Coherent states path int. $\mathcal{H} = \mathcal{H}(b^+, b)$

Fermions		Bosons (singl state)
$\hat{f}\|f\rangle = f\|f\rangle$	Basis (2nd quant.)	$\hat{b}\|b\rangle = b\|b\rangle$, $[\hat{b}, \hat{b}^+] = 1$
1. $\|f\rangle = e^{-f\hat{f}^+}\|0\rangle$ if $f^2 = 0$		1. $\|b\rangle = e^{b\hat{b}^+}\|0\rangle$
2. $\langle f\|f'\rangle = e^{\bar{f}f'}$	Scalar product	2. $\langle b\|b'\rangle = e^{\bar{b}b'}$ Campbell-Hausdorff
3. $\int d\bar{f}\, df\, e^{-\bar{f}f}\|f\rangle\langle f\| = \hat{1}$	Closure	3. $\int \frac{d\bar{b}\, db}{\pi} e^{-\bar{b}b}\|b\rangle\langle b\| = \hat{1}$
if $\int df = \int d\bar{f} = 0$, $\int df\, f = 1$ and everything anti-commutes		
4. $\int d\bar{f}\, df\, e^{-\bar{f}f}\langle -f\|\hat{A}\|f\rangle = \hat{1}$	Trace	4. $Tr\,\hat{A} = \int \frac{d\bar{b}\, db}{\pi} e^{-\bar{b}b}\langle b\|\hat{A}\|b\rangle$

$$\langle b|\hat{U}|b'\rangle = \int d^{N-1}b\, \Pi \langle b_i|b_{i-1}\rangle\, e^{-\bar{b}_i b_i}\, e^{-i\mathcal{H}(\bar{b}_i, b_{i-1})\Delta}$$

$$= \int \mathcal{D}b\, e^{\sum \bar{b}_i (b_{i-1} - b_i) - i\mathcal{H}_i \Delta}$$

$$= \int_{b_{fin} = b} \mathcal{D}b\, e^{i\int dt\, [\bar{b}\, i\partial_t b - \mathcal{H}]}$$

$$\langle \hat{T}\, \Pi\, \hat{b}(t_i)\, \hat{b}^+(t_i')\rangle = \int_{b_{in} = b_{final}}^{b_{in} = b'} \mathcal{D}b\, e^{i\int_K dt\, L}\, \Pi\, b(t_i)\, \bar{b}(t_i')$$

$$\langle T\, \Pi\, \hat{f}(t_i)\, \hat{f}(t_i')\rangle$$

$$= \int_{f_{in} = -f_{final}} \mathcal{D}f\, e^{iS_K}\, \Pi f(t_i)\, \bar{f}(t_i')$$

Fields $\{\hat{\psi}(x), \hat{\psi}^{+}(x')\} = \delta(x-x')$

Basis

$$|\psi\rangle = e^{-\int dx\, \psi(x)\hat{\psi}^{+}(x)}|0\rangle$$

$\psi(x)\psi(x') = -\psi(x')\psi(x) \ldots$

$\langle\psi|\psi'\rangle = e^{\bar{\psi}\psi'}, \quad \bar{\psi}\psi' = \int dx\, \bar{\psi}(x)\psi(x')$ Sc. prod.

$\int \mathcal{D}\psi\, e^{-\bar{\psi}\psi}|\psi\rangle\langle\psi| = \mathbb{1}$ Closure

$\int \mathcal{D}\psi\, e^{-\bar{\psi}\psi}\langle-\psi|\hat{A}|\psi\rangle = \operatorname{tr}\hat{A}$ Trace

$\langle T \prod \hat{\psi}(x_i t_i)\, \hat{\psi}^{+}(x_i' t_i')\rangle$

$= \int \mathcal{D}\psi\, e^{i\int_x L} \prod \psi\bar{\psi}, \quad L = \bar{\psi}\, i\partial_t \psi - \mathcal{H}(\bar{\psi}, \psi)$

$\psi_{final} = -\psi_{in}$

Fields $[\hat{\Phi}(x), \hat{\Phi}^{+}(x')] = \delta(x-x')$

$\hat{\phi}(x)|\phi\rangle = \phi(x)|\phi\rangle$

Cont. limit $(\hat{\phi}(x_i) \leftarrow \hat{b}_i)$

$\hat{\phi}_i\, e^{\sum \phi_j \hat{\phi}_j^{+}}|0\rangle = \phi_i\, e^{\sum \phi_j \hat{\phi}_j^{+}}|0\rangle$

1) $|\phi\rangle = e^{\int dx\, \phi(x)\hat{\phi}^{+}(x)}|0\rangle$ (Could be done via Campb.-Haus.)

