Calculations of polarized multiply scattered return signals of a space-based multichannel LIDAR and the retrieval of the extinction coefficient and the particle size distribution based on random search including a sensitivity analysis.[1]

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ABSTRACT

First, we present some examples of calculations of polarized multiply scattered return signals of a space–based multichannel LIDAR (such as the LITE) and compare them to return signals of a ground–based LIDAR. These methods are variance reduction Monte Carlo algorithms allowing for controlling the empirical variance. Second, we recall the method of random search shortly. Random search procedures are Monte Carlo algorithms which are designed to solve deterministic or stochastic optimization problems and, hence, also deterministic or stochastic equations. Third, we show how variance reduction Monte Carlo methods may be combined with properly chosen random search procedures to retrieve one or more environmental parameters from a given (measured or calculated) LIDAR return signal. Such a procedure is time consuming, of course, but it will allow for the retrieval of several parameters simultaneously and for a sensitivity analysis. Hence, a retrieval procedure based on the combination of variance reduction Monte Carlo methods and random search including such a sensitivity analysis gives much more information than the usual inversion procedures (e.g. based on integral equations and often uncheckable environmental assumptions). Fourth, we present some examples of the (simultaneous) retrieval of the extinction coefficient and the particle size distribution of a cloud from the multiply scattered return signal of a space–based LIDAR and a sensitivity analysis of this retrieval.

1. INTRODUCTION

We present some examples of calculations of polarized multiply scattered return signals of a space–based multichannel LIDAR (such as the LITE) obtained with the help of a variance reduction Monte Carlo code. Using this fast code, we are able to retrieve environmental parameters from a given (e.g. measured) LIDAR return signal by solving a minimization problem with a search strategy. This inversion method is based on five ingredients:

- the stochastic model for the transport of multiply scattered polarized light resulting in an exact LIDAR equation
- a variance reduction Monte Carlo method based on this model,
- a model for the scattering (and extinction) probabilities determined by the environmental parameters,
- a distance measure for the comparison of LIDAR return signals,
- a search strategy for the minimization of the distance between the simulated and the measured LIDAR return signal.

For each set of enviromental parameters the LIDAR return signal may be calculated with the help of our variance reduction Monte Carlo code. Using a properly chosen distance measure for the comparison of LIDAR return signals, this simulated signal may be compared to the measured signal. If the chosen parameter set is the true parameter set, the simulated and the measured signal must coincide (or coincide modulo some noise) and, hence, the distance between the two signals must be zero (or zero modulo some noise). If the inversion problem has a unique solution, the inversion problem may be solved by minimization of the distance between the simulated and mesured signal. To

[1] In: Lidar Techniques for Remote Sensing. Ed. Christian Werner. Proc. Europtro Series. Vol. 2581, 79–90. (Proceedings of "'The European Symposium on Satellite Remote Sensing"', Paris, September 25–29, 1995)

solve the minimization problem we may apply properly chosen search procedures such as uniform grid search or more effective random search procedures. Such a procedure is time consuming, of course, but

- it will allow for the retrieval of several parameters simultaneously,
- it will allow for a sensitivity analysis of the dependency of the LIDAR return signal on the enviromental parameters.

2. EXACT MULTIPLE SCATTERING LIDAR EQUATIONS AND VARIANCE REDUCTION MONTE CARLO METHODS

All the modeling techniques of multiple scattering are essentially based on the idea of iterating the single scattering process. Single scattering is characterized by the scattering distribution. The scattering distribution is defined by the phase function or by the scattering matrix in models with polarization. Our approach to simulate the transport of light is based on a stochastic model which describes the process of multiple scattering as a generalized Rayleigh's random walk, it is a general and precise mathematical description of the physical process of multiple scattering of light viewed as a "stochastic corpuscular multiple scattering process"[4–8,12] . This model is essentially equivalent to the radiative transfer theory (also called transport theory) of the theory of differential equations. We shall briefly review the stochastic process of corpuscular multiple scattering with a description of polarization. In the framework of this model we are able to derive exact multiple scattering LIDAR equations; we shall present an example of such an equation. Then we shall discuss some results of space–based LIDAR return signals obtained with variance reduction Monte Carlo methods based on these LIDAR equations and compare them with ground–based LIDAR return signals.

2.1 The stochastic model and an exact multiple scattering LIDAR equation

To introduce our stochastic model of the stochastic process of corpuscular multiple scattering, we assume that light is corpuscular and that photons, which are scattered by particles of the atmosphere, travel along straight lines from one point of scattering to the next.

For the description of polarized light we use modified Stokes vectors[2] : $ST := (I_\parallel, I_\perp, U, V)^T$ where $I := I_\parallel + I_\perp$ denotes the intensity. The Stokes vector is defined with respect to a reference plane. This plane of reference is given by the direction of propagation $\varphi \in S^2$ (where S^2 denotes the unit sphere in $\mathbb{R}^3$) and a reference vector $\rho \in S^2$ with $\rho \perp \varphi$. We define the normed Stokes vector:

$$st := (i_\parallel, i_\perp, u, v)^T \quad := \quad \begin{cases} (\frac{I_\parallel}{I}, \frac{I_\perp}{I}, \frac{U}{I}, \frac{V}{I})^T & \text{for } I \neq 0, \\ (0,0,0,0) & \text{for } I = 0. \end{cases}$$

