**Lectures on mesoscopic physics - Exercise sheet 3**

## 1. Wave packet dynamics

Given a wave packet $\psi_0(x)$ to the left of a scattering region, we can expand it in terms of right-going scattering states (assuming it contains only right-moving plane waves):

$$\psi_0(x) = \int_0^\infty dk\, c(k)\psi_k(x),$$

where the coefficients $c(k)$ can be obtained from an expansion in plane waves (assuming the scattering states to be normalized as "$1 \cdot e^{ikx}$"):

$$c(k) = \int_{-\infty}^{+\infty} \frac{dx}{2\pi} e^{-ikx}\psi_0(x).$$

The subsequent time-evolution is given by $\psi(x,t) = \int_0^\infty dk c(k)\psi_k(x)e^{-i\omega_k t}$, where $\omega_k = \hbar k^2/2m$. Use the scattering states obtained for any of our example scattering regions (two or three barriers or square well) to show and discuss the time-evolution for a wave-packet centered around $\bar{k}$, e.g. $c(k) = \theta(\delta k/2 - |k - \bar{k}|)$ (where we do not care about normalization here). Plot the cases $\delta kl \gg 1$ and $\delta kl \ll 1$, where $l$ is the size of the structure (e.g. distance of barriers of the Fabry-Perot). Compare with our discussion of the effect of energy-averaging of the transmission probability.

## 2. Transmission phase and time-delay

Consider a transmission $t(k) = |t|e^{i\varphi}$, whose magnitude can be assumed constant in the k-interval of interest. However, we want to take into account the dependence of the transmission phase on energy or wavenumber, $\varphi(k)$.

Look at the transmitted part of a wave packet, $\psi(x,\tau) = \int dk\, e^{ikx}t(k)c(k)$, where the coefficients $c(k)$ are assumed to be a smooth function centered around $\bar{k}$, with a width $\delta k$. Approximate $\varphi(k)$ to linear order in $(k - \bar{k})$, and use the method of stationary phase to find the center-of-mass position of the wave packet, $\bar{x}(\tau)$. This will deviate from the "free result" $\bar{x} = v\tau$ by an amount $\Delta x$ that can be expressed through the derivative of $\varphi$ with respect to $k$. Convert this spatial offset into a delay time $\Delta\tau$, and write that in terms of an energy-derivative of $\varphi$.

Plot the transmission phase and the delay time for an example scatterer (e.g. Fabry-Perot).