Lectures on mesoscopic physics - Exercise sheet 2

13.6.2014

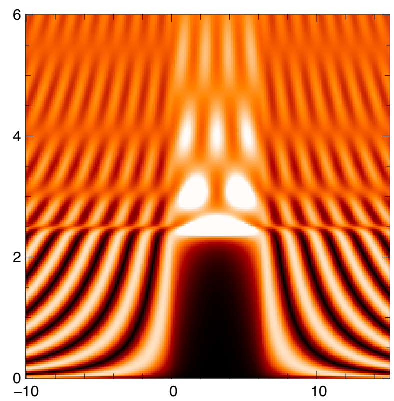
1. Local density of states from scattering states

A physical picture of the long-living states in an open quantum system (i.e. a system without true bound states) can be obtained by looking at the "local density of states". In general, it is defined by $\mathrm{DOS}(x,E) = \sum_n |\psi_n(x)|^2 \delta(E - E_n)$, where the ψ_n are the energy eigenstates belonging to the energies E_n (This is a density of states per unit energy interval and unit length). For a 1D scattering situation, it can be expressed as $\mathrm{DOS}(x,E) = \pi^{-1}\tilde{D}(k(E))(dk/dE)$, where $E = \hbar^2 k^2/2m$ is the free dispersion relation, $\hbar k(E) \equiv \sqrt{2mE}$, and the normalized DOS $\tilde{D}$ is related to the scattering states at that particular energy by

$$\tilde{D}(k) \equiv \frac{1}{2}(|\psi_k(x)|^2 + |\psi_{-k}(x)|^2).$$

Here $\psi_k(x)$ is a scattering state that is incoming from the left (right) for $k > 0$ ($k < 0$), and where the incoming wave has an amplitude 1, for correct normalization. With this definition, we would have $\tilde{D}(k) \equiv 1$ in the absence of any scatterer, i.e. $\tilde{D}$ measures the deviations from the unperturbed density of states of 1D electrons.

Calculate, plot and discuss $\tilde{D}$ for a scattering situation of your choice (e.g. delta barrier, Fabry Perot, or square barrier). Hint: For the square barrier it should look something like this (as a function of position, horizontal, and energy, vertical)



2. Transfer matrices, applied to Fabry Perot and three barriers

Write the amplitudes of right- and left-going plane waves e^{ikx} and e^{-ikx} as entries of a twodimensional vector. A transfer matrix M produces the amplitudes to the right of a barrier, when applied to those to the left of the barrier.

(i) Consider a single barrier, with transmission/reflection amplitude t/r (the same for both directions, i.e. for $k > 0$ and $k < 0$). Check that the transfer matrix (at the given energy) is

$$M = \begin{pmatrix} t - \frac{r^2}{t} & \frac{r}{t} \\ -\frac{r}{t} & \frac{1}{t} \end{pmatrix},$$

by using the relations $\begin{pmatrix} t \\ 0 \end{pmatrix} = M \begin{pmatrix} 1 \\ r \end{pmatrix}$ (should be clear) and $\begin{pmatrix} r \\ 1 \end{pmatrix} = M \begin{pmatrix} 0 \\ t \end{pmatrix}$ (why?).

(ii) Now treat the Fabry Perot by multiplying transfer matrices. Hint: The amplitudes of the right/left-moving plane waves are multiplied by a factor e^{ikl}/e^{-ikl}, when proceeding from the first barrier (with t_1, r_1 and corresponding M_1) to the second barrier (with M_2). Second hint: The total matrix M may still be written in the manner shown in (i), enabling you to read off t. Note: Your result will contain an extra factor e^{ikl} with respect to the definition used in our lecture, corresponding to the phase picked up in free propagation.

(iii) Now apply the same technique to the three-barrier problem (the "double quantum dot"). This is the simplest way to obtain the transmission amplitude t for three barriers with $t_{1,2,3}/r_{1,2,3}$ and distances l_1 (between 1 and 2) and l_2 (between 2 and 3). Plot the result and discuss it (e.g. what happens when $l_1 = l_2$? what happens as a function of l_2 for fixed l_1? what happens when you keep $t_{1,3}$ fixed and change t_2?). You might also (numerically) check that the k-average of $|t|^2$ is indeed smaller than the incoherent result, for almost any $l_{1,2}$.