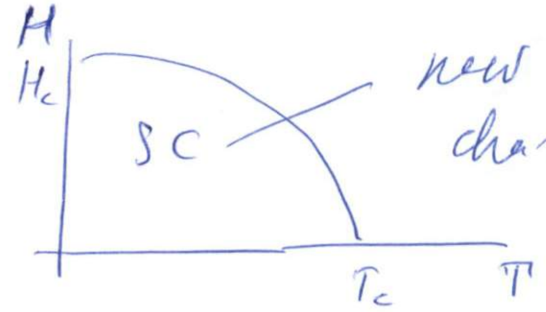
L 2 **Reminder**



new state of matter, characterized by $\begin{cases} R = 0 \\ \vec{B} = 0 \end{cases}$

Important that — SC $\neq$ " ideal Me " (no impurities)
$\Rightarrow$ we may expect the phase transition N-Me $\Leftrightarrow$ SC

— SC cannot be expained as superfluidity of conduction ē's since excitations ~~of~~ in the FS do not have properties which 'd allow one to use the Landau criterion.

Expectation if there is the phase-transit- we expect the entire rearrangement of $|GS\rangle$ + (probably) opening the gap the spectrum of excitations.

To explain this let's show that
— there is ph. mediated attraction
— the F-surface is unstable if even small attraction appears

Why phonons are important?

The substitution effect for different isotops

$$T_c \cdot \underset{\text{mass of ion}}{M^{\alpha}} = \text{const}$$

$\alpha \simeq 1/2$ for many materials (Zn, Sn, Hg, Pb, ...)

We conclude that the lattice is important for SC

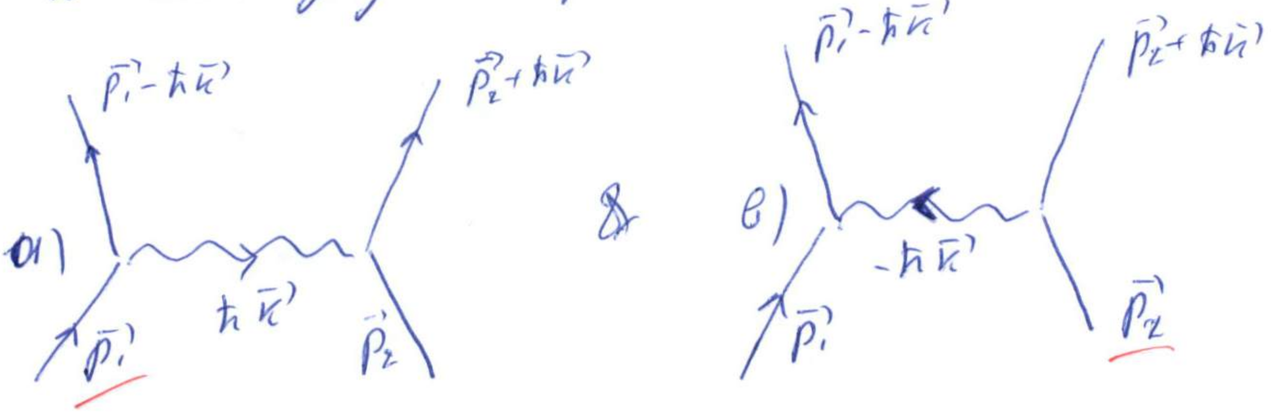
Ph-mediated attraction

Consider 2 ē's which propagate in the lattice and can emit/absorb phonons.

$$1\,\bar{e} \qquad \begin{array}{l} \vec{p}_1 \to \vec{p}_1 \mp \hbar\vec{k} \\ \vec{p}_2 \to \vec{p}_2 \pm \hbar\vec{k} \end{array}$$

initial state of electrons ($\vec{p}_1$, $\vec{p}_2$); single phonons ($\hbar\vec{k}$).

Calculate the leading correction to the energy of the electron which emits ~~due to exchanging~~ the phonon, assuming the same f-state.



$$a): \quad \frac{|V_k|^2}{\varepsilon(\vec{p}_1) - [\varepsilon(p_1 - \hbar\vec{k}) + \hbar\omega(\vec{k})]}$$

$$b.): \quad \frac{|V_{-k}|^2}{\varepsilon(\vec{p}_2) - [\varepsilon(\vec{p}_2 + \hbar\vec{k}) + \hbar\omega(-\vec{k})]}$$