The stochastic model is an expanded version of the model we have proposed and used earlier[9,11,12] . In this expanded model the path of a polarized photon is characterized by its starting point x_0, its starting direction φ_0, its state of polarization st_0 (st_0 is the normed Stokes vector), the reference vector ρ_0, and by the sequence of the time intervals t_n between the $(n-1)$–th and the n–th scattering event, the direction φ_n, the Stokes vector st_n and the reference vector ρ_n after the n–th scattering event. We describe the transition from the state after the n–th scattering $(t_n, \varphi_n, st_n, \rho_n)$ to the state after the $n+1$–th scattering $(t_{n+1}, \varphi_{n+1}, st_{n+1}, \rho_{n+1})$ using the extinction transition probability E_n and the scattering transition probability P_n. The transition probabilities E_n describing the extinction are the same as in the unpolarized case and are independent of the Stokes vectors:

$$E_n(x_0, \varphi_0, st_0, \rho_0; t_1, \varphi_1, st_1, \rho_1; \ldots; t_{n-1}, \varphi_{n-1}, st_{n-1}, \rho_{n-1}; \cdot) := E_n(x_0, \varphi_0; t_1, \varphi_1; \ldots; t_{n-1}, \varphi_{n-1}; \cdot).$$

As a consequence of the Beer–Lambert law E_n has the shape of a generalized exponential distribution[12] . The scattering transition probabilities P_n, however, depend on the Stokes vectors and the reference vectors:

$$P_n(x_0, \varphi_0, st_0, \rho_0; t_1, \varphi_1, st_1, \rho_1; \ldots; t_{n-1}, \varphi_{n-1}, st_{n-1}, \rho_{n-1}; t_n; \cdot).$$

For mixtures of spherical scatterers P_n may be calculated from the Mueller matrix, respectively the polarized phase function[4,5,7] , see also subsection 2.2 below.

In terms of the polarized stochastic multiple scattering process let us now state an exact multiple scattering LIDAR equation, the time dependent LIDAR equation for the polarized flux:
The transmitter of the LIDAR is characterized by a probability distribution on the $(x_0, \varphi_0, t_0, st_0, \rho_0)$–space, consisting of the initial space point x_0, direction φ_0, time t_0, Stokes vector st_0, and reference vector ρ_0. The receiver is supposed to have a conical field of view with an effective area A_R with center x_R, radius r_R, normal ν_R on the receiver plane A_R, and reference vector ρ_R. For sake of simplicity we assume that $x_R := (0,0,0)^T$. We denote by $\mathcal{R}$ the set of space–direction–time triples $(x, \varphi, t) \in \mathrm{IR}^3 \times \mathrm{S}^2 \times \mathrm{IR}_0^+$ such that x is in front of the receiver and the line from x to $x + ct\varphi$ is intersecting the receiver within the receiver field of view $\mathrm{FOV_R}$. $\mathcal{T}$ is the measuring time interval. Then the four-dimensional vector

$$f_{\mathcal{T}}(x, \varphi, st, \rho, t) := 1_{\mathcal{T}}\left(t + \frac{-<x, \nu_R>}{c<\varphi, \nu_R>}\right) \frac{<\varphi, \nu_R>}{<x, \nu_R>^2}\ \mathcal{L}(\Gamma(\varphi, \rho))\, st$$

is the polarized time dependent flux through the receiver at the time t from the space point x in the direction of φ with Stokes vector st and the reference vector ρ. Here $<.,.>$ is the Euclidian scalar product, $1_{\mathcal{T}}$ is the indicator function of the measuring time interval $\mathcal{T}$, $\Gamma(\varphi, \rho) := \arccos(<\nu_R \times \rho_R, \varphi \times \rho>)$ is the angle between reference plane of the photon at position x and the reference plane of the receiver, and $\mathcal{L}(\phi)$ is the matrix for rotation of the Stokes vector[2] . (This formula for the flux is true if certain conditions in the very near neighbourhood of the receiver are fulfilled; otherwise some technical conditions have to be added[4] .)

The Stokes vector of the polarized LIDAR signal which is received during the (e.g. small) measuring interval $\mathcal{T}$ is equal to $st_R(\tau; \mathcal{T}) = \sum_{n \in \mathrm{IN}} st_R(\tau; \mathcal{T}; n)$ where the integral

$$\begin{aligned} st_R(\tau; \mathcal{T}; n) := & \int_{\mathrm{IR}^3 \times \mathrm{S}^2 \times \mathrm{IR}^4 \times \mathrm{S}^2 \times \mathrm{IR}_0^+} \int_{\mathrm{IR}_0^+} \int_{\mathrm{S}^2 \times \mathrm{IR}^4 \times \mathrm{S}^2} \cdots \int_{\mathrm{S}^2 \times \mathrm{IR}^4 \times \mathrm{S}^2} \int_{\mathrm{IR}_0^+} \\ & f_{\mathcal{T}}(x_{(n)}, \varphi_n, st_n, \rho_n, \textstyle\sum_{i=0}^n t_n)\ 1_{\mathcal{R}}(x_{(n)}, \varphi_n, t_{n+1}) \displaystyle\prod_{i=0}^{n-1} 1_{\bar{\mathcal{R}}}(x_{(i)}, \varphi_i, t_{i+1}) \\ & \mathrm{E}_{n+1}((x_0, \varphi_0, t_0); t_1, \varphi_1; \ldots; t_n, \varphi_n; dt_{n+1})\ \mathrm{P}_n((x_0, \varphi_0, st_0, \rho_0, t_0); t_1, \varphi_1, st_1, \rho_1; \ldots; t_n; d(\varphi_n, st_n, \rho_n)) \ \ldots \\ & \ldots\ \mathrm{P}_1((x_0, \varphi_0, st_0, \rho_0, t_0); t_1; d(\varphi_1, st_1, \rho_1))\ \mathrm{E}_1((x_0, \varphi_0, t_0); dt_1) \quad \tau(d(x_0, \varphi_0, st_0, \rho_0, t_0)) \end{aligned}$$

is to be taken coordinatewise and represents the contribution of the n-th order of multiple scattering, $x_{(i)} := x_0 + \sum_{j=0}^{i} ct_j \varphi_{j-1}$ and $\bar{\mathcal{R}}$ is the complement of $\mathcal{R}$. These multiple integrals correspond to the contributions of the different orders of multiple scattering. The interpretation of these complicated integrals is directly related to the physical interpretation and the construction of the stochastic process of corpuscular multiple scattering. (Look at the integral $st_R(\tau; \mathcal{T})$ from the rear: The photon starts in x_0, is getting scattered after time t_1 into the direction φ_1 and so on.) It can be shown[4,11] that our stochastic approach is equivalent to the scattering density model and with some additional assumptions to the collision density model in the transport theory[10] .

