Retrieval of environmental parameters from multiply scattered return signals of ground-based and airborne LIDARs[1]

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ABSTRACT

We present some examples of calculations of polarised multiply scattered return signals of ground-based and airborne multichannel LIDARs (such as the Microlidar of the DLR) and we discuss the applied numerical methods. These methods are variance reduction Monte Carlo algorithms allowing for controlling the empirical variance. We show how variance reduction Monte Carlo methods may be combined with properly chosen uniform grid search or random search procedures to retrieve one or more environmental parameters from a given (measured or calculated) LIDAR return signal. Such a procedure is time consuming, of course, but it will allow for the retrieval of several parameters simultaneously and for a sensitivity analysis. A retrieval method based on Monte Carlo methods and random search including such a sensitivity analysis gives much more information than the usual inversion procedures (e.g. based on integral equations and often uncheckable environmental assumptions). We present some examples of the (simultaneous) retrieval of the extinction coefficient and the particle size distribution of a cloud from the multiply scattered return signal of ground-based and airborne LIDARs and a sensitivity analysis of this retrieval.

1.INTRODUCTION

A LIDAR is not only used for light detection and ranging, but nowadays it is one of the most powerful instruments for analysing the atmosphere. A LIDAR consists of a transmitter and a receiver. The transmitter is sending a fairly collimated and pulsed or modulated beam of monochromatic LASER light of a certain frequency into the atmosphere where this light is absorbed or scattered by the particles (molecules, droplets, ice crystals, dust) of the atmosphere. The highly sensitive receiver registrates light which is reflected by the atmosphere from a certain field of view (FOV) and with or without a certain state of polarisation.

The receiver and the transmitter may be situated in the same place (then we call the LIDAR monostatic) or they may be situated in different places (then we call the LIDAR bistatic). Furthermore, the FOV of the receiver may be directed in the same or in a different direction as the (FOV of the) transmitted beam. Very often the FOVs of the transmitter and of the receiver are coaxial cones; sometimes the receiver has several FOVs (e.g. a narrow central FOV and a ring-shaped outer FOV) and registers several states of polarisation (e.g. parallel and perpendicular to the direction of polarisation of the transmitted beam) for these FOVs even simultaneously. The Microlidar of the DLR is such a LIDAR. Furthermore, some LIDARs have the possibility of transmitting and receiving several frequencies for these FOVs almost simultaneously. LIDARs may be ground-based, airborne or space-based.

The goal of LIDAR sensing of the atmosphere is to retrieve meteorologically interesting parameters of the atmosphere from the received LIDAR signal. To be able to obtain such parameters, it is necessary to describe the received LIDAR signal physically sufficiently precisely by a mathematical formula and to solve this formula for the interesting parameters, if possible.

[1] Air Pollution and visibility Measurements. Eds. Peter Fabian, Volker Klein, Marus Tacke, Konradin Weber , Christian Werner: Proc. SPIE 2506, 624–635 (1995). (Proceedings of "European Symposium on Optics for Environmental and Public Safety. Munich, June 19–23, 1995)

In the early days the analysis of LIDAR signals was based on what we call the classical LIDAR equation which we present here in a very simple version for a monostatic LIDAR:

$$P_R = P_0 \frac{c\tau}{2} \frac{1}{r^2} \beta(r) A \, exp\left(-2 \int_0^r \sigma(s)\, ds\right)$$

where P_R is the received signal, P_0 the transmitted power at the time $t_0 = 0$, c the speed of light, τ the duration of the transmitted LASER pulse, β the backscatter coefficient, σ the extinction coefficient, r the range, and A the area of the receiver.

This equation is nothing but the well-known RADAR equation. The classical LIDAR equation describes the part of the LIDAR signal which is produced by single scattering only.

Very early phenomena like depolarisation by spherical particles, pulse broadening and lengthening, asphericity of LIDAR signals from dense clouds, from gaseous layers, and from far distant cirrus clouds were observed which were understandable only with the help of multiple scattering effects. Hence, it was necessary to make a better physical and mathematical model of the process LIDAR sensing of the atmosphere including multiple scattering and, based on such a model, to derive new LIDAR equations which describe the LIDAR signal more precisely by taking into account not only the part of the LIDAR signal which is due to single scattering, but also the part of the LIDAR signal which is due to some or all orders of multiple scattering.

2. LIDAR MULTIPLE SCATTERING

Different techniques of modeling the process of light propagation have been proposed and used to obtain formulas and algorithms to calculate the multiply scattered parts of the LIDAR signal. The modeling may be based on transport theory (integro-differential or integral transport equation; stationary or nonstationary), approximations of transport equations (e.g. small angle approximation), the physical Monte Carlo simulation, stochastic models (e.g. Rayleigh's random walk, transport processes), and iterated applications of the Maxwell equations; for more details and comparisons of calculations[1] . Using any of these approaches, the multiple scattering process can be formulated with or without polarisation.

All these modeling techniques of multiple scattering are essentially based on the idea of iterating the single scattering process. Again, the single scattering may be modeled using different techniques (e.g. linear optics, Maxwell equations, approximations of Maxwell equations). Single scattering is characterized by the scattering distribution, defined by the phase function or by the scattering matrix in models with polarisation.

