Density matrix, distr. function

Classically, prob. to be in a state with energy E_i

$$P_{cl}(E_i) = \frac{1}{Z} e^{-\beta E_i}$$

Thermodynamics $O = \sum_{conf} O(conf) e^{-\beta E(conf)}/Z$ ($conf = i$)

Stat. phys $O(t) = \sum_{conf} P(conf;t)\, O(conf)$

Quantum mechanics

$$O = \sum_i \langle i | \hat{O} | i \rangle e^{-\beta E_i}/Z = Tr\, \hat{\rho}\, \hat{O}$$

$$\hat{\rho} = \frac{1}{Z} e^{-\beta \hat{H}}, \quad Tr\,\hat{\rho} = 1 \Rightarrow Z = Tr\, e^{-\beta \hat{H}}$$

Partition function

In general: $\hat{\rho} = \frac{1}{Z} \sum |i\rangle \rho_i \langle i|$
(out-of-equilibrium)

2nd Quantization

$\{\hat{a}_\mu, \hat{a}^+_{\mu'}\} = \delta_{\mu\mu'}$ ← Creation/Anihilation in a single-particle state "μ".

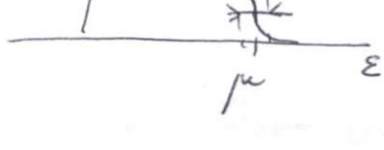
Fermions

Non-interacting particles ↙ number operator

$$\hat{H} = \sum \varepsilon_\mu \hat{a}^+_\mu \hat{a}_\mu = \sum \varepsilon_\mu \hat{n}_\mu \to \sum \xi_\mu \hat{n}_\mu, \quad \xi_\mu = \varepsilon_\mu - \mu$$

$$n_\lambda \equiv \frac{1}{Z} Tr\, \hat{n}_\lambda e^{-\beta \hat{H}} = \frac{\sum \langle n_1 \ldots | \hat{n}_\lambda e^{-\beta \sum \xi_\mu \hat{n}_\mu} | n_1 \ldots \rangle}{\sum \langle \{\} | \{\} \rangle}$$

$$= \frac{\prod_{\mu \neq \lambda} \langle n_\mu | e^{-\beta \xi_\mu \hat{n}_\mu} | n_\mu \rangle \cdot \langle n_\lambda | e^{-\beta \xi_\lambda \hat{n}_\lambda} | n_\lambda \rangle}{\prod_\mu \langle n_\mu | e^{-\beta \xi_\mu \hat{n}_\mu} | n_\mu \rangle} = f(\varepsilon_\lambda - \mu)$$



Distr. function

$$\langle \hat{a}^+_\mu \hat{a}_\lambda \rangle_{\mu\neq\lambda} = 0 \qquad \sum_{n=0,1} \langle n| e^{-\beta\xi \hat{n}} \hat{a} |n\rangle = \langle 1| e^{-\beta\xi}|0\rangle = 0$$

or $\hat{a}_\mu \to \hat{a}_\mu e^{i\varphi_\mu}$

$$\langle \hat{a}^+_\mu \hat{a}_\lambda \rangle = e^{i(\varphi_\lambda - \varphi_\mu)} \langle \hat{a}^+_\mu \hat{a}_\lambda \rangle_{\lambda\neq\mu} = 0$$

$$\boxed{\langle \hat{a}^+_\mu \hat{a}_\lambda \rangle = \delta_{\mu\lambda} f(\varepsilon_\mu)}$$

Scattering states

$$\hat{\psi}(x) = \sum_\mu \psi_\mu(x) \hat{a}_\mu \Rightarrow \{\hat{\psi}(x), \hat{\psi}^+(x')\} = \delta(x-x')$$

Compare
$\langle p,\alpha|p',\alpha'\rangle = 2\pi\hbar\,\delta_{\alpha\alpha'}\delta(p-p')$ for $p,p'>0$
$\langle p|p'\rangle = 2\pi\hbar\,\delta(p-p')$, $\forall p$

scat. state

$$\langle \varepsilon,\alpha|\varepsilon',\alpha'\rangle = \nu^{-1}_\alpha(\varepsilon)\delta_{\alpha\alpha'}\delta(\varepsilon-\varepsilon')$$

Creation op.

$$\langle 0| \hat{a}^+_\alpha(\varepsilon) \hat{a}_{\alpha'}(\varepsilon')|0\rangle = \nu^{-1}_\alpha(\varepsilon)\delta_{\alpha\alpha'}\delta(\varepsilon-\varepsilon')$$

$$\boxed{\{\hat{a}_\alpha(\varepsilon), \hat{a}^+_{\alpha'}(\varepsilon')\} = \nu^{-1}_\alpha(\varepsilon)\delta_{\alpha\alpha'}\delta(\varepsilon-\varepsilon')}$$

$$\hat{H} = \int dx\, \hat{\psi}^+(x) \hat{h} \hat{\psi}(x), \text{ for ex. } \int dx\, \hat{\psi}^+(x)\left[-\frac{\hbar^2}{2m}\partial_x^2 + V(x)\right]\hat{\psi}(x)$$

$$= \int dx \int d\varepsilon d\varepsilon' \sum_{\alpha\alpha'} \psi_{\alpha\varepsilon}(x)\psi_{\alpha'\varepsilon'}(x)\, \nu_\alpha(\varepsilon)\nu_{\alpha'}(\varepsilon')\, \varepsilon'\, \hat{a}^+_\alpha(\varepsilon)\hat{a}_{\alpha'}(\varepsilon')$$

$$\boxed{\hat{\psi}(x) = \sum_\alpha \int d\varepsilon\, \nu_\alpha(\varepsilon)\psi_{\alpha\varepsilon}(x)\hat{a}_\alpha(\varepsilon)} \text{ - over scat. states}$$

$$= \sum_\alpha \int d\varepsilon\, \nu_\alpha(\varepsilon)\,\varepsilon\, \hat{a}^+_\alpha(\varepsilon)\hat{a}_\alpha(\varepsilon)$$