At present it does not seem to be possible to calculate the iterated integrals describing the contributions of the higher orders of multiple scattering analytically or numerically exactly. These exact LIDAR equations allow for calculating the LIDAR signal by (direct) Monte Carlo methods, but in a very inefficient way only. To improve the efficiency of the Monte Carlo simulation we introduce variance reduction methods (such as local estimates, scattering splitting, and exponential transforms) and we switch from the stochastic model of the type of the generalized Rayleigh's random walk to a much more complicated stochastic model of a "stochastic multiple scattering cascade process" which describes the variance reduction methods, reproduces the exact multiple scattering LIDAR equations, and guarantees fast calculation of the LIDAR signal[1,4,8,11] . The precision of the calculation is controlled by the empirical variance of the Monte Carlo simulation; this variance may be chosen as small as it is wanted. Of course, the higher the demanded precision the more computer time is needed for the simulation of the LIDAR signal.

2.2 Extinction and scattering probabilities

The LIDAR return signals are determined by the geometry of the LIDAR transmitter and receiver, the cloud geometry, and the extinction and scattering properties of the atmosphere. In our stochastic aproach we modeled the extinction and scattering properties with transition probabilities. To connect the extinction and scattering transition probabilities with the environmental microphysical parameters, we need a model for the description of the cloud in terms of these parameters.

In this paper we assume that the cloud consists of spherical particles with the size distribution[3]

$$n(r) = a\, r^{\alpha} \exp(-br^{\gamma}), \qquad 0 \le r_c < \infty \text{ and } a, \alpha, b, \gamma \in \mathbb{R}^+$$

$$\text{with} \qquad a = N^{-1} \gamma^{\frac{\gamma-\alpha+1}{\gamma}} r_c^{-(\alpha+1)} \alpha^{\frac{\alpha+1}{\gamma}} (\Gamma(\frac{\alpha+1}{\gamma}))^{-1} \qquad \text{and} \qquad b = \frac{\alpha}{\gamma r_c^{\gamma}}.$$

r_c is the mode radius of the size distribution, N the number of particles per unit volume. The parameters γ and α determine the width and asymmetry of the distribution. In our calculations we use $\alpha = 6.0$ and $\gamma = 1.0$, the standard values for a Deirmendjian C1 cloud. Knowing these values and the refraction index $m = 1.33 - 0.0i$ and the wavelength $\lambda = 0.355$ [μm] we may calculate the extinction

$$\sigma \quad = \quad \int_0^{\infty} r^2 \pi \; n(r) K_{\text{ext}}(r) dr,$$

with the extinction efficiency $K_{\text{ext}}(r)$ [3] . The extinction transition probability E_n is determined by the extinction coefficient σ [12] . Similary we are able to calculate the normalized components of the Mueller matrix. We only show the first component,

$$\mathcal{M}_{11}(\Theta) \quad = \quad \frac{2\lambda^4}{\pi^3 \sigma} \int_0^{\infty} n(r) S_1(r, \Theta) S_1^{\star}(r, \Theta) dr,$$

where S_1 is one of the complex amplitudes used in the Mie series expansion[3] . With the Mueller matrix we are able to calculate the polarized phase function and therefore the scattering distribution P_n of the cloud[4,5,7] . Inserting σ in the equation for Mueller matrix we notice that $\mathcal{M}$ is independent of the number of particles N per unit volume. Since σ is linearly dependend on N we use σ instead of N as a model parameter.

Therefore we investigate the dependency of the LIDAR return signals on the extinction coefficient σ and the mode radius r_c under the assumption that our model of the cloud is accurate. We are not able to judge the model, but we are able to investigate how sensitive the model is towards variations of the parameters.

3. MULTIPLY SCATTERED RETURN SIGNALS OF A SPACE–BASED LIDAR

Using our variance reduction Monte Carlo methods, we simulated LIDAR return signals from 0.3 km thick C1–like clouds at a distance of 400 km (**"space–based"**) and for comparison at a distance of 1 km (**"ground–based"**). The transmitter beam of a monostatic LIDAR with conical and coaxial fields of view of the transmitter and the receiver is directed perpendicularly towards the cloud. In the space–based situation the transmitter operates at a wavelength of 0.355 μm with a spot size of 2 cm and a semi-aperture of 0.03 mrad. The receiver has a diameter of 98 cm and a semi-aperture of 0.5 mrad. The geometry of the ground–based LIDAR is described in Kerscher et al.[5] Before and behind the cloud no scattering particles are supposed to be, i.e. the extinction coefficient σ there is zero. Within the cloud we assume the extinction coefficient σ to be constant.

In Figure 1 and Figure 2 we show ground–based and space–based LIDAR return signals, respectively. The figures show the total intensities of the signals and the contributions of the different orders of muliple scattering. Although the cloud is in both cases identical (with $\sigma = 17.108\text{km}^{-1}$ and $r_c = 4.75\mu\text{m}$), the curves of the total intensities and the contributions of the different orders of multiple scattering are quite different in shape and in magnitude.
The space–based LIDAR return signal has a higher magnitude of the total intensity and the contributions of the different orders of multiple scattering and is delayed in time. These effects may be explained by simple geometric considerations, at least partially: the reflecting volume for a ground–based LIDAR is a pencil–like narrow cone, whereas for a space–based LIDAR it has the shape of a fairly broad disk of a diameter of 24 m and an area of more than 450 m^2. In the space–based case this disk is the base of the cone of the sounding volume at the surface of the cloud.

In Figure 3 and Figure 4 we show several ratios of the contribution of some orders of multiple scattering to the total intenstity of the return signal of a ground–based and a space–based LIDAR, respectively:

Figure 1:
The total intensity and the contributions of different scattering orders of a **ground–based** LIDAR return signal from a cloud with mode radius 4.75 μm and the extinction coefficient 17.108 km^{-1}.

