Experimental observation of quark mass effects in $e^+e^-$ annihilation

- Introduction: Successes of QCD
- Quark mass effects ?
- Quark mass effects !
- Determination of $b$ quark mass at $m_Z$
Introduction

- QCD very successful in describing strong interaction:
  - Colour factors $C_A, C_F, T_R$
  - Non-Abelian gluon self coupling
  - Running of strong coupling $\alpha_s$
    Meanwhile: $\Delta \alpha_s(M_Z) = 2.5$
    all thoroughly scrutinized by experiments

- a few missing pieces:
  - Effects due to finite quark masses:
    - "dead-cone" effect in $Q \rightarrow Q + g$
    - Parton multiplicity: $Q \bar{Q}$ vs. $q \bar{q}$
    - Running of quark masses (i.e., renormalized quark masses)
**Successes of QCD**

- gluon coherence
- LLA parton shower

average charged particle multiplicity:

![Graph](image-url)
Quark mass effects: \( n_{\text{ch}} \) \( \propto \alpha_s(q)^5 \cdot \exp\left[ \frac{c}{\sqrt{\alpha_s(q)}} \right] \)

Fit to exp. data \( Q = 12 \ldots 189 \) GeV:

\[ \rightarrow \text{NLLA overshoes data at high } Q \]

Higher order terms missing?

Impact of high precision \( Z \) peak data with large \( b \)-quark contrib.?
Branching ratio

\[ BR\left( e^+e^- \rightarrow \gamma/\zeta \rightarrow q\bar{q} \right) \]
Quark mass effects? --- fragmentation fct.s

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expect: higher q mass → harder fragmentation

observe: heavy quark: soft fragmentation

at Z peak:

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<table>
<thead>
<tr>
<th>Inclusive</th>
<th>uds</th>
<th>c</th>
<th>b</th>
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<tr>
<td>O A D M</td>
<td>O D</td>
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\[
\int \sigma_{\text{tot}} \, d\sigma/dk_T
\]

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\[
\text{low} \left( x_p \frac{2p_T}{E_{cm}} \right)
\]

---

due to chain decays \( b \to c \to s \)?
$b$ fragmentation function

$$\langle x_E^{(b)} \rangle = 0.715 \pm 0.015$$

**ALEPH**

- Peterson
- Kartvelishvili
- Lund symmetric

![Graph](image)

Figure 7: The acceptance corrected $x_E^{(b)}$ spectrum of the leading b-meson for $f_{uu}=27.9\%$ and $f_{uu}=30\%$, compared with the predictions of different fragmentation models. The smaller error bar is statistical. The larger one is the sum of statistical + systematic errors. The errors shown do not account for the point-to-point correlations induced by the deconvolution process.
Quark mass effects — $ln \frac{1}{x_p}$ distributions

Small $x_p$ fragmentation: $\xi_p = ln \frac{1}{x_p}$

NLLA: distorted Gaussian shape

($= \text{limiting spectrum of NLLA}$)

around maximum

![Graphs showing $\xi_p$ distributions for different event types: inclusive (a), uds events (b), c events (c), b events (d).](image-url)
Quark mass effects? — position of max

MLLA: position of max. $S_0 = Y \cdot \left( \frac{1}{2} + \sqrt{\frac{1}{2}(1 + Y)} \right)$
($Y \equiv \ln \left( \frac{Q}{2Q_0} \right)$)

fit to exp. data $Q = 14 \ldots 189 \text{ GeV}$:

→ MLLA undershoots data at high $Q$

→ higher order terms missing?

→ impact of high precision $Z$ peak data with large b-quark contrib?
Quark mass effects?