A cooperation of the Computing Centre (CC) of the Russian Academy of Sciences in Novosibirsk and the Institute of Mathematics of the L-M-University of Munich (LMU), which was supported by the DLR, DFG, and the DAAD, resulted in a refinement of a stochastic model for the description and simulation of the transport of polarised light and a new "cascade" model for the calculation of LIDAR signals for polarised light by variance reduction Monte Carlo methods. This was achieved by bringing together the different modeling and computing techniques used by the CC and the LMU.

The approach to simulate the transport of light used by the LMU is based on a stochastic model which describes the process of multiple scattering as a generalized Rayleigh's random walk, it is a general and precise mathematical description of the physical process of multiple scattering of light viewed as a "stochastic corpuscular multiple scattering process"[10,5,6,7,4] . Also this stochastic model allows for deriving an exact multiple scattering LIDAR equation. The total LIDAR signal may be split up into parts due to the different orders of multiple scattering. Again, these parts may be expressed by complicated iterated integrals. The original stochastic model included polarisation implicitely[10] ; a slight change of the model allows for introducing polarisation more explicitely and for easier calculation of the LIDAR signal with polarisation[5,6] .

We shall briefly review the stochastic process of corpuscular multiple scattering with explicite description of polarisation. Based on this model, we obtain exact multiple scattering LIDAR equations. This model and these formulas allow for calculating the LIDAR signal by (direct) Monte Carlo methods, but in a very inefficient way only. To improve

the efficiency of the Monte Carlo simulation we introduce variance reduction methods (such as local estimates, scattering splitting, and exponential transforms) and we switch from the stochastic model of the type of the generalized Rayleigh's random walk to a much more complicated stochastic model of a "stochastic multiple scattering cascade process" which describes the variance reduction methods, reproduces the exact multiple scattering LIDAR equations, and guarantees fast calculation of the LIDAR signal with controlled finite variance.

2.1 Stochastic process of polarised multiple scattering

To introduce our stochastic model of the stochastic process of corpuscular multiple scattering, we assume that light is corpuscular and that photons which are scattered by particles of the atmosphere travel along straight lines from one point of scattering to the next. Here we shall present our stochastic model for the transfer of polarised light and, for sake of simplicity, we shall restrict ourselves to spherical particles.

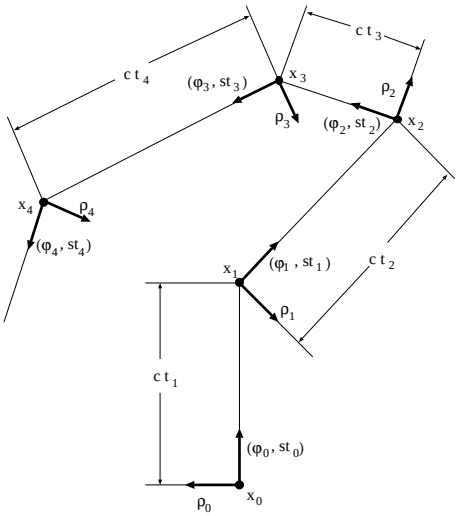
For the description of polarisation we use modified Stokes vectors[2] : $ST := (I_{\parallel}, I_{\perp}, U, V)^T$ where $I := I_{\parallel} + I_{\perp}$ denotes the intensity. The Stokes vector is defined with respect to a reference plane. This plane of reference is given by the direction of propagation $\varphi \in S^2$ (where S^2 denotes the unit sphere in $\mathbb{R}^3$) and a reference vector $\rho \in S^2$ with $\rho \perp \varphi$.

We define the normed Stokes vector:

$$st := (i_{\parallel}, i_{\perp}, u, v)^T \quad := \quad \begin{cases} (\frac{I_{\parallel}}{I}, \frac{I_{\perp}}{I}, \frac{U}{I}, \frac{V}{I})^T & \text{for } I \neq 0, \\ (0,0,0,0) & \text{for } I = 0. \end{cases}$$

The stochastic model is an expanded version of the model we have proposed and used earlier[10,8,9] . In the expanded model the path of a polarised photon is characterized by its starting point x_0, its starting direction φ_0, its state of polarisation st_0 (st_0 is the normed Stokes vector), the reference vector ρ_0, and by the sequence of the time intervals t_n between the $(n-1)$–th and the n–th scattering event, the direction φ_n, the Stokes vector st_n and the reference vector ρ_n after the n–th scattering event (see Figure 1).

Figure 1: Path of a photon starting in x_0 in the direction of φ_0 with normed Stokes vector st_0 and reference vector ρ_0, c denotes the speed of light.



The transition probabilities E_n describing the extinction are the same as in the unpolarised case and they are independent of the Stokes vectors:

$$E_n(x_0, \varphi_0, st_0, \rho_0; t_1, \varphi_1, st_1, \rho_1; \ldots; t_{n-1}, \varphi_{n-1}, st_{n-1}, \rho_{n-1}; \cdot) :=$$
$$:= E_n(x_0, \varphi_0; t_1, \varphi_1; \ldots; t_{n-1}, \varphi_{n-1}; \cdot).$$

As a consequence of the Beer–Lambert law E_n has the shape of a generalized exponential distribution[10] . The scattering transition probabilities P_n, however, depend on the Stokes vectors and the reference vectors:

$$P_n(x_0, \varphi_0, st_0, \rho_0; t_1, \varphi_1, st_1, \rho_1; \ldots; t_{n-1}, \varphi_{n-1}, st_{n-1}, \rho_{n-1}; t_n; \cdot).$$