2) $\langle\phi|\phi'\rangle = e^{\bar{\phi}\phi'}, \quad \bar{\phi}\phi' = \int dx\, \bar{\phi}(x)\phi'(x)$

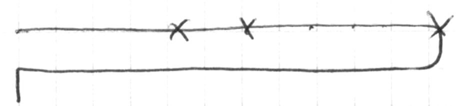
3) $\int \mathcal{D}\phi\, e^{-\bar{\phi}\phi}|\phi\rangle\langle\phi| = 1$

4) $\int \mathcal{D}\phi\, e^{-\bar{\phi}\phi}\langle\phi|\hat{A}|\phi\rangle = \operatorname{tr}\hat{A}$

$\langle T \prod \hat{\phi}(x_i t_i)\, \hat{\phi}^{+}(x_i' t_i')\rangle$

$= \int \mathcal{D}\phi\, e^{i\int_x dt\, L} \prod \phi_i \bar{\phi}_i, \quad L = \bar{\phi}\, i\partial_t \phi - \mathcal{H}(\bar{\phi}, \phi)$

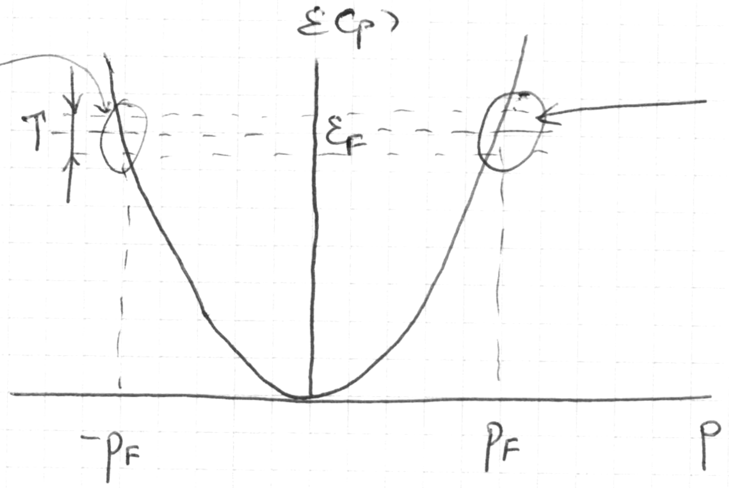
$\phi_{in} = \phi_{final}$



1D Fermions at low-T

$\mathcal{E}_L(p) \approx -v_F(p+p_F)$

$\hat{\mathcal{E}}_L = i v_F \partial_x$



$\mathcal{E}_R(p) \approx v_F(p-p_F)$

$\hat{\mathcal{E}}_R = -i v_F \partial_x$

Second quantization $\hat{\mathcal{H}}_R = -i v_F \int dx\, \hat{R}^+ \partial_x \hat{R}$

$$\{\hat{R}(x), \hat{R}^+(x')\} = \delta(x-x')$$

<u>Right-movers only</u> (at the moment)

$$S = \int dx\, dt \left[\bar{R}\, i\partial_t R - \mathcal{H}(\bar{R}, R) \right] = \int dx\, dt\, \bar{R}\, i\partial R$$

$$\partial \equiv \partial_t + v_F \partial_x$$

The Green function $\quad g(xt, x't') = -i \langle T_K \hat{R}(x,t) \hat{R}^+(x',t') \rangle$