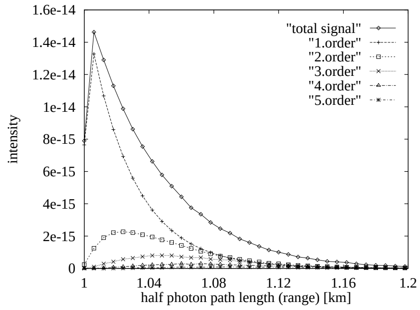


Figure 2:
The total intensity and the contributions of different scattering orders of a **space–based** LIDAR return signal from a cloud with mode radius 4.75 μm and the extinction coefficient 17.108 km^{-1}.

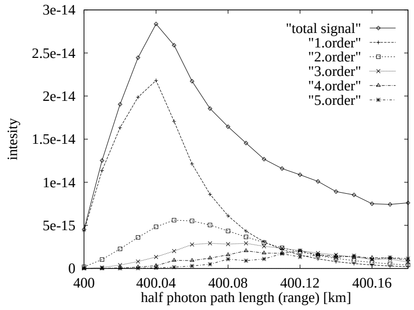


Figure 3: The ratios of the different scattering orders of a **ground–based** LIDAR return signal from a cloud with mode radius 4.75 μm and the extinction coefficient 17.108 km^{-1}.

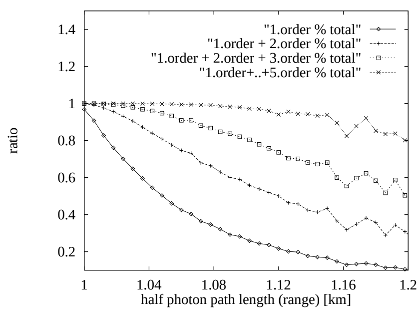


Figure 4: The ratio of the different scattering orders of a **space–based** LIDAR return signal from a cloud with mode radius 4.75 μm and the extinction coefficient 17.108 km^{-1}.

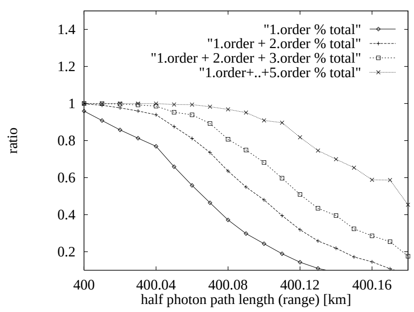


Figure 5: The depolarization of a **ground–based** and a **spaced-based** LIDAR return signal from a cloud with mode radius 4.75 μm and the extinction coefficient 17.108 km^{-1}.

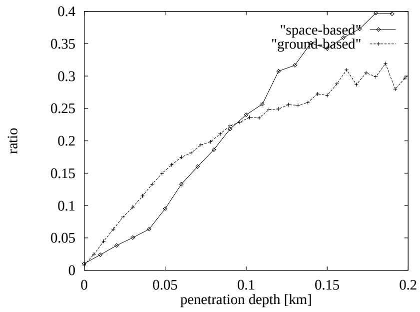


Figure 6: The parallel and perpendicular components of the Stokes vector of a **space–based** LIDAR return signal from a cloud with mode radius 4.75 μm and the extinction coefficient 17.108 km^{-1}: total signal, 1.order scattering, and 2.order scattering

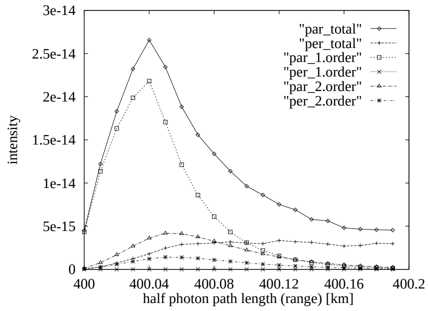


Figure 7: The total intensity of a **spaced–based** LIDAR return signal from clouds with different values of the mode radius and the extinction coefficient.

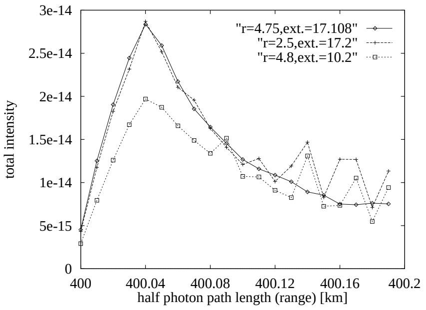
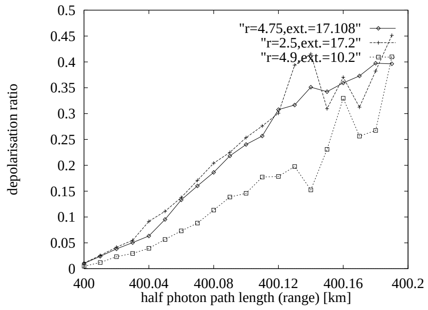


Figure 8: The depolarization of a **spaced-based** LIDAR return signal from clouds with different values of the mode radius and the extinction coefficient.



- first order versus total
- first and second order versus total
- first, second, and third order versus total
- first order up to fifth order versus total

Again in both cases we have the same identical parameters of the cloud. Yet the curves of the ratios for the ground–based and the space–based LIDAR are quite different.

In Figure 5 we show the linear depolarization ratio $I_\perp/(I_\| + I_\perp)$ of a ground–based and a space–based LIDAR. Again, in both cases we have the same idendical parameters of the cloud, but a totally different behaviour of the curves of these ratios.