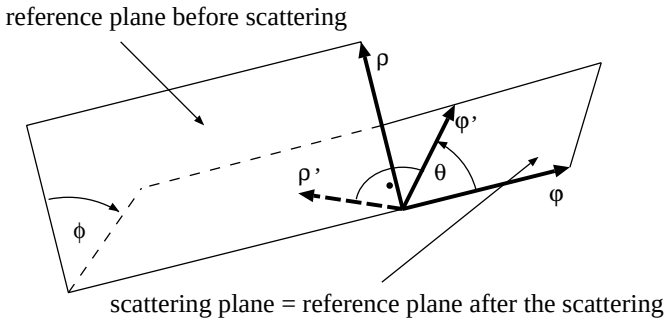
To show how this transition probability has to be changed with respect to the unpolarised case, we first discuss single scattering of polarised light with special attention to spherical particles.

The scattering of a polarised photon with reference plane (ρ, φ) and Stokes vector st at point x into direction φ' can be expressed by two matrix multiplications:

$$ST' \quad := \quad \mathcal{P}(\theta, \phi)\, \mathcal{L}(\phi)\, st,$$

where $\theta \in [0, \pi]$ is the elevation angle between the incident beam direction φ and the scattered direction φ' of the photon. $\phi \in [0, 2\pi]$ describes the azimuth angle between the plane of reference (ρ, φ) before the scattering and the plane of reference (ρ', φ') after the scattering, which is equal to the scattering plane (φ, φ'); see Figure 2. st is the Stokes vector for the incident and ST' for the scattered radiation. From Figure 2 we see that for a given reference plane (ρ, φ) and angles θ and ϕ the new direction φ' and the new reference vector ρ' are already determined.

Figure 2: The geometry for single scattering of polarised light.



$\mathcal{L}(\phi)$ denotes the matrix for the rotation of the Stokes vector[2] with

$$\mathcal{L}(\phi) \quad := \quad \begin{pmatrix} \cos^2(\phi) & \sin^2(\phi) & \frac{1}{2}\sin(2\phi) & 0 \\ \sin^2(\phi) & \cos^2(\phi) & -\frac{1}{2}\sin(2\phi) & 0 \\ -\sin(2\phi) & \sin(2\phi) & \cos(2\phi) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

$\mathcal{P}(\theta, \phi)$ is the so–called Mueller matrix for the transformation of the Stokes vector. For spherical particles the Mueller matrix is independent of ϕ and has the following shape[3] :

$$\mathcal{P}(\theta) \quad := \quad \begin{pmatrix} \mathcal{P}_1(\theta) & 0 & 0 & 0 \\ 0 & \mathcal{P}_2(\theta) & 0 & 0 \\ 0 & 0 & \mathcal{P}_3(\theta) & -\mathcal{P}_4(\theta) \\ 0 & 0 & \mathcal{P}_4(\theta) & \mathcal{P}_3(\theta) \end{pmatrix}.$$

We obtain the intensity $I' = I'_{\parallel} + I'_{\perp}$ after the scattering from iterated matrix multiplication:

$$I'(\theta, \phi, st) \quad = \quad \tfrac{1}{2}(\mathcal{P}_1(\theta) + \mathcal{P}_2(\theta)) + \tfrac{1}{2}(\mathcal{P}_1(\theta) - \mathcal{P}_2(\theta)) \, \Big((2i_{\parallel} - 1)\cos(2\phi) + u \sin(2\phi) \Big),$$

and with the normalization condition of the Mueller matrix for spherical particles[3] we get:

$$\int_0^{2\pi} \int_0^{\pi} I'(\theta, \phi, st)\, \sin(\theta)\, d\theta\, d\phi \quad = \quad 4\pi.$$

Now we are able to define the *polarised phase function* ph_{pol}:

$$ph_{pol}(\theta,\phi,st) \quad := \quad \frac{I'(\theta,\phi,st)}{4\pi},$$

which is the probability density of the scattering distribution, defining the scattering transition probability P_n. With the marginal density ph_1 and the conditional density ph_2 we are able to factorize ph_{pol} in the following way:

$$ph_{pol}(\theta,\phi,st) \quad = \quad ph_1(\theta)\cdot ph_2(\phi,st \mid \theta),$$

with

$$ph_1(\theta) \quad := \quad \int_0^{2\pi} ph_{pol}(\theta,\phi,st)\; d\phi = \frac{1}{4}\Big(\mathcal{P}_1(\theta)+\mathcal{P}_2(\theta)\Big),$$

and

$$ph_2(\phi,st \mid \theta) \quad := \quad \begin{cases} \dfrac{ph_{pol}(\theta,\phi,st)}{ph_1(\theta)} & \text{for } ph_1(\theta)\neq 0, \\ 0 & \text{for } ph_1(\theta)=0. \end{cases}$$

Especially for $ph_1(\theta)\neq 0$ we get

$$ph_2(\phi,st \mid \theta) \quad = \quad \frac{1}{2\pi}+\frac{1}{2\pi}\,\frac{\mathcal{P}_1(\theta)-\mathcal{P}_2(\theta)}{\mathcal{P}_1(\theta)+\mathcal{P}_2(\theta)}\Big((2i_{\|}-1)\cos(2\phi)+u\sin(2\phi)\Big).$$

For unpolarised scattering we use the well–known unpolarised phase function $ph_{unpol}(\theta) = ph_1(\theta)/(2\pi)$, which is independent of ϕ. We see that the deviation of ph_{pol} from ph_{unpol} is proportional to $(\mathcal{P}_1(\theta)-\mathcal{P}_2(\theta))\,/\,(\mathcal{P}_1(\theta)+\mathcal{P}_2(\theta))$ which may be in the magnitude of one[6] .