- massless QCD works fine in principle
- some experimental evidence for quark mass effects
- a few concrete examples
  - multiplicity differences: b vs. light
  - light vs. light
  - flavour (in-)dependence of strong coupling
  - b quark mass at Z mass scale
Experimental identification of quarks

- **b quarks**
  - semi-leptonic decays: $e, \mu$
  - decay vertices
  - impact parameter

- **c quarks**
  - $D^{*+}$ decays
  - semi-leptonic decays: $e, \mu$
  - decay vertices
  - impact parameter

- **u, d, s quarks**
  - leading particle effect

  e.g. $e^+ \rightarrow \mu^+ \bar{\nu}_e \bar{\nu}_\mu$ leading $K^-$
*b* quark tagging by vertices

decay length significance: \( \frac{L}{\sigma_L} \)

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**OPAL**

- OPAL
- MC, b quarks
- MC, c quarks
- MC, dus quarks

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\[ \frac{1}{\sigma} d\sigma / d(1/\sigma_L) \]

---

\( L/\sigma_L \)

decay length/measurement error

\[ \Rightarrow \]

Efficiency typ 70%  
Purity typ 90%
b quark tagging by impact parameter

$N_{\text{sig}} =$ no. of tracks with significant impact parameter $\delta$
Quark mass effects! - $<n_{Q\bar{Q}}>$ vs. $<n_{q\bar{q}}>$

- Flavour Independent Fragmentation (Kisselvet al.)

$<n_{Q\bar{Q}}(Q)> = 2 \cdot <n_{B}^{\text{decay}}>$ +

$+ \int d x_{b} d x_{\bar{b}} f(x_{b}) f(x_{\bar{b}}) \cdot <n_{q\bar{q}}(Q, \mathcal{B}-\frac{y}{\mathcal{B}})>$

b fragment. fct.

light flavour mult.

at reduced energy

$\Rightarrow$ $<n_{Q\bar{Q}}(Q)> \xrightarrow{Q \to \infty} <n_{q\bar{q}}(Q)>$

- QCD: MLLA calculation (eg. in Kuhn, Ochs, et al.)

$<n_{Q\bar{Q}}(Q)> = 2 \cdot <n_{B}^{\text{decay}}>$ + $<n_{q\bar{q}}(Q)> - <n_{q\bar{q}}(m_{b}, \mathcal{B})>$

$\Rightarrow \delta_{Q\bar{Q}} = <n_{Q\bar{Q}}(Q)> - <n_{q\bar{q}}(Q)> \propto \text{const.}(Q)$

- $= 2 \cdot <n_{B}^{\text{decay}}>- <n_{q\bar{q}}(m_{b}, \mathcal{B})> = 3.6-5.5$ (eg. Schiavone et al. PRL 69, 3075; Vetten et al. PLB 111, 95)
Quark mass effects! — \( \langle n_{\bar{u}u} \rangle \) vs. \( \langle n_{qq} \rangle \)

- QCD: \( S_{\text{be}} \approx 3 \) and const. \( (Q) \)!
- + \( S_{\text{be}} \) value depends on \( m_b \)

\[ Q^0 (\text{GeV}) \]
Quark mass effects! — $\langle n_{ud} \rangle$ vs. $\langle n_{ss} \rangle$?

- very difficult to distinguish between light flavours $u, d, s$

- leading particle effect (at $\pi$ peak):
  \[ K^0_s : s + (d) \]
  \[ K^\pm : s + (u) \]
  \[ \pi^\pm, K^\pm, \rho : u + d + s \]

- take care of bias due to leading particle

- statistical unfolding

\[
\begin{align*}
\langle n_s \rangle - \langle n_d \rangle &= -1.4 \pm 2.0 \\
\langle n_s \rangle - \langle n_u \rangle &= 2.3 \pm 1.5 \\
\langle n_d \rangle - \langle n_u \rangle &= 3.7 \pm 2.5
\end{align*}
\]

⇒ large errors
⇒ only a single energy $Q=316\text{GeV}$
light quark tagging by leading particle effect

\[ K^- \Rightarrow s + (d) \]
\[ K^\pm \Rightarrow s + (u) \]
\[ \pi^\pm, K^\pm, \rho^0 \Rightarrow u + d + s \]

\{ statistical unfolding \}

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\[ x_E(K^0_S \text{ cand.})_{\min} \]

\[ x_E(K^+ \text{ cand.})_{\min} \]

\[ x_E(\pi^+ K^\pm, \rho^0 \text{ cand.})_{\min} \]
Quark mass effects! — flavour (in-)dependence