In Figure 7 we show the total intensities of space–based LIDAR return signals for different values of the mode radius r_c (abbreviated by r in the plots) and the extinction coefficient σ (abbreviated by ext in the plots) of the cloud. The variance reduction Monte Carlo simulations were done for the parameters $r_c = 4.75\mu$m and $\sigma = 17.108\text{km}^{-1}$ with 10,000,000 starting photons and for the other pairs of parameters with 250,000 starting photons. The curves show that the variance of the Monte Carlo simulation is increasing with the photon phath length. Furthermore, the plots show that the variance of the Monte Carlo simulation is decreasing with the number of starting photons. Our code allows for controlling this variance at every path length; it allows for explicite calculation of the contributions of all orders of multiple scattering and their empirical variances.
In Figure 8 the linear depolarization ratios for space–based LIDAR return signals are plotted for the pairs of parameters and the numbers of starting photons of the simulations shown in Figure 7.
From the curves of Figure 7 and Figure 8 we see that a slight variation of the extinction coefficient has more influence on the shape of the curves of the total intensity and the depolarization than a much larger variation of the mode radius. This considerable effect will show up much clearer from the sensitivity analysis of the retrieval procedure based on uniform grid search; see subsection 4.2 and Figures 9–16.

4.RETRIEVAL OF ENVIRONMENTAL PARAMETERS

The ultimate goal of in situ or remote LIDAR sensing of the atmosphere is the determination of microphysical parameters which are important for meteorologists. For example, such parameters are the extinction, the absorption, the phase function, the particle number density, the particle size, shape and orientation.

The most simple way of evaluating multiply scattered LIDAR signals is to compare them with an appropriate set of calculated multiply scattered LIDAR signals. It is very hard, if not impossible, to draw conclusions on the wanted parameters only by such inspection. One reason for that is the huge number of parameters (e.g. of the phase function) entering the exact multiply scattering LIDAR equations.

Procedures which allow for extracting such parameters from one or several (e.g. FOV, wavelength, polarization) LIDAR signals are called inversion procedures. Usually these inversion procedures are obtained from solving an exact multiple scattering LIDAR equation for the wanted parameters. Of course, many parameters enter an exact multiple scattering LIDAR equation. Only under additional assumptions on the other environmental parameters this equation will be uniquely solvable for one or a few of these parameters. But usually in applications the values of these fixed parameters will not be known. Furthermore, these inversion procedures will be very sensitive to variations of the fixed parameters. The reason is that most of these inversion problems are "ill-posed" problems. Hence, these inversion methods are often extremely unreliable. A sensitivity analysis of the inversion procedure is needed to judge the quality of the inversion.

We have designed, implemented, and applied inversion algorithms which are based on uniform grid search and random search procedures and which include a sensitivity analysis:

Each set of environmental parameters is a vector p of some subset P of an n-dimensional Euclidian space. A random search procedure is a procedure which chooses a vector p in the set P at random according to some probability law (which may depend on the history of the search process). To this parameter p the multiple scattering LIDAR signal is calculated by variance reduction Monte Carlo simulation. Then this simulated signal is compared to a given (e.g. measured) multiple scattering LIDAR signal. For comparison we use the distance measures described below.

If we shall use an efficient random search procedure (e.g. uniform random search, uniform search on circles, uniform search on circles with adapted radii; genetic algorithms, Kiefer–Wolfowitz procedure) we shall find regions of parameter vectors which are "solutions" of our inversion problem.

Such random search procedures may be designed to be very efficient. Of course, many comparisons have to be made (by the computer). Much computing time is needed, since every single variance reduction Monte Carlo simulation of a multiple scattering LIDAR signal is time consuming (which can be reduced a little bit by starting simulations with higher variance and reducing the variance progressively). A by–product of the random search procedure (and even much more of the uniform grid search) is a sensitivity analysis of the dependence of the solution on the parameters. This is done by noting the progress of minimization of the distance between the measured and the simulated LIDAR signal for the inspected parameters during the search process. We shall present examples of inversions based on different search strategies and different distance measures.

4.1 Distance measures for the comparison of LIDAR return signals

We want to retrieve the environmental parameters from a given LIDAR return signal by comparing the given signal to simulated signals with known parameters. For this comparision we may use properly chosen distance measures. If we have a multichannel LIDAR, for this comparison we may use the

- total intensities $g : T \longrightarrow \mathbb{R}$ and $s_p : T \longrightarrow \mathbb{R}$
- parallel polarized intensities $g_{\|} : T \longrightarrow \mathbb{R}$ and $s_{p,\|} : T \longrightarrow \mathbb{R}$
- perpendicularly polarized intensities $g_{\perp} : T \longrightarrow \mathbb{R}$ and $s_{p,\perp} : T \longrightarrow \mathbb{R}$
- linear depolarization ratios $g_{\text{depol}} : T \longrightarrow \mathbb{R}$ and $s_{p,\text{depol}} : T \longrightarrow \mathbb{R}$

of the given signal and the simulated signal for the environmental parameter $p \in P$, respectively. Here T is the set of discrete time intervals (each of these is a measuring time interval) which corresponds to the photon phath length; $|T|$ denotes the number of such time intervals. For the comparision of the given and the simulated signals we may choose (e.g.) one pair (G, S_p) of the above mentioned functions. To measure the distance between these functions we may choose the L_2-distance

$$Q(G, S_p) := \frac{1}{|T|} \sqrt{\sum_{t \in T} |G(t) - S_p(t)|^2}$$

or some weighted L_2-distance

$$Q_v(G, S_p) := \frac{1}{|T|} \sqrt{\sum_{t \in T} |G(t) - S_p(t)|^2 \; v(t)}$$

where $v(t)$ is some properly chosen weight function. Since we use Monte Carlo simulation, at least $S_p(t)$ is a (realization of a) random variable whose empirical variance is determined as a by-product by our variance reduction Monte Carlo code. Therefore, the variance of $S_p(t)$ or the related variance of $S_p(t)$ may be used to define such a weight function $v(t)$; e.g.:

$$v(t) := \frac{1}{var(S_p(t))} \qquad \text{or} \qquad v(t) := \frac{\frac{1}{|T|}\sum_{s\in T} var(S_p(s))}{var(S_p(t))}.$$

For the L_2-distance Q and a weighted L_2-distance Q_v, for a function $G : T \to \mathbb{R}$, and a function $S : P \times T \to \mathbb{R}$ with $(p,t) \mapsto S_p(t)$ a function

$$\begin{array}{rcllll} Q_{G,S} : P & \to & \mathbb{R} & \text{with} & Q_{G,S}(p) := Q(G, S_p) & \text{and} \\ Q_{G,S;v} : P & \to & \mathbb{R} & \text{with} & Q_{G,S;v}(p) := Q_v(G, S_p) & \end{array}$$

is defined, respectively.