The Stokes vector st_n after the n–th scattering event is obtained from the photon history

$$(x_0,\varphi_0,st_0;t_1,\varphi_1,st_1;...;t_{n-1},\varphi_{n-1},st_{n-1};t_n,\varphi_n)$$

by the iterated transformations with $\mathcal{P}(\theta_i)\,\mathcal{L}(\phi_i)$. After each step the Stokes vector ST' has to be normed.

2.2 Multiple scattering LIDAR equation

For this expanded model we again obtain an exact multiple scattering LIDAR equation similar to the unpolarised LIDAR equation[9] . Signals may be calculated on the basis of the exact multiple scattering equation, e.g. by using variance reduction Monte Carlo simulations. The method of variance reduction Monte Carlo simulation for unpolarised LIDAR signals[1,9] , now includes these new scattering transition probabilities and new techniques of scattering splitting and point flux estimation for polarised light[4] .

In terms of the polarised stochastic multiple scattering process let us now state an exact multiple scattering LIDAR equation, the time dependent LIDAR equation for the polarised flux:

The transmitter of the LIDAR is characterized by a probability distribution on the $(x_0,\varphi_0,t_0,st_0,\rho_0)$–space, consisting of the initial space point x_0, direction φ_0, time t_0, Stokes vector st_0, and reference vector ρ_0. The receiver is supposed to have a conical field of view with an effective area A_R with center x_R, radius r_R, normal ν_R on the receiver plane A_R, and reference vector ρ_R. For sake of simplicity we assume that $x_R := (0,0,0)^T$.

We denote by $\mathcal{R}$ the set of space-direction-time triples $(x,\varphi,t)\in \mathbb{R}^3\times \mathrm{S}^2\times \mathbb{R}_0^+$ such that x is in front of the receiver and the line from x to $x+ct\varphi$ is intersecting the receiver within the receiver $\mathrm{FOV_R}$. $\mathcal{T}$ is the measuring time interval. Then the four-dimensional vector

$$f_{\mathcal{T}}(x,\varphi,st,\rho,t) := \frac{<\varphi,\nu_R>}{<x,\nu_R>^2}\;\; 1_{\mathcal{T}}\left(t+\frac{-<x,\nu_R>}{c<\varphi,\nu_R>}\right)\mathcal{L}(\Gamma(\varphi,\rho))\,st$$

is the polarised time dependent flux through the receiver at the time t from the space point x in the direction of φ with Stokes vector st and the reference vector ρ. Here $< .,. >$ is the Euclidian scalar product, $1_{\mathcal{T}}$ is the indicator function of the measuring time interval $\mathcal{T}$, $\Gamma(\varphi,\rho) := arccos(< \nu_R \times \rho_R, \varphi \times \rho >)$ is the angle between reference plane of the photon at position x and the reference plane of the receiver, and $\mathcal{L}(\phi)$ is the matrix for rotation of the Stokes vector. (This formula for the flux is true if certain conditions in the very near neighborhood of the receiver are fulfilled; otherwise some technical conditions have to be added[4] .)

The Stokes vector of the polarised LIDAR signal which is received during the (e.g. small) measuring interval $\mathcal{T}$ is equal to

$$st_R(\tau;\mathcal{T}) = \sum_{n\in\mathrm{N}} st_R(\tau;\mathcal{T};n)$$

where the integral

$$\begin{aligned} st_R(\tau;\mathcal{T};n) := & \int_{\mathrm{IR}^3\times\mathrm{S}^2\times\mathrm{IR}^4\times\mathrm{S}^2\times\mathrm{IR}_0^+} \int_{\mathrm{IR}_0^+} \int_{\mathrm{S}^2\times\mathrm{IR}^4\times\mathrm{S}^2} \dots \int_{\mathrm{S}^2\times\mathrm{IR}^4\times\mathrm{S}^2} \int_{\mathrm{IR}_0^+} \\ & f_{\mathcal{T}}(x_{(n)},\varphi_n,st_n,\rho_n,\textstyle\sum_{i=0}^n t_n)\ \ 1_{\mathcal{R}}(x_{(n)},\varphi_n,t_{n+1})\ \ \prod_{i=0}^{n-1} 1_{\bar{\mathcal{R}}}(x_{(i)},\varphi_i,t_{i+1}) \\ & \mathrm{E}_{n+1}((x_0,\varphi_0,t_0);t_1,\varphi_1;\dots;t_n,\varphi_n;dt_{n+1})\ \ \mathrm{P}_n((x_0,\varphi_0,st_0,\rho_0,t_0);t_1,\varphi_1,st_1,\rho_1;\dots;t_n;d(\varphi_n,st_n,\rho_n))\ \dots \\ & \dots\ \mathrm{P}_1((x_0,\varphi_0,st_0,\rho_0,t_0);t_1;d(\varphi_1,st_1,\rho_1))\ \ \mathrm{E}_1((x_0,\varphi_0,t_0);dt_1)\quad \tau(d(x_0,\varphi_0,st_0,\rho_0,t_0)) \end{aligned}$$

is to be taken coordinatewise and represents the contribution of the n-th order of multiple scattering, $x_{(i)} := x_0 + \sum_{j=0}^i ct_j\varphi_{j-1}$ and $\bar{\mathcal{R}}$ is the complement of $\mathcal{R}$.