- u, d, s multiplicities $\rightarrow$ flavour-dependent coupling

$$\langle n_{qq} \rangle = (c_{NLLA}^{s}(M_{B}))^{b_{c}} \cdot \exp \left[ \frac{c_{s}^{NLLA}(M_{B})}{d_{s}^{NLLA}(M_{B})^{b_{c}}} \right]$$

$\Rightarrow d_{s}^{NLLA}$ for flavour $q = u, d, s$

- ratios of $d_{s}^{g}/d_{s}^{g'}$ for better comparison

$$\begin{aligned}
d_{s}^{u}/d_{s}^{d} &= 0.88 \pm 0.08 \\
d_{s}^{s}/d_{s}^{d} &= 0.96 \pm 0.06 \\
d_{s}^{g}/d_{s}^{m} &= 1.09 \pm 0.06
\end{aligned}$$

$\Rightarrow$ flavour independence a: okay

but hadron decays may interfere
Quark mass effects! - flavour (in-)dependence

- high statistics Z peak data sample
  - vertex tag
  - $D^{*+}$ tag
  - vertex anti-tag
  - $b$ quark sample
  - $c$ quark sample
  - uds quark sample

- flavour dependent investigation of
  - event shapes ($A_T, H_T, B_w, C, y_3$)
  - jet rates ($E, E_T, p_T, D, G, ...$)

- and flavour dependent determination of $\lambda_s$
  - massless $O(\lambda_s^2)$ QCD ($\epsilon RT, NP B^{179}, (\theta) and others$
  - massive QCD predictions (full $O(\lambda_s^2)$ since 1997 available)
c quark tagging by $D^{*\pm}$ decays

$D^{*\pm} \rightarrow D^{0} \pi^{\pm}$

- $K^{-}\pi^{+}$
- $K^{-}\pi^{+}\pi^{0}$
- $K^{-}\pi^{+}\pi^{-}\pi^{0}$
- $K^{-}e^{+}\nu_{e}$
- $K^{-}\mu^{+}\nu_{\mu}$

$\rightarrow c.c.$

Efficiency typ. 2%

Purity typ. 60%

(but ~23% b -> c -> 0*)
Massive QCD predictions

- LO calculations:
  - Jaffe, PC 878 (1978) 277
  - Kramer et al., PR C41 (1980) 142
  - Lomnitz et al., PL 889 (1980) 225
  - Miller, PRL 45 (1980) 379

- NLO (without loops):
  - Ballesteros et al., PL 8294 (1992) 425
  - Ballesteros et al., NP B445 (1994) 265
  - Ballesteros et al., PL 6323 (1994) 53

- Full NLO calculation:
  - Bernreuther et al., PRL 79 (1997) 183 (PS mass)
  - Redivig et al., PRL 79 (1997) 193 (pole mass)
  - Hasegawa et al., NP A524 (1998) 637 (pole mass)
Notice: Small statistical uncertainties

Quark mass effects

Tiny mass effects

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2011 regime

Mass dependence
Quark mass effects! — flavour (in-)dependence

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2-jet regime

many-jet regime

uds-tag

Fit Range

uds-tag

Fit Range

uds-tag

QCD fit
incl.
mass
effects

Notice: ▲ small statistical uncertainties

▲ tiny mass effects
Quark mass effects! - flavour (in-)dependence

- results of fits:

OPAL  EPJ C 14, 643

\[ \frac{\alpha_s^Q}{\alpha_s^{uds}} \]

\( y_{23} \)  c massless

\( 1-T \)  b massive

\( M_H \)  c massive

\( B_W \)  b massless

\( C \)

\( \Rightarrow 2 - 7\% \) mass effects for c & b
Quark mass effects! — flavour (in-)dependence

Flavour independent coupling:
\[ \frac{\alpha_s^c}{\alpha_s^{uds}} = 0.977 \pm 0.050 \quad (5\% \text{ prec.}) \]
\[ \frac{\alpha_s^b}{\alpha_s^{uds}} = 0.993 \pm 0.015 \quad (1.5\% \text{ prec.}) \]

when quark masses considered!