To apply our inversion procedure, we have to minimize some of these functions $Q_{G,S}$ or $Q_{G,S;v}$ on the set P of environmental parameters. Here we want to demonstrate this procedure for the case that

$$P = [1.5, 5] \times [6, 30] \quad \text{where} \quad [1.5, 5] := \{r_c \in \mathbb{R} : 1.5 \le r_c \le 5.0\} \quad \text{and} \quad [6, 30] := \{\sigma \in \mathbb{R} : 6.0 \le \sigma \le 30\}$$

are the intervals for the mode radius r_c and the extinction coefficient σ, respectively. In this framework we denote:

$$\begin{array}{lllll}
Q & := Q_{G,S} & \text{for} & G(t) = g(t) & \text{and} \quad S((r_c,\sigma),t) = s_{(r_c,\sigma)}(t), \\
\text{i.e.} & Q := \frac{1}{|T|}\sqrt{\sum_{t\in T} \mid g(t) - s_{(r_c,\sigma)}(t) \mid^2}, & & & \\
Q_{\|} & := Q_{G,S} & \text{for} & G(t) = g_{\|}(t) & \text{and} \quad S((r_c,\sigma),t) = s_{\|(r_c,\sigma)}(t), \\
Q_{\perp} & := Q_{G,S} & \text{for} & G(t) = g_{\perp}(t) & \text{and} \quad S((r_c,\sigma),t) = s_{\perp(r_c,\sigma)}(t), \\
Q_{\text{depol}} & := Q_{G,S} & \text{for} & G(t) = g_{\text{depol}}(t) & \text{and} \quad S((r_c,\sigma),t) = s_{\text{depol}(r_c,\sigma)}(t), \\
Q_v & := Q_{G,S;v} & \text{for} & G(t) = g(t) & \text{and} \quad S((r_c,\sigma),t) = s_{(r_c,\sigma)}(t) \\
Q_{v,\|} & := Q_{G,S;v} & \text{for} & G(t) = g_{\|}(t) & \text{and} \quad S((r_c,\sigma),t) = s_{\|(r_c,\sigma)}(t), \\
Q_{v,\perp} & := Q_{G,S;v} & \text{for} & G(t) = g_{\perp}(t) & \text{and} \quad S((r_c,\sigma),t) = s_{\perp(r_c,\sigma)}(t), \\
Q_{v,\text{depol}} & := Q_{G,S;v} & \text{for} & G(t) = g_{\text{depol}}(t) & \text{and} \quad S((r_c,\sigma),t) = s_{\text{depol}(r_c,\sigma)}(t),
\end{array}$$

4.2 Search procedures for the minimization of the distance between LIDAR return signals

4.2.1 The uniform grid search

In order to make the main idea clear, let us consider an example. Furthermore, let us use the (not very efficient) uniform grid search instead of the (usually more efficient) random search:
We want to retrieve the extinction coefficient and the size distribution of the scattering particles of a homogeneous cloud from the multiply scattered LIDAR signal. Before and behind the cloud are no scattering particles, i.e. the extinction coefficient there is zero. Within the cloud the extinction coefficient σ is constant, we know that $6 \le \sigma \le 30$ [km^{-1}]. The cloud is supposed to consist of spherical particles with the Deirmendjian size distribution[3] , see subsection 2.2. To show the main ideas of retrieval algorithms based on uniform grid (and similarly on random) search, let us assume that we are in the following situation:
Somebody has calculated the parallel and the perpendicularly polarized multiply scattered LIDAR return signal from a C1–like cloud at a distance of 400 km

with the mode radius $r_c = 4.75\,\mu$m and the extinction coefficient $\sigma = 17.108\,\text{km}^{-1}$

which are known only to him. He asks us to find the parameters σ and r_c of this LIDAR signal. To answer this question we could proceed as follows:
We subdivide the r_c – interval [1.5,5.0] by steps of 0.1 μm and the σ – interval [6,30] by steps of 0.7 km^{-1}. Doing this we get

$$35 \times 34 = 1190 \text{ grid points } (r_c, \sigma) \text{ on the } r_c - \sigma - \text{rectangle } [\,1.5,5.0] \times [6,30]\ .$$

For every such grid point (r_c, σ) we simulate the polarized multiply scattered return signal by variance reduction Monte Carlo simulation with 250,000 starting photons for a space–based LIDAR. From the Stokes vector of the simulated

LIDAR signal we may obtain the parallel and the perpendicularly polarized part of the LIDAR signal and, hence, the total intensity of the LIDAR signal. We want to retrieve the parameters of the given LIDAR signal by comparing this given signal with the simulated signals for all the 1190 points of the uniform grid on the parameter space. We may use the total signal or the parallel or the perpendicular component of the polarized signal for comparison. Furthermore, as an outcome of a Monte Carlo simulation the value of a simulated signal at a certain time point (which corresponds to a photon path length or a "maximal range") is a realization of a random variable; the empirical variance of this random variable is obtained as a by–product from our program. For comparison we could also use this type of information about the randomness of the simulated signal. Hence, we have several possibilities to compare the given signal with the simulated signal. We shall see that the result and the quality of the retrieval methods may depend on the chosen method of comparison.