Where $V_k \equiv \langle \vec{p} - \hbar\vec{k} | \hat{V} | \vec{p} \rangle$ - matrix element of ē-ph interaction

which can be estimated as

$$V_k \sim -i \frac{p_F}{\sqrt{Vol}} \sqrt{\frac{\hbar\omega(k)}{n \cdot M}}$$

of ions

of ē'

$\varepsilon(\vec{p})$ - energy of $\bar{e}$
$\hbar\omega(k)$ - —"— ph
} even functions $\Rightarrow |V_k| = |V_{-k}|$

Add the energy conservation law

$$\varepsilon(\vec{p}_1) + \varepsilon(\vec{p}_2) = \varepsilon(\vec{p}_1 - \hbar\vec{k}) + \varepsilon(\vec{p}_2 + \hbar\vec{k})$$

$$\Rightarrow \quad \varepsilon(\vec{p}_1) - \varepsilon(\vec{p}_1 - \hbar\vec{k}) = \varepsilon(\vec{p}_2 + \hbar\vec{k}) - \varepsilon(\vec{p}_2) \overset{def}{\equiv} \Delta\varepsilon$$

The total correction to energy of 2 $\bar{e}$s due to both processes:

$$|V_k|^2 \left[\frac{1}{\underbrace{\varepsilon(\vec{p}_1) - \varepsilon(\ldots)}_{\Delta\varepsilon} - \hbar\omega(k)} + \frac{1}{\underbrace{\varepsilon(\vec{p}_2) - \varepsilon(\ldots)}_{-\Delta\varepsilon} - \hbar\omega(k)} \right]$$

$$= -\frac{2|V_k|^2 \hbar\omega(k)}{-(\Delta\varepsilon)^2 + (\hbar\omega(k))^2}\Big|_{\text{insert } V_k} = -\frac{2p_F^2}{Vnm} \frac{(\hbar\omega(k))^2}{(\hbar\omega(k))^2 - (\Delta\varepsilon)^2}$$

Estimate $n \sim p_F/\hbar \Rightarrow \frac{p_F^2}{Vm}\frac{1}{n} \sim \frac{p_F^2}{Vm}\left(\frac{\hbar}{p_F}\right)^3 = \frac{\hbar^3}{Vp_F m}$

$$\Rightarrow \quad \Delta E \sim -\underbrace{\frac{\hbar^3}{Vp_F m}}_{\Delta E_0 \equiv g/V \text{ - coupling const}} \frac{(\hbar\omega(k))^2}{(\hbar\omega(k))^2 - (\Delta\varepsilon)^2}$$

If $\Delta\varepsilon = \varepsilon(\vec{p}_1) - \varepsilon(\vec{p}_1 - \hbar\omega(\vec{k})) \ll \hbar\omega(\vec{k}) \Rightarrow$

$$\Delta E \simeq -\underbrace{\Delta E_0}_{\text{const}} < 0 \Rightarrow \text{attraction!}$$

The range $\Delta\varepsilon \ll \hbar\omega(\vec{k})$ is the most important for the BCS

Analysis of this answer

1) Δk is angle independent (the so-called interaction in the s-state)

$$\Psi_{\bar{e}+\bar{e}} = \Psi_{coord} \cdot \sigma_{spin} \text{ - must be}$$

antisymmetric wrt $\bar{e}_1 \leftrightarrow \bar{e}_2$ since $\bar{e}$s are fermions

But we expect Ψ_{coord} to be symmetric since Δk is const (angle independent) $\Rightarrow$ we must choose antisymmetric σ_{spin} which is possible if we consider the interaction of $\bar{e}$s with $\uparrow\downarrow$

2) DoS of phonons $\propto k^2 \frac{dk}{d\omega} \Rightarrow$ max contribution comes from max $\underbrace{k = k_D = \pi/a}_{\text{D-model}} \Rightarrow$ substitute $\hbar\omega_D \to \hbar\omega(\dots)$

3.) Competition of two forces

ph. mediated attraction

Coulombic repulsion

Assume $V = 1\,cm^3$. Screened C-potential

$$V_c(q) = \frac{4\pi e^2}{q^2 + k_s^2}$$

~~that~~ with $\lambda_s^{-1} = k_s \sim 1/a$ – Thomas-Fermi screening w-vector

$$\Rightarrow \max V_c \sim (ea)^2$$

Estimate the ratio:

$$\frac{\max V_c}{g} \sim \frac{e^2 a^2}{\hbar^3/p_F m} \quad \frac{m p_F}{\hbar^3} \sim e^2 \left(\frac{\hbar}{p_F}\right)^2 \frac{p_F m}{\hbar^3} \sim \frac{e^2}{m^{\prime} p_F \hbar} \sim \frac{e^2}{v_F \hbar}$$