3.EXAMPLES OF POLARISED LIDAR SIGNALS

From the transport theoretic and from the stochastic approach to multiple scattering various types of exact LIDAR equations may be obtained. All these equations are very complicated: The multiply scattered LIDAR signal is obtained as an infinite sum of iterated integrals. These multiply scattered integrals correspond to the contributions of the different orders of multiple scattering. At present it seems not to be possible to calculate the i terated integrals describing the contributions of the higher orders of multiple scattering analytically or numerically exactly. But starting from these exact LIDAR equations, it is possible to design variance reduction Monte Carlo methods in such a way that it is possible to calculate the LIDAR signals fairly fast and as precisely as wanted. The precision of the calculation is controlled by the empirical variance of the Monte Carlo simulation; this variance may be chosen as small as it is wanted. Of course, the higher the demanded precision the more computer time is needed for the simulation of the LIDAR signal.

We have designed very effective computer codes for the variance reduction Monte Carlo simulations. For unpolarised multiply scattered LIDAR signals we have such programs e.g. for a monostatic LIDAR with conical and coaxial fields of view of the transmitter and the receiver which are directed perpendicularly to homogeneous slabs of aerosols with scatterers which have scattering distributions which are rotational invariant with respect to the incident beam (e.g. mixtures of spherical particles, randomly oriented nonspherical particles). For polarised multiply scattered LIDAR signals we have such programs too, but with the additional restriction that the scatterers are mixtures of spherical particles[5,6,7,9,4] . These programs are about to be generalized.

Figure 3: The total intensity for different values of the mode radius and the extinction.

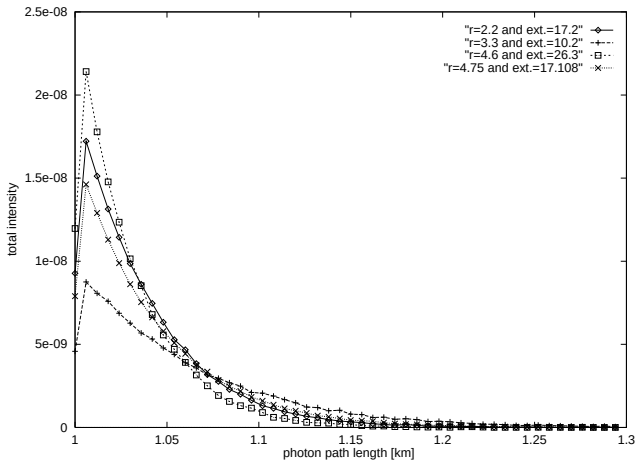
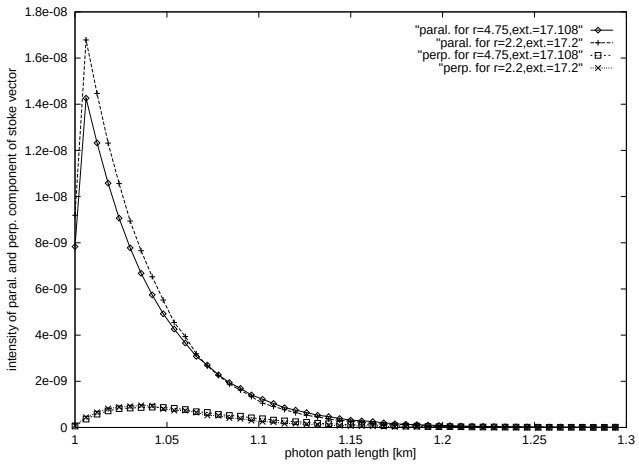


Figure 4: The parallel and perpendicular intensities for different values of the mode radius and the extinction.



4. RETRIEVAL OF PARAMETERS FROM LIDAR SIGNALS BY SEARCH PROCEDURES

The ultimate goal of in situ or remote LIDAR sensing of the atmoshere is the determination of microphysical parameters which are important for meteorologists. For example, such parameters are the extinction, the absorption, the phasefunction, the particle number density, the particle size, shape and orientation.

The most simple way of evaluating multiply scattered LIDAR signals is to compare them with an appropriate set of calculated multiply scattered LIDAR signals. It is very hard, if not impossible, to draw conclusions on the wanted parameters only by such inspection. One reason for that is the huge number of parameters (e.g. of the phase function) entering the exact multiply scattering LIDAR equations.

Procedures which allow for extracting such parameters from one or several (e.g. FOV, wavelength, polarisation) LIDAR signals are called inversion procedures. Usually these inversion procedures are obtained from solving an exact multiple scattering LIDAR equation for the wanted parameters. Of course, many parameters enter an exact multiple scattering LIDAR equation. Only under additional assumptions on the other environmental parameters this equation will be uniquely solvable for one or a few of these parameters. But usually in applications the values of these fixed parameters will not be known. Furthermore, these inversion procedures will be very sensitive to variations of the fixed parameters. The reason is that most of these inversion problems are "ill-posed" problems. Hence, these inversion methods are often extremely unreliable. A sensitivity analysis of the inversion procedure is needed to judge the quality of the inversion.