Each distance measure $Q_{\dots}$ (see subsection 4.1) for the comparison of the given and the simulated LIDAR return signal is evaluated for each grid point of the here two–dimensional rectangular parameter space and defines here a three–dimensional "mountain area" on the $r_c - \sigma$–rectangle. Since the grid points are fairly dense, the values of $Q_{\dots}$ for the grid points determine the contour lines of this mountain area. Figures 9–16 show these contour lines for the different distance measures. All the variance reduction Monte Carlo simulations where done with 250,000 starting photons and therefore had considerable variance, especially for the more distant measuring intervals; see also Figure 7. First of all, the contour lines show a very low sensitivity of the space–based LIDAR return signal towards the variation of the mode radius r_c and much higher sensitivity of the space–based LIDAR return signal towards the variation of the extinction coefficient σ. For all the distance measures the extinction coefficient was retrieved pretty precisely; see Figures 9–16. Because of the very low sensitivity of r_c and because of the considerable noise of the simulation of the space–based LIDAR return signal, the true value of r_c was not retrieved sufficiently precisely by the uniform grid search for all the distance measures; see Figures 9–16. The weighted distance measures $Q_{v,\dots}$ give slightly better results. As we have shown earlier,[5] under the same conditions the retrieval of both the parameters was precise for ground–based LIDAR return signals, although in that case the sensitivity towards variations of r_c was low, too. To get better results for the space–based case, one could do the simulations with more starting photons. A more efficient way is to use random search procedures.

4.2.2 Adaptive step size random search

Now, we shall apply a random search procedure to solve the minimization problem stated in subsection 4.2.1 more efficiently. For that purpose we design a random search procedure which is a mixture of a uniform random search and Guseva's adaptive step size random search. Such a procedure is one out of many random search procedures[13–15] . The uniform random search is selecting pairs (r_c, σ) of parameters according to the (normed) uniform distribution on the parameter rectangle $[1.5, 5] \times [6, 30]$. The adaptive step size random search is selecting pairs (r_c, σ) of parameters at random according to the (normed) uniform distribution on the circle around the last minimizing search point with given radius; the radius is adapted according to the progress of the search. We mix the two types in the following manner:
We used the uniform random search for the first 150 steps of the minimization. For these 150 steps we did the simulation of the LIDAR return signals with 500,000 starting photons.
Then we used the adaptive step size random search starting from the parameter point obtained from the uniform random search minimization. For the first 100 steps we selected parameter points (r_c, σ) according to the uniform distribution on the circle with radius 0.1 arround the normed minimizing parameter point obtained so far. Again, we used 500,000 starting photons for the Monte Carlo simulation of the LIDAR return signal belonging to the selected parameter point. Next, we repeated this procedure 50 times with a radius decrementation step size of 0.01 and 650,000 starting photons. Next, 25 times with a decrementation step size of 0.001 and 850,000 starting photons. Finally, 12 times with a decrementation step size of 0.0001 and 1,000,000 starting photons.

Figure 17 shows all the parameter points selected during our adaptive step size random search. The pair of parameters $r_c = 4.747\,\mu\text{m}$ and $\sigma = 17.914\text{km}^{-1}$ retrieved by this random search procedure is fairly close of the pair of the true parameters $r_c = 4.75\,\mu\text{m}$ and $\sigma = 17.108\text{km}^{-1}$. Figure 18 illustrates the speed of the minimization process for the retrieval of the environmental parameters based on the adaptive step size random search.

CONCLUSIONS

Using variance reduction Monte Carlo methods, we simulated polarized multiply scattered return signals from C1–like clouds for a ground–based and a space–based LIDAR. The two types of signals show essential differences with respect to the magnitude and the contributions of the different orders of multiple scattering. These differences show the necessity of new inversion procedures for retrieving microphysical parameters from multiply scattered LIDAR return signals, especially for space–based LIDARs. Our proposed retrieval algorithms are based on variance reduction Monte Carlo methods and on the minimization of some properly chosen distance measure for the comparison of LIDAR return signals via uniform grid search or some properly chosen random search. These algorithms need lots of computer power and cannot be operated on a PC, but need a powerful network of workstations. Yet they have a number of good properties:

- they may be very efficient,
- they are very generally applicable, e.g. without questionable assumptions on the environment,
- they are equally appropriate for all kinds of retrieval problems,
- they are applicable for larger sets of parameters,
- they give empirical information on the uniqueness of the solution of the retrieval problem,
- they yield a sensitivity analysis of the retrieval problem as a by–product.

Especially this sensitivity analysis is equally valuable for

- the comparison of retrieval algorithms,
- the judging of the importance of model assumptions,
- the interpretation of results of the retrieval,
- the designing of instrumental equipment.

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To Figures 9–16:
Comparision of given and simulated space–based LIDAR return signals form C1–like clouds with mode radius $1.5 \leq r_c \leq 5$ and extinction coefficient $6 \leq \sigma \leq 30$. Uniform grid search: 1190 grid points. True parameters: $r_c = 4.75\,\mu\text{m}$ and $\sigma = 17.108\,\text{km}^{-1}$. Monte Carlo simulation with 250,000 starting photons.

Figure 9:
Contour lines for distance measure Q. Result of retrieval: $r_c = 2.2\,\mu\text{m}$ and $\sigma = 18.6\,\text{km}^{-1}$.

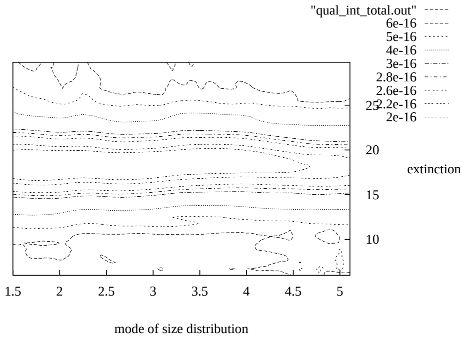

Figure 10:
Contour lines for distance measure $Q_{\parallel}$. Result of retrieval: $r_c = 2.2\,\mu\text{m}$ and $\sigma = 18.6\,\text{km}^{-1}$.

