Lectures on mesoscopic physics - Exercise sheet 4

4.7.2014

1. Connecting the scattering and the sequential tunneling pictures

Consider two barriers in one dimension, where we found a transmission probability of the form (neglecting an irrelevant phase):

$$\mathcal{T} = \frac{\mathcal{T}_L \mathcal{T}_R}{1 + \mathcal{R}_L \mathcal{R}_R - 2\sqrt{\mathcal{R}_L \mathcal{R}_R}\cos(2kl)},$$

where L, R refer to the left and the right barrier, l is the distance and k the wavenumber. Expand this expression around a k-value that belongs to a maximum in $\mathcal{T}$, to quadratic order in k (in the denominator), and convert k into energy E by the approximation $k - k_0 \approx (E - E_0)/\hbar v_F$. Now rewrite everything in terms of the rates

$$\Gamma_{L,R} \equiv \frac{v_F}{2l} \mathcal{T}_{L,R},$$

and interpret these rates physically.

Obtain the current I flowing through one such transmission resonance (at $T = 0$) according to the Landauer formula (for one spin direction), for a voltage window that extends over "the whole resonance" (approximate the resulting integral by taking the limits to $\pm\infty$). Show that it is exactly equivalent to the result one obtains by solving the master equation for sequential tunneling transport through one level, with the given rates.

Now calculate the current that flows if the small voltage window does not contain the resonance (and we can nevertheless still neglect other resonances), as a function of the energetic distance between the Fermi energy and the discrete level that forms the resonance. Show that it is compatible with the result from a co-tunneling picture, i.e. calculating a total tunneling rate through the level with the help of Fermi's Golden Rule in second order (as discussed in the lectures).