EPJ CM, 643
Quark mass effects! = flavour (in-)dependence

- Flavour independent coupling when quark masses considered

precision: $\delta_c = 6\%$, $\delta_b = 4\%$

Figure 6. (a) The measured ratios $R^q/R^uds$, and (b) the corresponding translated ratios $\alpha^q/\alpha^uds$, $q = c,b$. The arrows in (a) indicate the range of the theoretical prediction described in the text for values of the $b$-quark mass in the range $2.5 < m_b < 3.5$ GeV/$c^2$, with the arrow pointing towards the lower mass value.
Determination of $b$ quark mass

- Some theoretical issues on quark masses:
  - NLO calculations use different mass definitions
- Pole mass $M_b$
  - From pole in quark propagator $\frac{1}{q^2 - M_b^2}$
  - Independent of renormalization scheme
  - On-shell quark propagator has no pole (quarks are confined, though not free)
  - Mass only in pert. theory defined
  - Suffers nonpert. infrared effects in QCD
- Renormalized mass $\bar{m}_b$ (eg. $\bar{m}_S : \bar{m}_b$)
  - Depends on renormalization scheme
  - Mass is scale dependent $\to$ running
  - Theoretically well-defined concept ($\bar{m}_{\Lambda}$)
  - $\equiv$ mass in Lagrangian (current mass)

Both masses are related in pert. theory
(relation known in $O(a_s^3)$ Chetyrkin et al. 9311454
Kotlubin et al. 9312204)
Determination of $b$ quark mass

- Assume flavour independent coupling ($=\overline{QCD}$)

  \[
  \frac{d\sigma^b_{uds}}{d\sigma^b_s} = 1 \quad \rightarrow \quad M_b \quad \text{or} \quad \overline{m}_b(Q)
  \]

- Sensitive observable:

  \[
  R_3^{bl} = \frac{R_3^b (Y_{unt})}{R_3^{uds} (Y_{unt})} \quad \text{ratio of 3-jet rates}
  \]
  (also called $r^b$ and $B_3$)

  with a simple relation with $\overline{m}_b$ ($M_b$):

  \[
  R_3^{bl} = a_0 + a_2 \cdot \overline{m}_b^2 \quad (+ \text{negligible low mass terms})
  \]

- NLO calculations of $R_3^{bl}$ vs $\overline{m}_b$ ($M_b$)

  for many jet finders:
  JADE, E6, E1, F0, P, Durham, Geneva, Cambridge
  [program Abbott of Nasr and Cleo to integrate]
  [massive matrix element for every observable]
Determination of $b$ quark mass

LO + NLO calculation for $R^b = \frac{\langle Y_3^b \rangle}{\langle Y_3^\alpha \rangle}$ using ZEUS:

**Pole mass**

**MS mass**

- MC Integration $Y_3$
- Parametrisation, pole scheme

Identical

- MC Integration $Y_3$
- Parametrisation, running scheme

Different

$\Rightarrow$ NLO contribution typ. $\leq 3\%$
Determination of $b$ quark mass

Comparison $H_{pole}$ vs. $\overline{MS} - m_b$: $H_{pole} = 56 GeV; \overline{m}_b(m_Z) = 36 GeV$

$\Rightarrow$ large differences $NLO - H_b \leftrightarrow NLO - \overline{m}_b(m_Z) \rightarrow$ theory uncertainty?
Determination of $b$ quark mass

- data vs. theory
  - determine $\bar{m}_b(M_b)$ at a fixed $Y_{cut}$

$Y_{cut} = 0.02 \implies \bar{m}_b(m_Z) = 2.8$ GeV

Figure 3: Corrected data values of $R^\ell_\ell$ using DURHAM algorithm compared with the theoretical predictions from reference [10] at LO and NLO in terms of the pole mass $M_b = 4.6$ GeV/c$^2$ (dashed lines) and in terms of the running mass $m_b(M_Z) = 2.8$ GeV/c$^2$ (solid lines).
Determination of $b$ quark mass

- choice of $\gamma_{cut}$ to extract $\bar{m}_b$

![Graph showing statistical and systematic uncertainties of $R_b$]

- DELPHI
- Statistical and Systematic
- Uncertainties of $R_b$

- DURHAM algorithm

- Data
- Simulation
- Fragment Model
- Mass Ambiguity
- Fragment Tuning
- $\mu$-scale