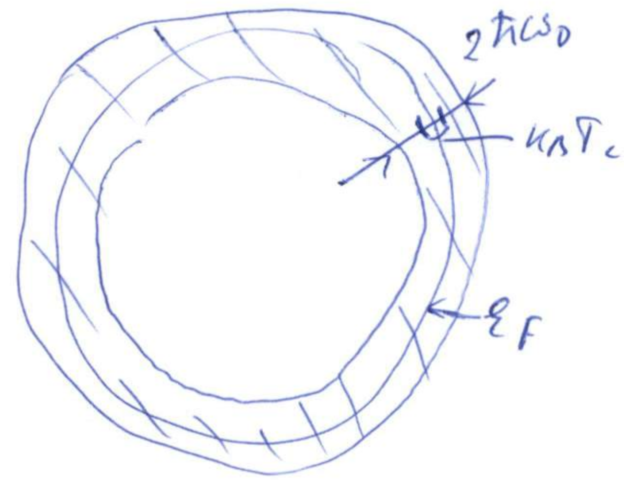
~ 1 at $v_F \sim 10^8$ cm/s (typical for good Me)

I.e. we come accross a competition of two forces of the same order which explains the smallness of T_c w.r.t. ε_F.

The hierchy of energy scales:

$$\underbrace{k_B T_c}_{1 \div 10\,K} \ll \underbrace{\hbar\omega_D}_{10^2 K} \ll \underbrace{\varepsilon_F}_{10^4 K}$$

allows one to introduce the effective model for the attraction



effective!! (rep-attr)

$\begin{cases} -g & \text{inside the layer} \\ 0 & \text{otherwise} \end{cases}$

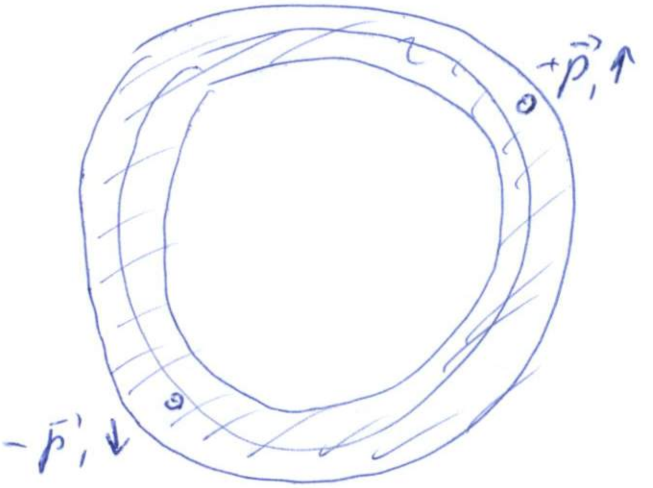
Justification $k_B T_c$ is the relevant scale for SC which is small due to the competition and we are deep inside the layer.

Cooper pairs

We hope to get „boson" = the bound state of 2 fermions to remove the contradiction with the L-criterion which is possible in 3d iff $g > g_c$ while $g_{(eff)}$ is small.

BUT the presence of the F-sphere changes everything (12)

Consider the toy model: 2 $\bar{e}$ + the blocking |FS⟩



$\vec{p}-\vec{p}=0$ for the s-state, the lowest total momentum

$\uparrow$ & $\downarrow$ - the singlet state

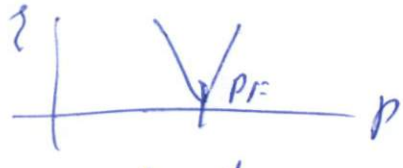
Additional ~~attractions~~ assumptions: 1) the in-layer attraction

2.) these 2 $\bar{e}$s interact <u>only</u> with each other.

<u>Reminder:</u> energy of q-particles in the F system

$$\xi = V|p-p_F|$$

V – (renormalized) F-velocity



The Hamiltonian

$$\hat{H} = \underbrace{\hat{H}_0(r_1) + \hat{H}_0(r_2)}_{\text{single-particles}} + \overbrace{U(r_1, r_2)}^{\text{interactions}}$$

in q-space: $\hat{H}_0 \to \xi(\vec{p})$; $U = \begin{cases} -g & \text{inside the layer} \\ 0 & \text{other-wise} \end{cases}$

