<u>Note:</u> **On Monday, November 30, the class will take place in room B052, and on Wednesday, December 2, the class will take place in room B349!**

Exercise 1 – Open string trajectory

Consider the following classical trajectory of an open string,

$$\begin{aligned} X^0 &= 3A\,\tau\ , \\ X^1 &= A\,\cos(3\tau)\,\cos(3\sigma)\ , \\ X^2 &= A\,\sin(a\tau)\,\cos(b\sigma)\ , \end{aligned} \tag{1}$$

and assume the conformal gauge.

a) Determine the values of a and b so that the above equations describe an open string that solves the constraint $T_{\alpha\beta} = 0$. Express the solution in the form

$$X^\mu = X_L^\mu(\sigma^+) + X_R^\mu(\sigma^-)\ . \tag{2}$$

Determine the boundary conditions satisfied by this field configuration.

b) Plot the solution in the (x^1, x^2)-plane as a function of τ in steps of $\pi/12$.

c) Compute the center-of-mass momentum and angular momentum and show that they are conserved. What do you obtain for the relation between the energy and angular momentum of this string?

Exercise 2 – Rotating open string in the light-cone gauge

Consider string motion defined by $x^- = x^I = 0$, and the vanishing of all the coefficients α_n^I with the exception of

$$\alpha_1^2 = (\alpha_{-1}^2)^* = a \quad , \quad \alpha_1^3 = (\alpha_{-1}^3)^* = ia\ . \tag{3}$$

Here a is a dimensionless real constant. We want to construct a solution that represents an open string that is rotating in the (x^2, x^3)-plane.

a) What is the mass (or energy) of this string?

b) Construct the explicit functions $X^2(\tau, \sigma)$ and $X^3(\tau, \sigma)$. What is the length of the string in terms of a and α'?

c) Calculate the $L_n^\perp$ modes for all n. Use your result to construct $X^-(\tau, \sigma)$.

d) Determine the value of p^+ by demanding that $X^1(\tau, \sigma) = 0$ (as we want to describ a string rotating in the (x^2, x^3)-plane). Find the relation between t and τ. What is the frequency of the rotation?

e) Calculate the angular momentum $J = |M_{23}|$ and compare it with the square of the energy.

Exercise 3 – Action of $L_0^\perp - \tilde{L}_0^\perp$

In class we showed that in the classical theory $L_0 - \tilde{L}_0$ generates σ-translations, via the Poisson-bracket. In the quantum theory the Poisson-bracket is replaced by the commutator. In light-cone gauge, this implies

$$\left[L_0^\perp - \tilde{L}_0^\perp, X^I(\tau,\sigma)\right] = i\frac{\partial X^I}{\partial \sigma} \ . \tag{4}$$

a) Using (4) and the definition $P \equiv L_0^\perp - \tilde{L}_0^\perp$, show that

$$e^{-iP\sigma_0} X^I(\tau,\sigma) e^{iP\sigma_0} = X^I(\tau,\sigma+\sigma_0) \ . \tag{5}$$

b) Use the result of part a) to calculate $e^{-iP\sigma_0}\alpha_n^I e^{iP\sigma_0}$ and $e^{-iP\sigma_0}\tilde{\alpha}_n^I e^{iP\sigma_0}$.

c) Consider the state

$$|U\rangle = \alpha_{-m}^I \tilde{\alpha}_{-n}^J |p^+, p^I\rangle \quad , \quad m,n>0 \ . \tag{6}$$

Use the result of part b) to calculate $e^{-iP\sigma_0}|U\rangle$. What is the condition that makes the sate $|U\rangle$ invariant under σ-translations?