Functional integral representation:

$$g(xt, x't') = -i \int \mathcal{D}\bar{R}\mathcal{D}R\; R(xt)\bar{R}(x't')\, e^{i\int_K \bar{R}\, i\partial R\, dx\, dt}$$

$$R(t_0) = -R(t_0 - i\beta)$$

$$\hat{g} = (i\partial)^{-1}$$

Fourier transform

$$\hat{g}_p = (i\partial_t - v_F p)^{-1}$$

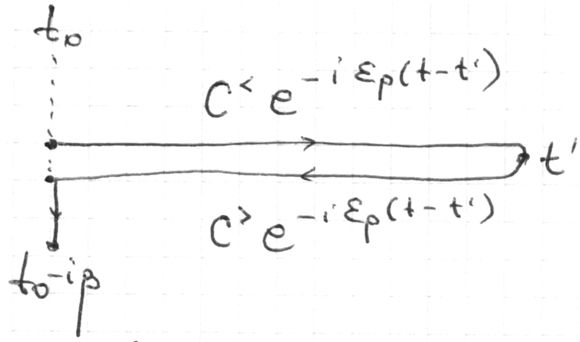
$$i\partial\,\hat{g} = 1$$

$$i(\partial_t + v_F\partial_x)\, g(xt, x't') = \delta(x-x')\,\delta(t-t')$$

$$(i\partial_t - v_F p)\, g_p(t,t') = \delta(t-t')$$

on Keldysh!

Inversion of ∂_t on Keldysh contour



discontinuity around t=

$$\begin{cases} g(t=t'+0) - g(t=t'-0) = -i \\ g(t_0, t') = -g(t_0 - i\beta, t') \end{cases}$$

or

$$\begin{cases} g^>(t\to t') - g^<(t\to t') = -i \\ g^<(t_0, t') = -g^>(t_0 - i\beta, t') \end{cases}$$

$$\begin{cases} C^> - C^< = -i \\ C^< = -C^> e^{-\beta\varepsilon_p} \end{cases}$$

$$\begin{cases} C^> = -i \dfrac{1}{1+e^{-\beta\varepsilon_p}} = -i(1-f_p) \\ C^< = i \dfrac{e^{-\beta\varepsilon_p}}{1+e^{-\beta\varepsilon_p}} = i f_p \end{cases}$$

Fermi dist. func:

$$f_p = \frac{1}{e^{\beta\varepsilon_p}+1}$$

$$g(x-x'; t, t') = -\frac{T}{2v_F} \sinh^{-1}(z - z' + i0\cdot S_{tt'})$$

$$z = \pi T\left(t - \frac{x}{v_F}\right), \quad S_{tt'} \equiv \mathrm{sgn}_K(t, t')$$

$$(i\partial)^{-1} \longleftrightarrow g(x-x'; t, t') \text{ on Keldysh}$$

This expression is valid <u>ONLY</u> for equilibrium.

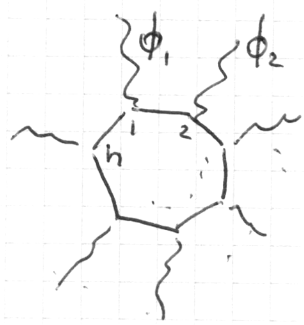
Loop cancellation

$$\frac{\int \mathcal{D}R\, e^{i\bar{R}(i\partial - i\phi)R}}{\int \mathcal{D}R\, e^{i\bar{R}(i\partial)R}} = \frac{\det(\partial-\phi)}{\det \partial} \qquad \Big| \; \int dx\, \phi(x,t) = 0$$

$$= e^{\mathrm{Tr}[\ln(\partial-\phi) - \ln \partial]} = e^{\mathrm{Tr}\ln(1-\partial^{-1}\phi)}$$

$$= e^{-\sum_{n=1}^{\infty} \frac{1}{n} \mathrm{Tr}[(i\partial)^{-1} i\phi]^n}$$

Each term in the exponent



$$\Gamma_n \sim \int d^n x\, d^n t \prod_{i=1}^{n} \phi(x_i,t_i) \prod_{i=1}^{n} g(z_i - z_{i+1})$$

$$z_i = \pi T\left(t_i - \frac{x_i}{v_F}\right)$$

$$g(z-z') \sim \frac{1}{\sinh(z-z')} \sim \frac{e^{+(z_1+z')}}{e^{2z}-e^{2z'}} = \frac{e^{+(z+z')}}{\xi - \xi'}$$

$$\Gamma_n \sim \int d^n x\, d^n t \prod_{i=1}^{n} \phi_i \prod_{i=1}^{n} \frac{1}{\xi_i - \xi_{i+1}}$$ ← Only symmetric part contributes.

$$\prod_{i=1}^{n} \frac{1}{\xi_i - \xi_{i+1}} = \frac{A_n(\xi_1 \ldots \xi_n)}{\prod_{i<j}(\xi_i - \xi_j)}$$

← anti-symm. polinomial

← common denominator

Δ_n = Van der Mond → $\frac{n^2-n}{2}$ order in ξ

A_n must be $\frac{n^2-n}{2} - n \geq 0$ (true for $n \geq 3$)

Minimum order of anti-sym. polinomial $= \frac{n^2-n}{2}$ (like VdM)

All symmetric form with $n>2$ are 0!

