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MÜNCHEN

Lehrstuhl für Theoretische Nanophysik **SS 2014**
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9th Exercise Sheet Many-Body Physics

Will be discussed on Fri June 20th.

Exercise 1: BCS pairing amplitudes FW 10.7

Compute the pairing amplitudes $F_k^* \equiv \langle \mathbf{0}|a^\dagger_{\boldsymbol{k},\uparrow}a^\dagger_{-\boldsymbol{k}\downarrow}|\mathbf{0}\rangle$ and $F_k \equiv \langle \mathbf{0}|a_{\boldsymbol{k},\uparrow}a_{-\boldsymbol{k}\downarrow}|\mathbf{0}\rangle$ in the BCS ground state. Sketch their behavior as a function of k, and show that they vanish in the normal ground state.

Exercise 2: BCS wave function partly FW 10.8

The superconducting ground state was orginally derived with a variational principle by considering the state

$$|\varphi\rangle = \prod_{\boldsymbol{k}} (u_{\boldsymbol{k}} + v_{\boldsymbol{k}} a^\dagger_{\boldsymbol{k}\uparrow} a^\dagger_{-\boldsymbol{k}\downarrow})|0\rangle, \tag{1}$$

where the product is over all $\boldsymbol{k}$ and $|0\rangle$ is the no-particle state.

a. Show that $|\varphi\rangle$ is normalized if $u_k^2 + v_k^2 = 1$.

b. By varying $u_{\boldsymbol{k}}$ and $v_{\boldsymbol{k}}$ subject to the constraint $u_{\boldsymbol{k}}^2 + v_{\boldsymbol{k}}^2 = 1$, derive the gap equation as the condition for minimum thermodynamic potential.

c. Construct the creation and annihilation operators for the Bogoliubov quasiparticles by the transformation

$$\begin{aligned} a_{\boldsymbol{k}\uparrow} &= u_{\boldsymbol{k}}\gamma_{\boldsymbol{k}0} + v_{\boldsymbol{k}}\gamma^\dagger_{\boldsymbol{k}1} \\ a^\dagger_{-\boldsymbol{k}\downarrow} &= -v_{\boldsymbol{k}}\gamma_{\boldsymbol{k}0} + u_{\boldsymbol{k}}\gamma^\dagger_{\boldsymbol{k}1} \end{aligned} \tag{2}$$

Show that this transformation is canonical, *i.e.*, that the fermionic anticommutation relations are preserved.

d. Show that Eq. 1 is the ground state of the Bogoliubov quasiparticles.

e. Analyze the gap equation for (i) $T \to 0$, and (ii) $T \to T_c$.