Compendium of lectures to be offered for the LMU elite masters program in ”Theoretical and mathematical physics”

Synopsis

[*new lectures specially conceived for this elite masters program are printed in italics*]

Basic Concepts (BC): in Physics: Classical mechanics; Classical electrodynamics; Quantum mechanics I; Statistical mechanics; **in Mathematics:** Functional analysis; Complex analysis; Manifolds and differential forms; Stochastics.

Mathematical Methods (MM): Partial differential equations; Lie groups and their representation theory; Topology; Probability theory; *Stochastic processes (Markov processes, martingales; Game theory*

Classical Mechanics (CM): *Symplectic geometry I; Symplectic geometry II (→FT,ST); Dynamical systems and nonlinear physics*

Quantum mechanics (QM): Quantum mechanics II, Many body theory (→FT*);* Mathematical quantum mechanics I, *Mathematical quantum mechanics II;*

Applied quantum mechanics (APM): Solid state theory; Quantum phase transitions; Quantum optics; Mesoscopic physics; Nonrelativistic quantum electrodynamics (→FT I); Superfluidity and Bose-Einstein-Condensation*; Theory of open quantum systems; Quantum information processing*

Statistical Physics (SP): Soft condensed matter theory; Theoretical biological physics; *Stochastic processes in physics and biology; Mathematical statistical mechanic; Stochastic integration and stochastic differential equations; Statistical mechanics (rigorous results)*

Field Theory (FT): Quantum field theory I (introduction); Quantum field theory II (standard model); Supersymmetry*;* Conformal field theory (→ST); *Renormalization group methods in condensed matter theory; Statistical field theory and renormalization; Mathematical gauge theory (→ST); Quantization/ axiomatic quantum field theory*

Relativity, Cosmology (RC): General relativity; Cosmology; Quantum theory in external fields; *Black holes; Riemannian and Semi-Riemannian Geometry (→FT, ST)*

String theory (ST): Conformal field theory (→FT); Introduction to string theory*; Complex and Kähler geometry; Moduli spaces (and strings)*

Course Contents (P: prequisites; L: possible lecturers)

Basic Concepts (Theoretical Physics)

Classical mechanics: Newton’s laws, mechanics of a point mass; Galilei invariance; inertial forces; two body- and central force problem; Lagrangian mechanics; symmetries and Nöther’s theorem; Hamiltonian mechanics; rigid body; normal modes; relativistic mechanics, 4-vectors; chaos, nonintegrable systems (L: all theoretical physicists)

Classical Electrodynamics: Vector calculus, Maxwell equations in vacuum and in matter, electro- and magnetostatics, covariant formulation, electromagnetic waves, radiation (P: Classical Mechanics, Calculus; L: all theoretical physicists)

Quantum mechanics I: Wave mechanics; harmonic oscillator; basic principles, Hilbert space formulation; symmetries, generators; angular momentum, spin; hydrogen atom; Wigner-

Eckart theorem; perturbation theory, golden rule; scattering theory; identical particles (P: Classical Mechanics; L: all theoretical physicists)

Statistical Physics: Laws of classical thermodynamics; Thermodynamic potentials; Ensembles in statistical mechanics; Statistical entropy; Cluster - and virial expansion; Monte Carlo integration; Quantum statistical mechanics. Fermi - and Bose gases. Superfluidity. Quasi particles. Blackbody radiation. Phase transitions, Landau theory. Renormalization group. (P: classical mechanics, quantum mechanics; L: Frey, Sachs, von Delft, Zwerger)

Basic Concepts (Mathematics)

Introduction to functional analysis: Completeness, Riesz-Fischer. Hilbert and Banach spaces. Sobolev spaces. Dual space, Radon-Nikodym, Hahn-Banach, Weak and weak* convergence, Banach-Steinhaus, Banach-Alaouglu, Spectral theory of compact operators, Fredholm alternative. Trace. (P: Analysis I-III, Linear algebra, L: all mathematicians)