We have designed, implemented, and applied inversion algorithms which are based on uniform grid search and random search procedures and which include a sensitivity analysis:

Each set of environmental parameters is a vector p of some subset P of an n-dimensional Euclidian space. A random search procedure e.g. is a procedure which chooses a vector p in the set P at random according to some probability law (which may depend on the history of the search process). To this parameter p the multiple scattering LIDAR signal is calculated by variance reduction Monte Carlo simulation. Then this simulated signal is compared to a given (e.g. measured) multiple scattering LIDAR signal. For comparison we can use functionals describing the distance between the measured and the simulated signal appropriately (e.g. by a Chebyshev norm for some interval, a weighted L_2–norm).

If we shall use an efficient random search procedure (e.g. uniform random search, uniform search on circles, uniform search on circles with adapted radii; genetic algorithms, Kiefer–Wolfowitz procedure) we shall find regions of parameter vectors which are "solutions" of our inversion problem.

Such random search procedures may be designed to be very efficient. Of course, many comparisons have to be made (by the computer). Much computing time is needed, since every single variance reduction Monte Carlo simulation of a multiple scattering LIDAR signal is time consuming (which can be reduced a little bit by starting simulations with higher variance and reducing the variance progressively). A by–product of the random search procedure (and even much more of the uniform grid search) is a sensitivity analysis of the dependence of the solution on the parameters. This is done by noting the progress of approximation of the distance between the measured and the simulated LIDAR signal for the inspected parameters during the search process.

5. EXAMPLE OF A SEARCH PROCEDURE

In order to make the main idea clear, let us consider an example. Furthermore, let us use the (not very effective) uniform grid search instead of the (usually more effective) random search:
We want to retrieve the extinction coefficient and the size distribution of the scattering particles of a homogeneous cloud from the multiply scattered LIDAR signal. The cloud is at a distance of 1 km from the monostatic LIDAR which has a conical and coaxial transmitter and receiver field of view with a full angle of aperture of 0.1 mrad and 10 mrad, respectively. The receiver telescope has the radius of 1 m. The transmitted beam has the wavelength of $\lambda = 1.064 \mu$m and is directed perpendicularly towards the cloud which is 0.3 km thick. Before and behind the cloud are no scattering particles, i.e. the extinction coefficient there is 0. Within the cloud the extinction coefficient σ is constant, we know that

$$6 < \sigma < 30 \qquad [\mathrm{km}^{-1}].$$

The cloud is supposed to consist of spherical particles with the normed size distribution[3]

$$n(r) = a\, r^{\alpha}\, \exp(-br^{\gamma}), \qquad 0 \le r_c < \infty \ \text{ and } \ a, \alpha, b, \gamma \in \mathbb{R}^{+}$$

$$\text{with} \qquad a = \gamma^{\frac{\gamma-\alpha+1}{\gamma}} r_c^{-(\alpha+1)} \alpha^{\frac{\alpha+1}{\gamma}} (\Gamma(\frac{\alpha+1}{\gamma}))^{-1} \qquad \text{and} \qquad b = \tfrac{\alpha}{\gamma r_c^{\gamma}}.$$

r_c is the mode radius of the size distribution. Furthermore, we assume that

$$\alpha = 6.0, \quad \gamma = 1.0, \qquad \text{and} \quad 1.5 \le r_c \le 5.0 \quad [\mu\mathrm{m}].$$

Knowing these values and the refraction index $m = 1.33 - 0.0i$ and the wavelength $\lambda = 1.064$ [μm] we can calculate the phase function from the Mueller matrix and, hence, the scattering distribution of the cloud.

To show the main ideas of retrieval algorithms based on uniform grid (and similarly on random search), let us assume that we are in the following situation:
Somebody has calculated the parallelly and the perpendicularly polarised multiply scattered LIDAR signal for the values

$$r_c = 4.75\,\mu\mathrm{m} \qquad \text{and} \qquad \sigma = 17.108\,\mathrm{km}^{-1}$$

which are known only to him. He asks us to find the parameters σ and r_c of this LIDAR signal. To answer this question we could proceed as follows:
We subdivide the r_c – interval [1.5,5.0] by steps of 0.1 μm and the σ – interval [6,30] by steps of 0.7 km^{-1}. Doing this we get

$$35 \times 34 = 1190 \text{ grid points } (r_c, \sigma) \text{ on the } r_c - \sigma \text{ – rectangle } [\ 1.5{,}5.0] \times [6{,}30]\ .$$

For every such grid point (r_c, σ) we calculate the polarised multiply scattered LIDAR signal by variance reduction Monte Carlo simulation. From the Stokes vector of the simulated LIDAR signal we may obtain the parallelly and the perpendicularly polarised part of the LIDAR signal and, hence, the total intensity of the LIDAR signal. We want to retrieve the parameters of the given LIDAR signal by comparing this given signal with the simulated signals for all the 1190 points of the uniform grid of the parameter space. We may use the total signal or the parallel or the perpendicular component of the polarised signal for comparison. Furthermore, as an outcome of a Monte Carlo simulation the value

of a simulated signal at a certain time point (which corresponds to a photon path length or a "maximal range") is a realization of a random variable; the empirical variance of this random variable is obtained as a by–product from our program. For comparison we could also use this type of information about the randomness of the simulated signal. Hence, we have several possibilities to compare the given signal with the simulated signal. We shall see that the result and the quality of the retrieval methods depend havily on the chosen method of comparison.