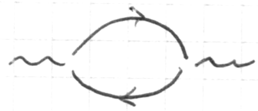
Math excercise: Jacobian

$$\int \mathcal{D}R\, e^{i\bar{R}(i\partial - i\phi)R} = J_\phi[\theta] \int \mathcal{D}R\, e^{i\bar{R}(i\partial - \partial\theta - i\phi)R}$$

$$R \to R e^{i\theta}$$

$$J_\phi^{-1}[\theta] = \frac{\int \mathcal{D}R\, e^{i\bar{R}(i\partial - \partial\theta - i\phi)R}}{\int \mathcal{D}R\, e^{i\bar{R}(i\partial - o\phi)R}} = \frac{J_o^{-1}[\theta + i\partial^{-1}\phi]}{J_o^{-1}[i\partial^{-1}\phi]}$$

$$J_o^{-1}[\theta] = e^{-\frac{i}{2}\partial\theta\, \Pi\, \partial\theta},$$



$$= \Pi^R = \frac{1}{2\pi}\frac{q}{\omega - v_F q}$$

$$J_o[\theta] = e^{-\frac{i}{4\pi}\partial\theta \cdot \partial_x\theta} \qquad \hat{\Pi} = -\frac{1}{2\pi}\partial_x \partial^{-1}$$

$$\Downarrow$$

$$J_\phi[\theta] = e^{iS[\theta] + \phi\frac{1}{2\pi}\partial_x\theta}$$

Change of variables $R = \chi e^{i\theta}$

$$\int \mathcal{D}R\, e^{i\bar{R}(i\partial - i\phi)R} = \int J_\phi[\theta]\, \mathcal{D}\theta\, \underbrace{\int \mathcal{D}\chi\, e^{-\chi\partial_t\chi}}_{\text{cancelled from average}}$$

$\theta(xt)$ – ordinary field $\qquad \theta_{in} = \theta_{final}$ – Bose

$\chi(t)$ – real Grassmann field $\qquad \chi_{in} = -\chi_{final}$ – Fermi

$$J_\phi[\theta] = e^{iS[\theta] + \phi\frac{1}{2\pi}\partial_x\theta}$$

$R = \chi e^{i\theta}$: $S_F[R] \to S[\theta]$, $\quad n(xt) \to \frac{1}{2\pi}\partial_x\theta$

$$S[\theta] = \frac{1}{4\pi}\theta\,\partial\partial_x\theta$$

Luttinger Liquid

Right- and left-movers:

$$S_0 = \frac{1}{4\pi}\left[\theta_R \partial_R \partial_x \theta_R - \theta_L \partial_L \partial_x \theta_L\right]$$

$$\partial_R = \partial_t + v_F \partial_x$$
$$\partial_L = \partial_t - v_F \partial_x$$

$$S_{int} = -\frac{V_0}{2}(n_R + n_L)^2 = -\frac{V_0}{2(2\pi)^2}\left[\partial_x(\theta_R - \theta_L)\right]^2$$

Action is quadratic $\Rightarrow$ Gaussian integrals.

The Green function

$$iG_{\eta\eta'}(xt, x't') = \int \mathcal{D}R\, \mathcal{D}L \;\; \psi_\eta(xt)\, \bar{\psi}_{\eta'}(x't')\, e^{i(S_0 + S_{int})}$$

$$\eta = R \;:\; R(xt) = \chi_R(t)\, e^{i\theta_R(xt)}$$

$$\eta = L \;:\; L(xt) = \chi_L(t)\, e^{i\theta_L(xt)}$$

$$n_R(xt) = \frac{1}{2\pi}\partial_x \theta_R(xt), \quad n_L(xt) = -\frac{1}{2\pi}\partial_x \theta_L(xt)$$