Start with free particles (skip „–>")

$$\psi_p(r) = \frac{1}{\sqrt{V}} \exp\left(i\frac{pr}{\hbar}\right)$$

and try to find a solution in the form

$$\Psi(1,2) = \sum_p c_p \cdot \psi_{p,\uparrow}(r_1)\, \psi_{-p\downarrow}(r_2)$$

The stationary Sch- eq.

$$\hat{H}\Psi(1,2) = E\,\Psi(1,2)$$

Insert the trial $\Psi(1,2)$ and go to the q-space:

$$\Rightarrow \quad 2\xi(p)\, c_p + (-g)\sum_{p'\in \text{layer}} c_{p'} = E\, c_p \quad \text{at } p\in\text{layer}$$

Find the unknown coefficient

$$\sum_{p\in L} \Big| \quad c_p = g\,\frac{\sum_{p'\in L} c_{p'}}{2\xi(p) \underbrace{-E}_{2\Delta}} \equiv \frac{g}{2}\,\frac{\sum_{p'\in L} c_{p'}}{\xi(p)+\Delta}$$

$$\Rightarrow \quad \cancel{\sum_{p\in L} c_p} = \frac{g}{2}\sum_{p\in L}\frac{1}{\xi(p)+\Delta}\,\cancel{\sum_{p'\in L} c_{p'}}$$

Change $\sum_p \to \overset{\text{const}}{\frac{\nu}{2}} \int d\xi$ with $\frac{\nu}{2}$ – the DoS at ε_F per one spin

Note that we have effectively reduced the system dimensionality since the SC occurs only close to ε_F

$$\Rightarrow \quad 1 = \frac{g\nu}{2}\,\frac{1}{2}\int_{-\hbar\omega_D \text{ – for } h^+}^{\hbar\omega_D \text{ – for } \bar{e}} \frac{d\xi}{|\xi|+\Delta} = \frac{g\nu}{2}\log\frac{\hbar\omega_D+\Delta}{\Delta}$$

Δ – the new scale due to attraction is expected to be $\sim k_B T_c \ll \hbar\omega_D$

$$\Rightarrow \quad \log\frac{\hbar\omega_D}{\Delta} \simeq \frac{2}{g\nu}; \quad \boxed{\Delta \simeq \hbar\omega_D \exp\left(-\frac{2}{g\nu}\right)}$$

Typically, $gV \le 0.3 \Rightarrow \Delta \ll \hbar\omega_D$ - verified (14)

And we found the 2-particle state with

$$E = -2\Delta < 0 \, !$$

Note that the negative energy is required for b-state

We see that $\xi \to \xi + \Delta$ gap in the spectrum.

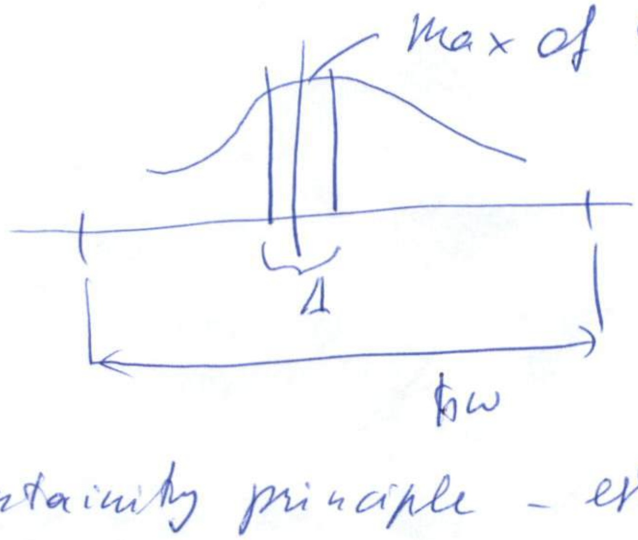
Properties

1) Δ is nonperturbative in g and appears even at small g (no condition like $g > g_c$)

2) $\Delta \propto \hbar\omega_D$ - the gap knows about the phonons, as expected

3.) solution for the C_p

$$C_p \sim \frac{1}{\xi(p) + \Delta}$$ - is again angle-independent (s-state) and $\searrow$ at $\xi(p) > 0$



$\Rightarrow$ indeed, we need only small energies, i.e. the model is verified

4.) uncertainty principle - estimate the size of the pair

$$\Delta X \sim \Delta t \cdot V_F \sim \frac{\hbar}{\Delta} V_F \sim \frac{\hbar}{k_B T_c} V_F \sim 1\,\mu m \gg a$$

$\Rightarrow$ the pair is macro-object and there are many overlap. many pairs, the MBP is further needed.