Complex analysis [Funktionentheorie]: Moebius Transformations, Holomorphic functions, Cauchy integral formula, Taylor/Laurent series, Residues, Harmonic functions, Maximum principles, Conformal mappings, Riemann mapping, Weierstrass factorization, Analytic continuation. (P: Analysis I-II; L: all mathematicians)

Manifolds and differential forms: topological spaces, manifolds, vector fields and flows, differential forms, integration and Stokes' theorem, applications. (P: Analysis I-II, Linear algebra; L: all mathematicians)

Partial Differential Equations: Separation of variables, Fourier Method, Method of characteristics, Heat equation, Maximum principle, Wave equation, Finite speed of propagation, Huygens principle, Poisson equation, Perron Method, Variational problems. (P: Analysis I & II; L: Erdös, Kalf, Siedentop)

Stochastics:. Probability models, random variables, cumulative distribution function, densities, standard probability models, elementary conditional expectations, independence, expectation and variance, generating functions, weak law of large numbers, central limit theorem for iid random variables, random walks, Markov chains on finite state spaces, statistics: statistical models, exponential families, estimators, hypothesis testing: Neyman Pearson lemma, special tests, confidence intervals. (P: analysis I-II, linear algebra, L: Dürr, Filipovic, Georgii, Merkl, Pruscha, Oppel, Winkler)

Mathematical Methods (MM):

Advanced partial differential equations: Fourier transform, distributions, Sobolev spaces, applications hyperbolic and parabolic equations, variational methods, applications to linear elliptic equations and non-linear integro-differetnial equations in mathematical physics as density functionals or Hartree-Fock equations. (P: functional analysis, partial differential equations; L: Erdös, Kalf, Siedentop)

Lie groups and their representation theory: Lie groups and Lie algebras, homogeneous spaces, maximal tori, roots and weights, semisimple Lie algebras, classification theory, representations of the classical groups (P: Analysis I & II, elementary algebra; L: Cieliebak, Kotschick, Schottenloher)

Topology: homology and cohomology, differential topology, vector bundles, characteristic classes (P: Manifolds and Differential Forms; L: Cieliebak, Kotschick, Leeb, Schottenloher)

Probability theory: conditional expectations and stochastic kernels, regular conditional probabilities, gambling systems and stopping times, martingales (mostly in discrete time), super- and submartingales, Borel-Cantelli, Kolmogorov's 0-1 law, strong law of large numbers, central limit theorem: triangle schemes, Berry-Esséen, large deviations, law of iterated logarithm (P: Introduction to stochastics, measure theory; L: Dürr, Filipovic, Georgii, Merkl, Pruscha)

Stochastic processes (Markov processes, martingales): weak convergence, compactness criteria, Markov processes: recurrence and transience, harmonic functions stationary processes, ergodic theorem for Markov chains, Stochastic processes in continuous time: renewal processes, Poisson process, Levy processes, Brownian motion, Donsker's invariance principle, Martingales and stopping times in continuous time, stochastic integral with Brownian motion as integrator. Ito formula. (P: probability theory; L: Dürr, Filipovic, Georgii, Merkl, Pruscha)

Game theory: Elements of Noncooperative Game Theory; Evolutionary Stability Criteria, Replicator Dynamics and Deterministic Models; Permanence and Stability; Stochastic Games and Mixed-Srategy Equilibria; Multipopulation Models; Population Genetics and Game Dynamics (P: Elementary probabilty theory, Ordinary differential equations; L: Frey, Schottenloher)

Classical Mechanics (CM)

Symplectic geometry I: symplectic and Poisson manifolds, Hamiltonian systems, symmetries and the moment map, symplectic reduction, completely integrable systems, toric manifolds, Duistermaat-Heckmann theorem. (P: Manifolds and differential forms, L: Cieliebak, Kotschick, Schottenloher)

Symplectic geometry II: (→FT,ST) almost complex structures, holomorphic curves, Gromov-Witten invariants, quantum cohomology, Floer homology, symplectic field theory. (P: Symplectic geometry I, L: Cieliebak, Kotschick)

