Lectures on mesoscopic physics - Exercise sheet 1

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Scattering from a square barrier and Landauer conductance

Consider a particle (mass m) being scattered from a square barrier: $V(x) = U > 0$ for $0 < x < a$ and $V(x) = 0$ otherwise. Show that the transmission amplitude t is given by

$$t = 2ie^{-ika}\frac{f}{1-f^2}\cdot\frac{1}{\sin(\tilde{k}a + i\lambda)},$$

where k is the wavenumber of the particle (coming in from the left), $f \equiv \frac{\tilde{k}}{k}$, $e^{-\lambda} \equiv \frac{1-f}{1+f}$, and $\tilde{k} = \sqrt{k^2 - k_{\mathrm{thr}}^2}$ is the wavenumber inside the barrier, with a threshold k_{thr} to be determined from m, U.

Hint: Proceed by first writing down the scattering state (incoming from the left) in the three regions $x < 0$, $0 < x < a$, $x > a$. Assume that $k > k_{\mathrm{thr}}$ (the formula above will also be valid for the opposite case, if continued analytically). Then write down the conditions for continuity of the wavefunction and its derivative at $x = 0$ (Eqs. (1) & (2)) and $x = a$ (Eqs. (3) & (4)). Eliminate the transmission and reflection amplitudes t and r by combining Eqs. (1)&(2) and (3)&(4). Use this to obtain the coefficients of the right- and left-moving plane waves inside the barrier, A and B, and from them the transmission amplitude t. Introduce abbreviations $z = e^{i\tilde{k}a}$ and f from the start to keep the formulas brief. In the end, use $ze^{-\lambda} - z^{-1}e^{\lambda} = 2i\sin(\tilde{k}a + i\lambda)$.

Now discuss the resulting transmission probability $\mathcal{T} \equiv |t|^2$: Show that it can be written purely as a function of k/k_{thr}, with the single dimensionless parameter $k_{\mathrm{thr}}a$. Plot $\mathcal{T}(k/k_{\mathrm{thr}})$ for different values of $k_{\mathrm{thr}}a$ (possibly with the help of a computer), and explain the physical regimes described by these values. At which points does one get transmission resonances above the barrier?

Assume that this barrier is inserted inside an otherwise clean quantum wire. Sketch the total conductance I/V for the situation $\mu_L = \mu + eV$, $\mu_R = \mu$, as a function of μ, for infinitesimal voltage V and for rather "large voltage" (compared to what?)! We are assuming a model of non-interacting electrons and zero temperature. What would be the effect of raising the temperature?