First, we want to compare the total intensities of the given and the simulated LIDAR signals. The total intensity of the given LIDAR signal and the simulated LIDAR signal for a pair (r_c, σ) of parameters is represented by a function

$$g : \mathrm{T} \longrightarrow \mathbb{R} \qquad \text{and} \qquad s_{(r_c,\sigma)} : \mathrm{T} \longrightarrow \mathbb{R},$$

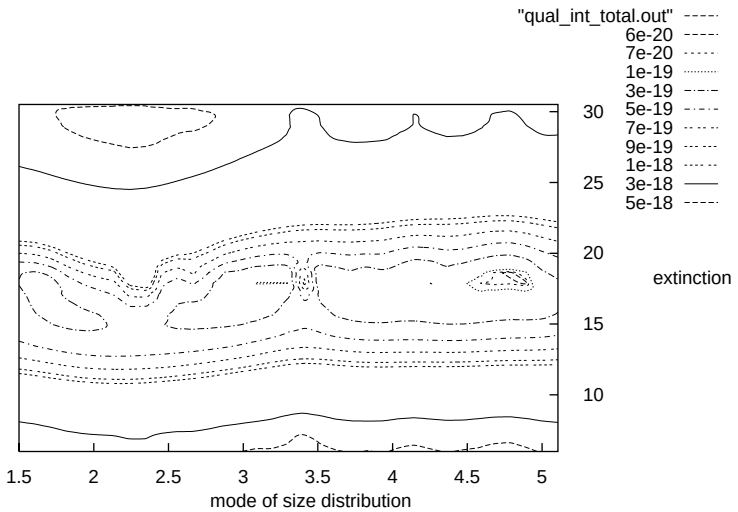
respectively. Here T is a set of discrete time intervals (each one of these is a measuring time interval) which corresponds to the photon path length; |T| then denotes the number of such time intervals. To measure the distance between the given and the simulated signal, we may choose the L_2 – distance

$$Q : [1.5, 5.0] \times [6, 30] \longrightarrow \mathbb{R}_0^+ \qquad \text{with} \qquad (r_c, \sigma) \longrightarrow Q(r_c, \sigma) := \frac{1}{|\mathrm{T}|} \sum_{t \in \mathrm{T}} |\mathrm{g(t)} - \mathrm{s}_{(r_c,\sigma)}(\mathrm{t})|^2.$$

This value will be 0 for $r_c = 4.75$ and $\sigma = 17.108$ (if we have calculated exactly, i.e if the variance is reduced "numerically sufficiently"). Maybe it will be 0 for other points, too. In this case the inversion is not unique. If it is only 0 for $r_c = 4.75$ and $\sigma = 17.108$, the inversion is unique. Usually, we don't know this.

The function Q on the (here) two-dimensional parameter rectangle defines a (here) three-dimensional "mountain area" on the $r_c - \sigma$ – rectangle. Since the grid points are fairly dense, the values of Q for the grid points determine the contour-lines of this mountain area.

Figure 5: contour lines of the "mountain area" for values of Q with the total intensity.



The contour-lines in Figure 5 show that the true pair of parameters $r_c = 4.75\mu\mathrm{m}$ and $\sigma = 17.108\mathrm{km}^{-1}$ was retrieved by this primitive procedure pretty precisely. In addition the procedure shows that the sensitivity of the total intensity towards a variation of the parameter r_c (and hence towards a variation of the size distribution) is quite different from the sensitivity of the signal towards a variation of the extinction coefficient σ. Obviously, such an information about the sensitivity of the signal is very valuable.

Now, we want to compare the parallelly and the perpendicularly polarised intensities of the given and the simulated LIDAR signals. The parallel and the perpendicular intensity of the given LIDAR signal and the simuluated LIDAR signal for a pair (r_c, σ) of parameters is represented by a function

$$\begin{array}{lllll} g_{\parallel}: & \mathrm{T} \longrightarrow \mathrm{R} & \text{and} & s_{\parallel(r_c,\sigma)}: \mathrm{T} \longrightarrow \mathrm{R} & \text{and} \\ g_{\perp}: & \mathrm{T} \longrightarrow \mathrm{R} & \text{and} & s_{\perp(r_c,\sigma)}: \mathrm{T} \longrightarrow \mathrm{R}, & \end{array}$$

respectively.