Dynamical systems and nonlinear physics: Dynamical Systems and Nonlinear Physics; Finite-Difference Equations; Bifurcations and Cycles; Boolean Networks and Cellular Automata; Self-Similarity and Fractal Geometry; Nonlinear Differential Equations; Time-Series Analysis; Chaos (P: Analysis I-II, statistical physics; F: Frey)

Quantum mechanics (QM)

Quantum mechanics II: Heisenberg equation. Schrödinger picture. Perturbation theory. Interaction picture. Atom in a radiation field. Relativistic corrections. Foundation of quantum mechanics. Measuring process. Scattering theory. bound states. Cross section. Many body quantum mechanics. Density matrix. Interactions. Non-relativistic quantum field theory. Wick theorem. Holes. Superfluidity. Path integrals. Feynman-Kac formula. Partition functions.

(P: Classical mechanics, Quantum mechanics I; L: any theoretical physicist)

Many body theory: (→FT) (real- and imaginary time) Green's functions; diagrammatic perturbation theory, functional integrals; Fermi liquid theory; superconductivity, Landau-Ginzburg and BCS-theory; ferro- and antiferromagnetism, Hubbard model; Kondo effect; Anderson localization, weak localization; quantum Hall effect (P: quantum mechanics I & II, statistical mechanics; von Delft, Zwerger, Marquardt)

Mathematical quantum mechanics I: Unbounded operators; Coulomb-, Schrödinger- und Dirac-operators; perturbation theory; many body systems, stability of matter, quantisation; scattering theory (P: Analysis I-III, quantum mechanics I; L: Dürr, Erdös, Kalf, Siedentop)

Mathematical quantum mechanics II: selected topics from (→FT):
- Mathematical description of solids: Spectral theory of Schrödinger operators with periodic, quasiperiodic and random coefficients,
- Nonperturbative methods in quantum field theory, in particular, non-relativistic QED
- Models in relativistic N-particle quantum mechanics

- Derivation of effective one-particle equations (stationary and time-dependent) from the underlying microscopic theory, e.g., Thomas-Fermi theory, kinetic equations etc.
- Constructive quantum field theory
- Multi-particle scattering theory

(L: Erdös, Siedentop)

Applied quantum mechanics (APM)

Solid state theory: Theory of crystals, reciprocal lattice, phonons; electrons in a lattice; Bloch electrons, theory of metals, Fermi liquid theory; thermodynamic, magnetic and electrodynamic properties of metals; transport, Boltzmann equation; semiconductors; phase transitions, Landau theory (P: quantum mechanics, electrodynamics, statistical mechanics; L: von Delft, Frey, Marquardt, Zwerger)

Quantum phase transitions: Transverse Ising model, quantum criticality, Quantum-Antiferromagnets and nonlinear sigma-models, renormalization group and 1/N-expansion, Hertz-Millis theory of quantum ferromagnets, Skyrmions, quantum phase transitions of cold atoms in optical lattices (P: Quantum mechanics I & II, Statistical mechanics, L: Zwerger)

Theoretical quantum optics: quantisation of the electromagnetic field, nonclassical field states and photon statistics, phase space representations, light-matter interactions, Jaynes-Cummings-Model, master equations, resonance fluerescence, photon entanglement and quantum communication, laser theory, traps and cooling methods for atoms (P: quantum mechanics I; L: Cirac, Schenzle, Hornberger)

Mesoscopic physics: Electronic transport as a scattering problem, Quantum Hall effect, Quantum dots,Disorder effects, Shot noise and full counting statistics, dephasing, mesoscopic superconductivity, interacting electrons in one dimension (Luttinger liquid), spin effects, connections to quantum optics and cold atoms (P: quantum mechanics I & II, statistical mechanics; L: von Delft, Zwerger, Marquardt)