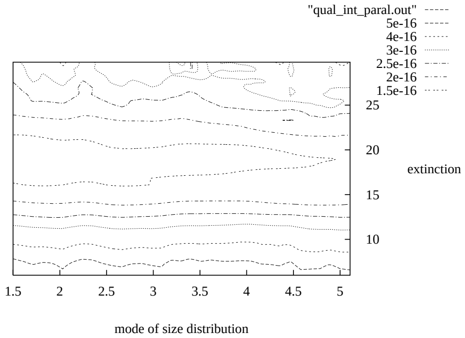

Figure 11:
Contour lines for distance measure $Q_{\perp}$. Result of retrieval: $r_c = 1.6\,\mu\text{m}$ and $\sigma = 13.0\,\text{km}^{-1}$.

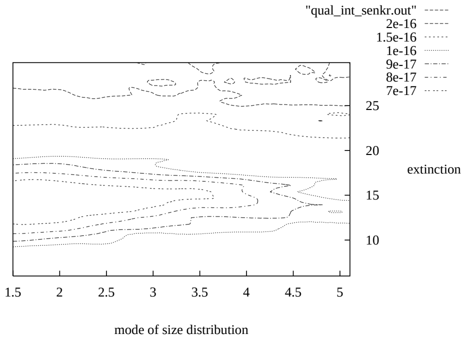

Figure 12:
Contour lines for distance measure $\mathbf{Q}_{\text{depol}}$. Result of retrieval: $r_c = 1.6\,\mu\text{m}$ and $\sigma = 15.1\,\text{km}^{-1}$.

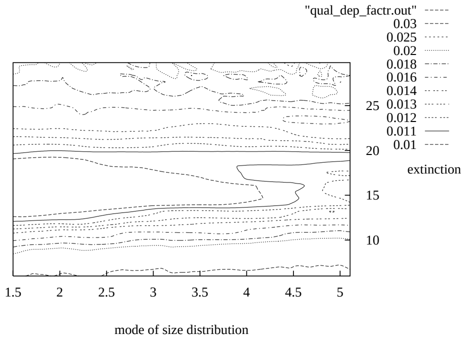

Figure 13:
Contour lines for distance measure Q_v. Result of retrieval: $r_c = 3.1\,\mu$m and $\sigma = 16.5\,\mathrm{km}^{-1}$.

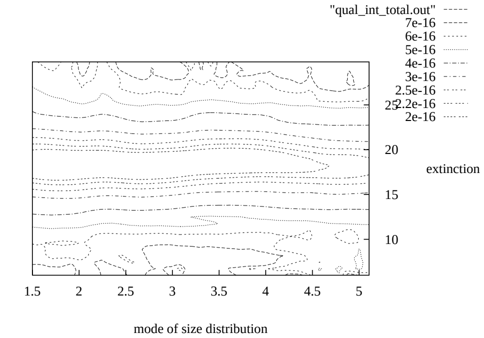

Figure 14:
Contour lines for distance measure $Q_{v,\parallel}$. Result of retrieval: $r_c = 1.8\,\mu$m and $\sigma = 19.3\,\mathrm{km}^{-1}$.

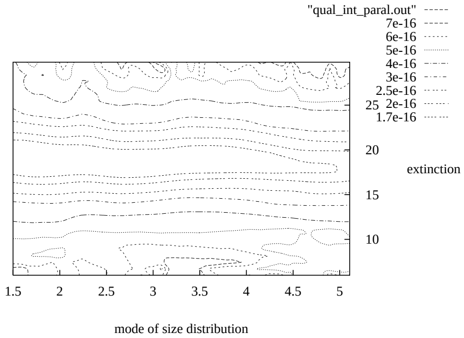

Figure 15:
Contour lines for distance measure $Q_{v,\perp}$. Result of retrieval: $r_c = 2.1\,\mu$m and $\sigma = 15.8\,\mathrm{km}^{-1}$.

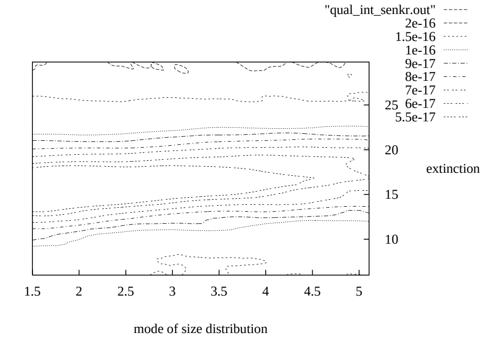

Figure 16:
Contour lines for distance measure $\mathbf{Q}_{v,\mathrm{depol}}$. Result of retrieval: $r_c = 1.6\,\mu$m and $\sigma = 14.4\,\mathrm{km}^{-1}$.

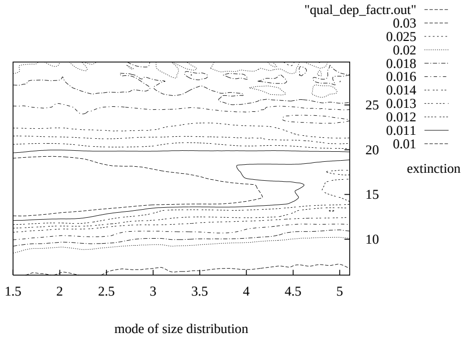

Figure 17:
Adaptive step size random search for the retrieval of environmental parameters by minimization based on the distance function Q.

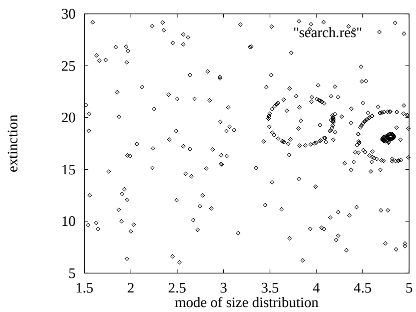

Figure 18:
Speed of the minimization of the distance function Q based on adaptive step size random search.