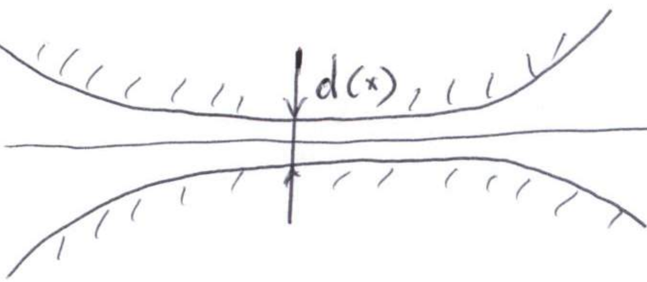
Ballistic Transport

"Waveguide" connecting 2 reservoirs

$$-\frac{\hbar^2}{2m}\Delta\psi = \varepsilon\psi$$



$$\psi(x,y) = \chi_n(y, d(x))\cdot\tilde{\psi}(x)$$

$$\chi_n \sim \begin{cases} \sin\frac{2\pi n}{d(x)}y \\ \cos\frac{(2n+1)\pi}{d(x)}y \end{cases}$$

Typical scale of the change $d(x)$ is L.

Assuming $\frac{\partial_x\chi}{\chi} \ll \frac{\partial_x\tilde{\psi}}{\tilde{\psi}}$ (smooth boundary)

$$-\frac{\hbar^2}{2m}\left(\partial_x^2+\partial_y^2\right)\chi_n(y,d(x))\tilde{\psi} = \varepsilon\chi_n(y,d(x))\tilde{\psi}$$

$$-\frac{\hbar^2}{2m}\partial_x^2(\chi_n\tilde{\psi}) + \varepsilon_n(x)\chi_n\tilde{\psi} = \varepsilon\chi_n\tilde{\psi}$$

$$\approx \quad -\frac{\hbar^2}{2m}\partial_x^2\tilde{\psi}_n + \varepsilon_n(x)\tilde{\psi}_n = \varepsilon\tilde{\psi}_n$$

$p_n \sim \sqrt{2m(\varepsilon-\varepsilon_n)}\cdot L \gg 1$ $(\lambda_n \ll L)$ means WKB!

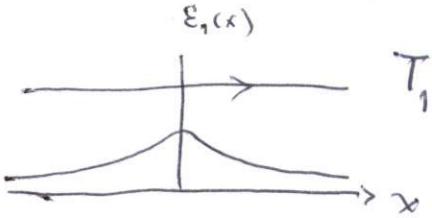
$$\tilde{\psi}_n(x) = \frac{1}{\sqrt{v_n(x)}}\begin{cases} e^{\pm\frac{i}{\hbar}\int^x dx'\sqrt{2m(\varepsilon-\varepsilon_n(x'))}}, & \varepsilon > \varepsilon_n(x) \\ e^{\pm\frac{1}{\hbar}\int^x dx\sqrt{2m(\varepsilon_n(x)-\varepsilon)}}, & \varepsilon < \varepsilon_n(x) \end{cases}$$

$$\boxed{\psi_n(x,y) \approx \chi_n(y, d(x))\cdot\tilde{\psi}_n(x)}$$

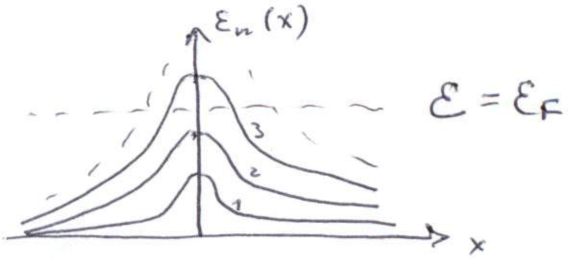
$$\varepsilon_n(x) = \left[\frac{\pi n}{d(x)}\right]^2\frac{\hbar^2}{2m}$$

BT - 2

(1)

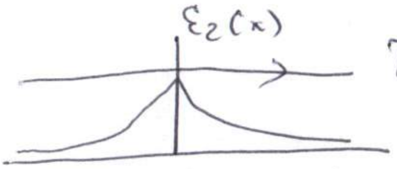


$T_1 = 1$

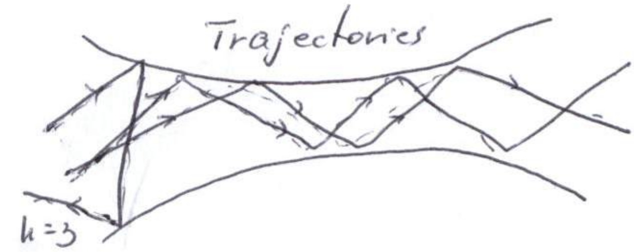


$\mathcal{E} = \mathcal{E}_F$

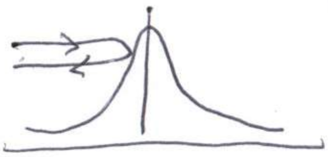
(2)



$T_2 = 1$



(3)



$T_3 = 0$

⋮

$T_{n \geq 3} = 0$

$$G = G_Q \operatorname{Tr} \hat{T}(\mathcal{E}_F) = G_Q \cdot N$$

$N = \#$ of "open" channels

i.e. $T_i(\mathcal{E}_F) = 1$

Applying gate voltage we change width of the waveguide. Open channels $\mathcal{E}_F > \frac{\hbar^2}{2m}\left(\frac{\pi n}{d_{min}}\right)^2$, $d_{min} = d_{min}(V_g)$