Nonrelativistic Quantum Electrodynamics: Abraham model and relativistic variants, effective motion of charges, radiative friction, quantization, existence of ground states, motion under slowly varying potentials, space-adiabatic theory, g-factor, energy and mass renormalization (P: quantum mechanics, electrodynamics, functional analysis; L: Spohn)

Superfluidity and Bose-Einstein-Condensation: Ideal and weakly interacting quantum gases, BCS and Bogoliubov-theory, BCS-BEC-crossover, Exact solutions of 1D models, Mott-superfluid phase transitions, Hubbard-model (P: Quantum mechanics I & II, statistical mechanics; L: Zwerger)

Theory of open quantum systems: Master equations for dissipative quantum systems; Quantum-Markov process and Lindbladt's theorem; decoherence; phase space representations and Quantum-Fokker-Planck equations; stochastic Schrödinger equations; environment-induced selection rules for pointer states (P: quantum mechanis I, statistical mechanics; L: von Delft, Hornberger, Marquardt)

Quantum measurement theory: Operational formulation of quantum mechanics, generalized measurements, positive operator measures and Kraus-Operators, indirect and which-path measurements, continuous measurements: standard quantum limit and Zeno effect, quantum optics applications: photon counting, state tomography, gravitational wave detection (P: quantum mechanics I, statistical mechanics; L: Cirac, Schenzle, Hornberger)

Quantum information processing: entanglement; quantum communication (teleportation, cryptography); quantum computation; decoherence; error correction; quantum channels. (P: Quantum mechanics I, Linear algebra; L: Cirac)

Statistical Physics (SP)

Soft Condensed Matter Theory: Review of Statistical Mechanics; Mean-Field Theory and Variational Principles; Density Functional Theory; Renormalization Group Theory; Liquid Crystals, Polymers and Membranes; Polyelectrolytes; Wetting and Thin Films; Correlations and Response; Hydrodynamics; Topological Defects; Walls, kinks and solitons; Disordered Systems (P: statistical physics; L: Frey)

Theoretical Biological Physics: Theory of Stochastic Processes; Fokker-Planck-Equations; Nonlinear Dynamics; Brownian Dynamics Simulations; Hydrodynamics at Low Reynolds Number; Entropic Forces; Molecular Engines; Polymers and Membranes; Gene Regulation; Driven Diffusive Systems; Neural Networks; System Theory and Networks (P: statistical physics; L: Frey)

Stochastic Processes in Physics and Biology: Review of Probability Theory; Discrete Time Markov Chains; Discrete Time Branching Processes; Continuous Time Markov Chains; Continuous Time Birth and Death Chains; Epidemic, Competition, Predation and Population Genetic Processes; Diffusion Processes and Stochastic Differential Equations; Driven Diffusive Systems (P: statistical physics; L: Frey)

Mathematical Statistical Mechanics: Newton dynamics for infinite volume, particle systems. Gibbs measures: DLR conditions, existence, uniqueness: Dobrushin's theorem. Phase transitions, extremal decomposition, absence of spontaneous symmetry breaking in 2 dimensions. Ising model: high temperature phase, Peierls argument, cluster expansion, FK representation, extremal Ising measures in 2 dimensions, FKG inequality. Spontaneous symmetry breaking in continuous models. Percolation; conformal invariance in 2 dimensions and stochastic Löwner evolution. (P: stochastic processes; Georgii, Merkl)

Stochastic integration and stochastic differential equations: Doob inequalities, Doob-Meyer decomposition in continuous time, quadradic variation and covariation, Ito isometry and stochastic integral with semimartingales as integrator. Ito formula in the general case. Stratonovich integral. Ito calculus. Stochastic treatment of parabolic and elliptic PDEs. Lévy's theorem. Random time changes in stochastic integrals. Change of measure: Girsanov's theorem. White noise. Stochastic differential equations: existence and uniqueness of strong solutions, weak solutions. (P: stochastic processes; Dürr, Filipovic, Georgii, Merkl, Pruscha)