- 4-jet rate less than 2%

- all choices in this range OKAY

- (DELPHI: $\gamma_{cut} = 0.02$)

- $\bar{m}_b$ vs. $M_b$
- JETSET vs. HERWIG
- JETSET parameters renormalization scale
Determination of $b$ quark mass

- renormalization scale dependence

\[
\text{Observable} = R_3^{(\text{DURHAM})} \quad \text{ALEPH}
\]

\[
\chi^2 \approx 0.5 \ldots 2
\]

\[
\overline{m}_b \ast \chi^2 \ast \mu/M_z
\]

- range spanned by scheme uncertainty

\[
\text{Observable} = Y_3, \quad \frac{\langle Y_b \rangle}{\langle Y_s \rangle} \quad \text{ALEPH}
\]

\[
m_b [\text{GeV}/c^2]
\]

- range spanned by scheme uncertainty
Determination of b quark mass

- Jet finders show different sensitivity to hadronization

\[ \Delta R_{3\text{D}} \]

Hadronization Uncertainty

- CAMBRIDGE
- DURHAM

\[ \Delta R_{\text{DURHAM}} \]

Cambridge allows for smaller \( \Delta R \) cut

\[ \Delta R_{\text{CAMBRIDGE}} \]

\( \Rightarrow \) reduced statistical uncertainty
Determination of $b$ quark mass

$Y_{cut} = 0.015 \quad \Rightarrow \quad \overline{m}_b(m_Z) = 2.6 \text{ GeV}$

data prefer $\overline{m}_b$ over $M_b$

$\overline{m}_b(M_Z) = 2.8 \text{ GeV/c}^2$

$M_b = 4.6 \text{ GeV/c}^2$

due to nonperturbative effects of $M_b$?
Fig. 25. At fixed $y_{cut}$, the theoretically expected dependence of the 3-jet rate ratio, $r_b = R_b/R_{b}^{d}$, for bottom and light quarks on the bottom quark mass is shown for various jet finders as points with error bars and parameterized by a curve. The gray bands represent the experimentally measured values of the ratio and the respective statistical errors as obtained by the SLD collaboration. The figure is taken from [156].
Summary of $m_b$ measurements

Compilation by Gimenez et al. and J. Fuster

Averaging method by M. Schmelling

- Devices 84
- Gimenez 97
- Marzocchi 98
- Ali Kahn 99
- Gimenez 2000
- Neron 99
- Velekhi 99
- Fabozzi 99
- Kurn 99
- Jamin 99
- Meinhold 99
- Heong 99
- Bonato 99
- Penin 99
- Yndurain 2000
- Rodrigo 97
- DELPHI 99
- Broderick 99
- ALEPH 2000
- DELPHI 2000 (Prel.)
- OPAL 2001 (Prelim.)

$m_b(M_Z) = 2.81 \pm 0.43 \text{ GeV}$

$m_b(M_Z) = 2.98 \pm 0.09 \text{ GeV}$

$m_b(M_Z) [\text{ GeV}]$
Determination of $b$ quark mass

- Compilation of $\overline{m}_b (m_b)$ determinations

$\Rightarrow \overline{m}_b (m_b) = 2.80 \pm 0.45 \text{ GeV}$

\[ \overline{m}_b (Q) \text{ at 4-loop precision} \]

$\Rightarrow \overline{m}_b (\overline{m}_b) = 4.11 \pm 0.66 \text{ GeV}$

$\overline{m}_b^{\text{PDG}} - \overline{m}_b (m_b) = 1.40 \pm 0.49 \text{ GeV} \ (\sigma; 3\sigma)$
Summary

- Quark mass effects start getting important in experimental studies
  - multiplicity
  - hard & soft fragmentation
  - flavour (in-) dependence

- NLO calculations on the market since about 4 years
  - flavour independence
  - determination of b quark mass

- (too?) many quark mass definitions
  - \( \bar{m}_b \) vs. \( M_{\text{pole}} \) a reliable test of higher orders?
  - choice of renormalization scale a significant uncertainty

- running of \( \overline{\text{MS}} \) b quark \( \bar{m}_b(m_Z) = 2.80 \pm 0.06 \text{GeV} \)
  - intermediate determination wishful to prove running unambiguously