Distr. function (for scat. states)

To reduce to the known form rescale

$$\nu_\alpha^{1/2}(\varepsilon)\,\hat{a}_\alpha(\varepsilon) \to \hat{a}_\alpha(\varepsilon)$$

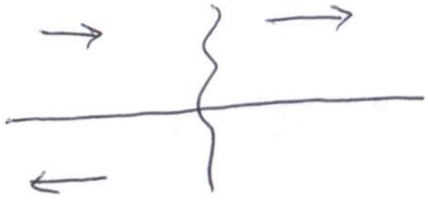
Result

$$\begin{cases} \hat{\psi}(x) = \sum_\alpha \int d\varepsilon\, \nu_\alpha^{1/2}(\varepsilon)\, \psi_{\alpha\varepsilon}(x)\, \hat{a}_\alpha(\varepsilon) \\ \{\hat{a}_\alpha(\varepsilon), \hat{a}^+_{\alpha'}(\varepsilon')\} = \delta_{\alpha\alpha'}\delta(\varepsilon-\varepsilon') \\ \mathcal{H} = \sum_\alpha \int d\varepsilon\, \varepsilon\, \hat{a}^+_\alpha(\varepsilon)\hat{a}_\alpha(\varepsilon) \end{cases}$$

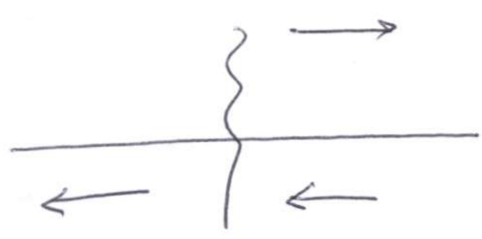
Therefore $\langle \hat{a}^+_\alpha(\varepsilon)\hat{a}_{\alpha'}(\varepsilon')\rangle = \delta_{\alpha\alpha'}\delta(\varepsilon-\varepsilon')\, f_\alpha(\varepsilon)$

distr. function

2 channels $\Leftrightarrow$ 2 scat. states (labelled with α)



$\alpha=1$



$\alpha=2$

reservoirs at $\pm\infty$

Current op.

$$\hat{j} = \frac{e\hbar}{2mi} \hat{\psi}^{+} \partial_x \hat{\psi} + h.c.$$

region 1 region 2

$$\hat{\psi}(x<1) = \int d\varepsilon \sqrt{\frac{v(\varepsilon)}{2}} \cdot \left[\hat{a}_1(\varepsilon)\left(e^{ipx} + r e^{-ipx}\right) + \hat{a}_2(\varepsilon)\, t' e^{-ipx} \right]$$

$$= \int d\varepsilon \sqrt{\frac{v}{2}} \left[\hat{a}_1 e^{ipx} + \hat{b}_1 e^{-ipx} \right]$$

$$\psi(x<2) = \int d\varepsilon \sqrt{\frac{v(\varepsilon)}{2}} \cdot \left[\hat{a}_1(\varepsilon)\, t e^{ipx} + \hat{a}_2\left(e^{-ipx} + r' e^{ipx}\right) \right]$$

$$= \int d\varepsilon \sqrt{\frac{v}{2}} \left[\hat{a}_2 e^{-ipx} + \hat{b}_2 e^{ipx} \right]$$

$$\hat{b}_2 = t\hat{a}_1 + r'\hat{a}_2$$

$$\hat{b}_1 = r\hat{a}_1 + t' a_2$$

$$\begin{pmatrix} \hat{b}_1 \\ \hat{b}_2 \end{pmatrix} = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix} \begin{pmatrix} \hat{a}_1 \\ \hat{a}_2 \end{pmatrix}, \quad \hat{b}_\alpha = \sum_\beta S_{\alpha\beta} \hat{a}_\beta$$

$$\hat{j}(x<1) = \frac{1}{2}\int d\varepsilon\, d\varepsilon' \sqrt{vv'}\, \frac{ev}{2} \left[\hat{a}_1 e^{ipx} + \hat{b}_1 e^{-ipx} \right]^{+} \left[\hat{a}_1' e^{ip'x} + \hat{b}_1' e^{-ip'x} \right] + h.c.$$

$$j(x<1) = \frac{e}{2} \int d\varepsilon\, \nu(\varepsilon) v(\varepsilon) \left[f_1 - \sum_\beta |S_{\alpha\beta}|^2 f_\beta \right]$$

$$= \frac{e}{2\pi\hbar} \int d\varepsilon\, T(\varepsilon)\left(f_1 - f_2\right)$$

$$\text{If } T(\varepsilon) \approx T(\mu): \quad j = \frac{e}{2\pi\hbar} T(\bar{\mu})(\mu_1 - \mu_2) = \underbrace{\frac{e^2}{2\pi\hbar} T(\mu)}_{G = G_Q \cdot T(\mu)} \cdot V$$