Statistical Mechanics (rigorous results) lattice models, one-dimensional models and Markov chains, DLR equations, exact solution: 2D Ising model, transfer matrix, Bethe ansatz, high- and low temperature expansions, Peierls argument and phase transitions, decay of correlations, critical behavior. (P: statistical mechanics, probability theory; L: Spohn)

Field Theory (FT)

Quantum field theory I (QED): Relativistic field theory, canonical quantization, fermions, gauge principle and QED Lagrangian, Feynman diagrams, elementary processes, radiative corrections, running coupling (P: Electrodynamics, quantum mechanics, special relativity; L: Buchalla, Fritzsch, Dittmaier)

Quantum field theory II (standard model): Relativistic quantum field theory, path integral quantization, QED, QCD, Higgs mechanism, electroweak interactions, flavour physics (P: Electrodynamics, quantum mechanics, special relativity; L: Buchalla, Fritzsch, Dittmaier)

Supersymmetry: Review of quantum field theory. No-go theorem. Supersymmetry algebra.Representation theory. Superfields. Supersymmetric actions. Noether currents. Supersymmetry breaking. Feynman rules. Effective actions. Non-renormalization theorems. Supersymmetric standard model. Supergravity. (P: Classical mechanics, Quantum mechanics; L: Lüst, Sachs, Blumenhagen, Dittmaier, Erdmenger, Seiler)

Conformal field theory: (→ST) Conformal Transformations. Critical Phenomena / Statistical Mechanics. Introduction to Conformal Field Theory. Virasoro Algebra and Representations / Verma Modules / Highest Weight Representations. Kac-table / Minimal Models. Conformal Ward Identities / BPZ - equations /Conformal Blocks. Toroidal Compactifications. Boundary CFT, Perturbed CFT. Affine Kac-Moody Algebra/ Lie Group Symmetries. Superconformal Field Theory. CFT in Higher Dimensions (P: Classical mechanics, Quantum mechanics; L: Lüst, Sachs, Blumenhagen, Erdmenger, Seiler, Schottenloher)

Renormalization group methods in condensed matter theory: Ising model, Landau theory, ε- and 1/N-expansions, Kosterlitz-Thouless transition, numerical RG, density matrix RG, Fermi liquid theory, Kondo model, Anderson model, Anderson localization, quantum phase transitions, flow equation RG (P: Quantum mechanics I & II, statistical mechanics; L: von Delft, Marquardt, Zwerger, Frey)

Statistical Field theory, Renormalization: Landau-theory, spontaneous symmetry breaking, Goldstone modes and Mermin-Wagner-Hohenberg theorem, Lattice models, transfer matrix, finite size scaling, renormalization group, scaling, universality, epsilon-expansion, random-walk-representations of euclidean field-theories (P: Quantum mechanics I & II, statistical mechanics; L: Zwerger)

Mathematical gauge theory (→ST): geometry and topology of fiber bundles, connections and curvature, gauge transformations, gauge invariant functionals on spaces of connections, second order Yang-Mills equations, self-duality on 4-manifolds and the SDYM equations, Seiberg-Witten Theory (P: Manifolds and Differential Forms; L: Kotschick, Schottenloher)

Quantization / Axiomatic quantum field theory: Dirac quantization of fields (constraints), BV-quantization, BRST-quantization, deformation quantization, geometric quantization; -- Wightman axioms, Osterwalder Schrader axioms, axioms of conformal field theoryP: Quantum mechanics, quantum field theory, functional analysis, manifolds and differential forms (P: quantum mechanics I, field theory I; L: Sachs, Schottenloher)

Relativity, Cosmology (RC)

General Relativity: Introduction to differential geometry. Special relativity. Einstein equations. Schwarzschild solution. Post Newtonian approximation. Gravitational waves/radiation. Classical test of general relativity. Black holes. Carter-Penrose Diagram. Elements of general relativistic hydrodynamics. Introduction to cosmology. (P: Classical mechanics; L: Mukhanov, Sachs, Lüst)

