Introduction to String Theory
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Assignment # 1

(Due May 8, 2006)

1) Consider the action of a massive relativistic point particle:

$$S = -mc \int_{\tau_0}^{\tau_1} d\tau \sqrt{-\dot{x}^\mu \dot{x}_\mu}.$$

a) Using $x^0 \equiv ct$, show the equivalence of S to the action

$$\hat{S} = \int_{t_0}^{t_1} dt L(t) \equiv -mc^2 \int_{t_0}^{t_1} dt \sqrt{1 - \frac{\vec{v}^2}{c^2}},$$

where $\vec{v} = \vec{v}(t)$ denotes the ordinary velocity of the particle with respect to the physical time t.

b) Verify that for small velocities, $|\vec{v}| \ll c$, $L(t)$ reduces to the standard form of a Lagrange function, i.e., kinetic minus potential energy. What plays the rôle of the potential energy in this case?

2) The advantage of the action S over the action $\hat{S}$ is that it treats time x^0 and the space coordinates $\vec{x}$ on an equal footing, making Poincaré invariance manifest. This comes at the expense of a new, unphysical, parameter, τ. Verify that the covariant action S is indeed invariant under changes of this unphysical parameter, i.e., under reparameterizations

$$\tau \longrightarrow \tilde{\tau}(\tau).$$

3) Consider now the action

$$S' = \frac{1}{2}\int_{\tau_0}^{\tau_1} d\tau (e^{-1}\dot{x}^\mu \dot{x}_\mu - em^2c^2).$$

a) How does e have to transform under the reparameterization $\tau \longrightarrow \tilde{\tau}(\tau)$ in order to ensure the reparameterization invariance of S'?

b) Find the equation of motion for e by varying S'. Insert the resulting equation into S' and verify that S' is classically equivalent to the action S of Problem 1).

4) The Nambu-Goto action of a one-dimensional object is given by:

$$S_{NG} = -T\int d\tau d\sigma \sqrt{-\det(\partial_a X^\mu \partial_b X^\nu \eta_{\mu\nu})}.$$

a) Check the invariance under Poincaré transformations of the "target space", i.e. under

$$X'^\mu(\tau,\sigma) = \Lambda^\mu{}_\nu X^\nu(\tau,\sigma) + a^\mu,$$

where $\Lambda^\mu{}_\nu$ denotes a constant (pseudo-)orthogonal matrix with respect to the metric $\eta_{\mu\nu}$, and a^μ is a constant vector.

b) Show the invariance of S_{NG} under arbitrary reparameterizations of the worldsheet

$$(\tau,\sigma) \longrightarrow (\tilde{\tau}(\tau,\sigma), \tilde{\sigma}(\tau,\sigma)).$$