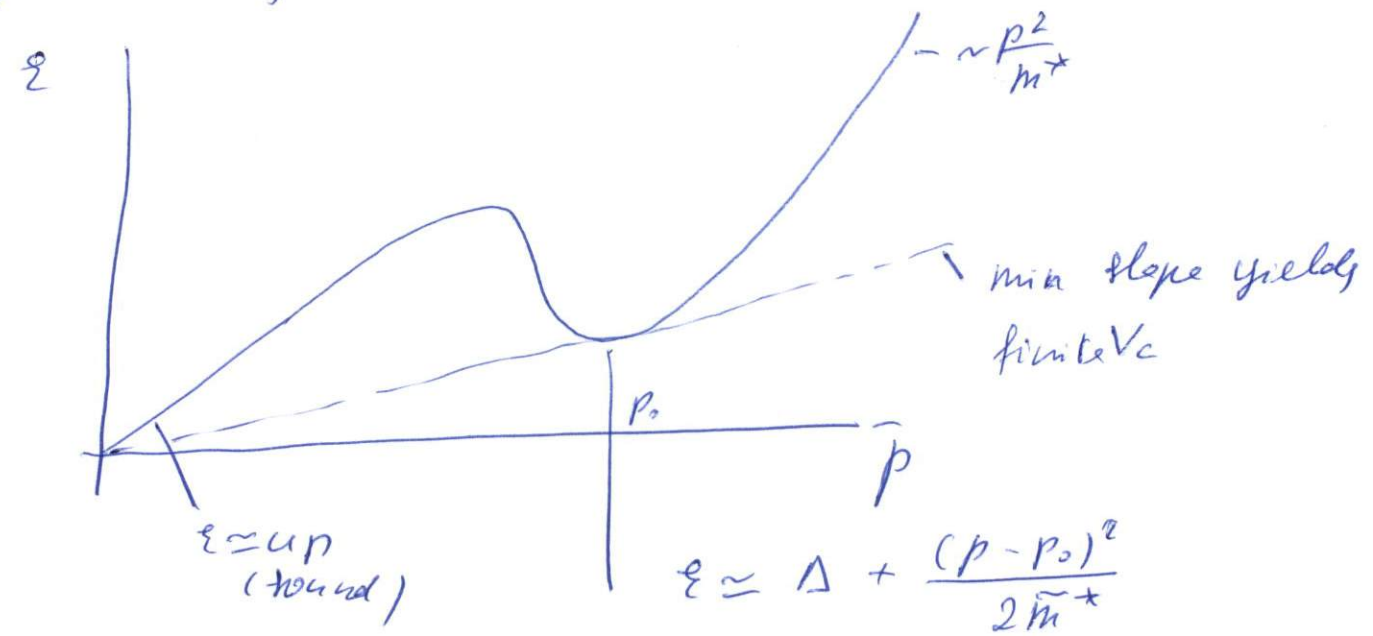
which is possible for $\vec{p} \uparrow\downarrow \vec{V}_D$

$$\varepsilon(\vec{p}) - p \cdot V_D < 0 \Rightarrow V_D > \varepsilon(p)/p$$

and we can estimate the critical velocity as

$$V_c \sim \left(\varepsilon(p)/p\right)_{min.} \text{ – Landau criterion}$$

Spectrum of excitations in He^4

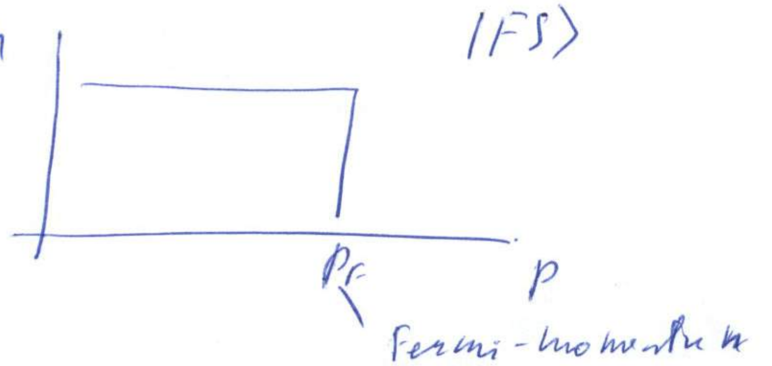


Δ – the gap. If $\begin{cases} \Delta \to 0 \Rightarrow \text{no superfluidity} \\ \text{or} \\ \varepsilon \sim p^2 \text{ at } p \to 0 \end{cases}$

He^4 is superfluid because it is the Bose-liquid and $\Delta \neq 0$

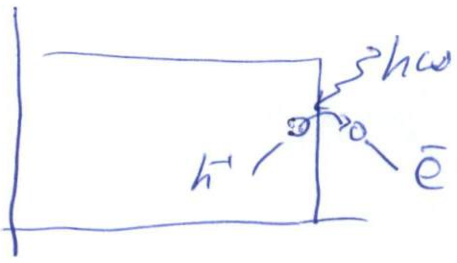
Can we explain SupCond as „superfluidity of e's"?

Excitations in FS at $T = 0$



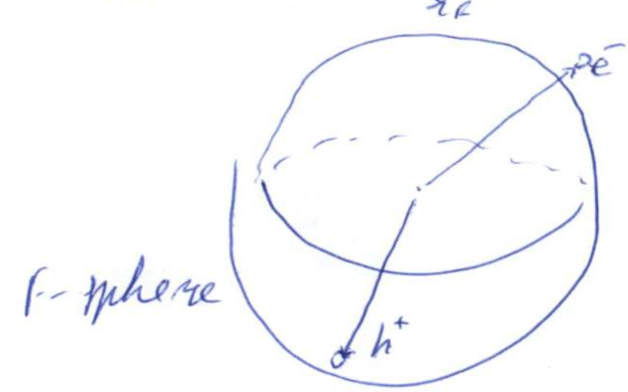
$n = 1, \ |\vec{p}| \leq p_F$

$n = 0, \ |\vec{p}| > p_F$



$\bar{e}$-h^+ pair is the elementary excitation in Fermi-system.

3d case



$\bar{e}$-h^+ can be generated such that

1) particles have (almost) opposite momentum
2) are very close to the F-surface

$$\Rightarrow \quad \varepsilon(p) \ll \varepsilon_F \quad (\text{actually} \to 0)$$

$$p \to 2p_F$$

$$\Rightarrow \quad V_c^{(\text{Landau})} \to 0$$

<u>Conclusion</u> $\bar{e}$'s in N-Me <u>cannot</u> show superfluidity, we need the complete change of the basic system properties.

<u>Announcement</u> weak attraction (mediated by phonons) result in a formation of bound states

$(\uparrow)\bar{e} + (\downarrow)\bar{e} \to$ „boson"

The <u>gap opens</u> in the spectrum of excitations since the F-surface is unstable wrt the attraction + entirely different $|GS\rangle$

This is N-Me to Sup Cond phase transition.