Again, to measure the distance between the given and the simulated signal, we may choose the L_2–distance

$$Q_{\parallel}: [1.5, 5.0] \times [6, 30] \longrightarrow \mathbb{R}_0^+ \quad \text{with} \quad (r_c, \sigma) \longrightarrow Q_{\parallel}(r_c, \sigma) := \frac{1}{|\mathrm{T}|} \sum_{t \in \mathrm{T}} |g_{\parallel}(t) - s_{\parallel}(t)|^2,$$

$$Q_{\perp}: [1.5, 5.0] \times [6, 30] \longrightarrow \mathbb{R}_0^+ \quad \text{with} \quad (r_c, \sigma) \longrightarrow Q_{\perp}(r_c, \sigma) := \frac{1}{|\mathrm{T}|} \sum_{t \in \mathrm{T}} |g_{\perp}(t) - s_{\perp}(t)|^2,$$

$$Q_{\mathrm{depol}}: [1.5, 5.0] \times [6, 30] \longrightarrow \mathbb{R}_0^+ \quad \text{with} \quad (r_c, \sigma) \longrightarrow Q_{\mathrm{depol}}(r_c, \sigma) := \frac{1}{|\mathrm{T}|} \sum_{t \in \mathrm{T}} |\frac{g_{\parallel}(t)}{g_{\perp}(t)} - \frac{s_{\parallel}(t)}{s_{\perp}(t)}|^2,$$

respectively, and the aggregated L_2 -distance

$$Q_{\parallel,\perp}: [1.5, 5.0] \times [6, 30] \longrightarrow \mathbb{R}_0^+ \quad \text{with} \quad Q_{\parallel,\perp}(r_c, \sigma) := Q_{\parallel}(r_c, \sigma) + Q_{\perp}(r_c, \sigma).$$

All the calculations were done with the program `pbs` recently published[7,4] . Each Monte Carlo simulation was done with 1 million starting photons. Obviously, such a huge number of time consuming Monte Carlo simulations can only be done on powerful workstations or on a network of such workstations, but never on a PC in acceptable time.

We have seen that our proposed retrieval algorithm is a minimization procedures combined with a (stochastic or deterministic) calculation program for multiply scattered LIDAR signals. The more parameters have to be retrieved the higher dimensional is the space on which this minimization problem has to be solved. In higher dimensions a uniform grid search is not possible in acceptable time. Therefore, we have to switch to more effective search algorithms such as random search[12,11,13] .

6. CONCLUSIONS

Concerning the proposed retrieval algorithms for microphysical parameters from polarised multiply scattered LIDAR signals, we arrive at the following conclusions:

- they cannot be operated on a PC
- they need a network of powerful workstations
- they are very generally applicable e.g. without questionable assumptions on the environment
- they are equally appropriate for all kinds of retrieval problems
- they are applicable for larger sets of parameters
- they give empirical information on the uniqueness of the solution of the retrieval problem
- they yield a sensitivity analysis of the retrieval problem as a by–product

Especially this sensitivity analysis is equally valuable for the interpretation of results of the retrieval, the comparison of retrieval algorithms, and for the designing of instrumental equipment.

Figure 6: "mountain area" for values of $Q_{\parallel}$.

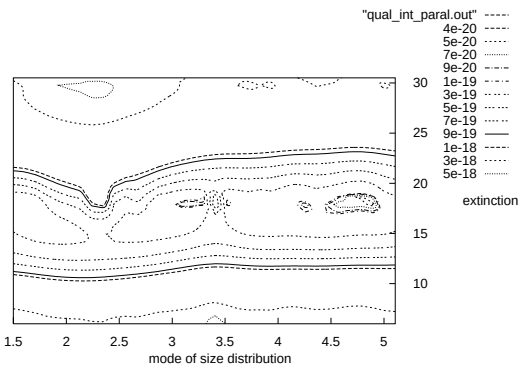


Figure 7: "mountain area" for values of $Q_{\perp}$.

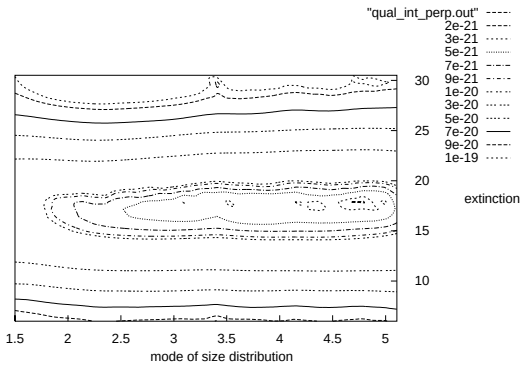


Figure 8: "mountain area" for values of Q_{depol}.

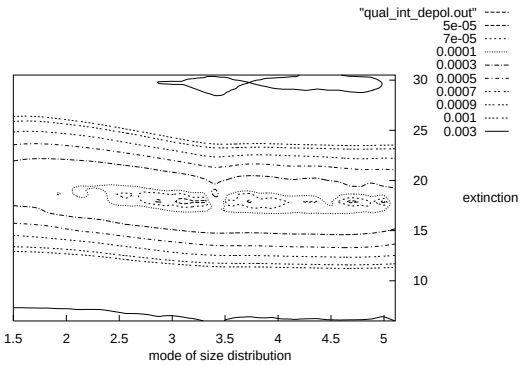
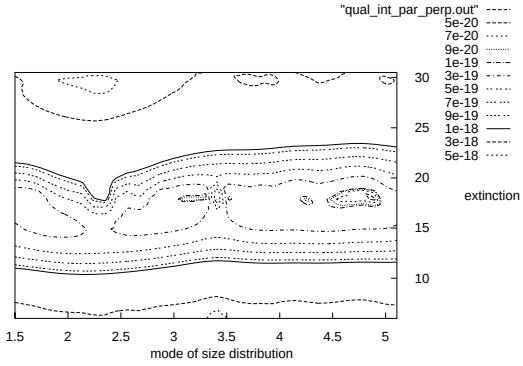


Figure 9: "mountain area" for values of $Q_{\parallel,\perp}$.



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