Cosmology: Friedmann Universe: Newtonian analysis, relativistic theory, thermal history, nucleosynthesis, necombination. Gravitational instability: Jeans' theory, Newtonian theory in expanding universe, general relativity, gauge-invariant formalism. Topological defects. Inflation. (P: classical mechanics, quantum mechanics I; L: Mukhanov, Sachs)

Quantum field theory in external fields: Review of classical - and quantum mechanics. Harmonic oscillator. Quantization of fields. Quantum fields in an expanding Universe and de Sitter space-time. Unruh effect. Hawking radiation. Thermodynamics of black holes. (P: classical -and quantum mechanics; L: Mukhanov, Sachs)

Black holes: Classical black hole solutions; Kruskal diagrams; Laws of black hole mechanics; Thermodynamics of black holes: the Unruh effect, Hawking radiation, the area law, Wald's law; Microscopic derivation of black hole entropy in the context of string theory. (P: general relativity, quantum field theory I; L: Cardoso, Sachs, Mukhanov)

Riemannian and Semi-Riemannian Geometry: (→FT, ST) Riemannian and Semi-Riemannian metrics, the Levi-Civita connection, geodesics and completeness, sectional curvature, the Riemann and Ricci tensors, scalar curvature, the Einstein equations and the Hilbert action, normal forms for the curvature tensor, Riemannian and Lorentzian examples of Einstein metrics (Manifolds and Differential Forms; L: Cieliebak, Kotschick, Leeb)

String theory (ST)

Conformal field theory: (→FT)

Introduction to String theory: Polyakov action. Conformal field theory. Spectrum of the string. S-matrix. String theory in general backgrounds. Beta-functions. Effective actions. String field theory. Superstrings. GSO-Projection. T- and S- Duality. D-branes. Supergravity. M-theory. (P: classical -and quantum mechanics; L: Lüst, Sachs, Blumenhagen, Erdmenger, Seiler)

Complex and Kähler geometry: Algebraic and differential geometry of complex manifolds, Kähler metric, Hodge decomposition, Lefschetz theorems, Hodge diamond, Calabi-Yau manifolds, connections on holomorphic vector bundles, Chern classes, Hyperkähler manifolds and supersymmetry, deformation of complex structures, Kodaira vanishing theorem and embedding (P: Complex Analysis, differential geometry; Schottenloher, Cieliebak, Kotschick)

Moduli spaces (and strings): Classification of compact 2dimensional manifolds, elliptic curves and their parametrization, Teichmüller space and moduli space of smooth curves over C , i.e. Riemann surfaces, mapping class group, degeneration of curves, moduli space of stable curves, fusion rules of conformal field theory and strings, moduli spaces of higher dimensional complex manifolds and of holomorphic vector bundles. (P: Complex analysis and complex geometry; L: Cieliebak, Kotschick, Schottenloher)

Lectures from Erlangen

Classical mechanics: Hamiltonian differential equations and dynamical systems; symplectic geometry, basic concepts of ergodic theory; variational principles, geodetic flow on manifolds, integrable systems and perturbations theory.

Statistical mechanics: Basic concepts of probability theory, entropy, classical spin systems and Ising models, thermodynamic limit, Gibbs measures, phase transitions, correlation inequalities, statistical quantum mechanics

Differential topology: Differential manifolds, bundles, embeddings, immersions, theorems of Morse and Sard, transversality, vector bundles, degree of mapping and Euler characteristics, Morse theory, cobordisms]

Functional analysis I: Hilbert and Banach spaces (with examples), operators on abstract spaces (fixed point theorem, continuous operators, closed operators), linear operators, linear functionals, weak topologies, compact operators, Ritz variational principle, Fredholm alternative, trace class operators

Functional analysis II: Banach algebras and C*-Algebras, self-adjoint operators, spectral theory of self-adjoint operators, perturbation theory, applications in numerical mathematics, pde in physics (in particular quantum mechanics)