D-Geometry As A Model For Quantum Space-Time

Robert C. Helling

Albert-Einstein-Institut

Max-Planck-Institut für Gravitationsphyisk

Am Mühlenberg 1

14476 Potsdam

helling@x4u2.desy.de

Although with the development of quantum physics and the theory of General Relativity great advances in our understanding of nature have been accomplished in the twentieth century, their unification is still an open and probably the most ambitious problem in theoretical physics. Right at the heart of this problem is the question on the nature of space-time at very short length scales. In General Relativity, space-time is mathematically described as a pseudo-Riemannian manifold, some space that locally looks like a neighborhood in $\mathbb{R}^{1,3}$. But from the basic principles of quantum physics we know that at very short distances, there are large uncertainties in momentum and therefore in energy. These quantum fluctuations of energy tend to curve space-time so that it probably loses its smooth structure at very small length scales and dissolves in a more and more foamy structure. A mathematical formulation of a structure with those properties would archive a major step towards the final theory of quantum gravity.

String theory, the theory of tiny threads that replace point particles as basic constituents of matter, is known for a long time to contain gravity as an effective theory in a low energy limit. For many researchers it is today's main candidate for the long searched theory of quantum gravity. As such, it should entail the the mathematical description of space-time at very small distances.

Since strings are not longer point-like but have a finite extension it seems impossible to use them to probe space-time at lengths much smaller than the string itself. But only recently, it became clear that besides strings themselves there are further, non-perturbative states in the spectrum of the theory. These so called D-branes can be thought of as topological defects in spacetime much like domain walls in solid state physics on which open strings can end. These D-branes exist in various dimensionalities and, ironically, there are again point particles called D0-branes among them. The D-branes are dynamical themselves and can be scattered amongst each other. But since they are much better localized than strings they can be used to probe space-time at much shorter scales.

D-brane states are especially well suited for study since their dynamics is described by a class of well known field theories, namely gauge theories. An ensemble of N D0-branes for example is captured by the dimensional reduction of $U(N)$ super Yang-Mills theory to 0+1 dimensions, i.e. a model in which the degrees of freedom are time dependant matrices. The potential energy of a configuration is given by the trace of the square of the commutator of those matrices.

In the low energy regime, these matrices have to commute and can therefore be assumed to be diagonal. They are determined by their N eigenvalues. To come back to the D-particle picture, one should interpret these N eigenvalues as space-time coordinates of the N particles. The off-diagonal matrix elements are approximated by harmonic oscillators with frequencies proportional to the distance between the D0-particles. So for generic,

macroscopic configurations of these D0-particles, all off-diagonal degrees of freedom are frozen into their ground states. But if two particles get closer than the Planck-length, these additional degrees of freedom become dynamical and the notion of space-time coordinates breaks down. At these short distances the dimensionality of space-time jumps and the classical manifold develops a singularity.

Although the classical geometric description becomes useless, the gauge theory description is defined at all energies and can therfore be used to define, what we mean by space-time at short distances.

Even if this particular example so far can only describe perturbations of very simple space-times, it can teach us an important lesson: It can be useful to take space-time not as given *a priori* but rather start with some field theory in an auxiliary space. Then, the true physical space-time arrises as a derived concept, namely as the moduli space of vacua of this auxiliary field theory. These moduli spaces usually have exactly the properties we were looking for: At generic points they look like smooth manifolds but at special points that correspond to objects getting very close they are highly quantum with jumping dimensionalities and singularities of other kinds.

Further research in this direction should therefore explore these moduli spaces and understand the geometrical properties of quantum spaces defined in this way. Especially, a better understanding of the scattering theory of D-branes will shed more light on the properties of short distance physics. More generic backgrounds for these scattering events will correspond to more complicated auxiliary field theories with a rich geometrical structure. On the other hand, the connection between (quantum-) gravity and gauge theories can be inverted to extract information on strongly coupled gauge theories like QCD from a dual gravity description.

So far, in all known examples of this correspondence, super-symmetry plays an important rôle in establishing the correspondence. It would be beneficial to understand this mechanism in more detail.

For quite some time, people have conjectured that at scales comparable to the Planck length, the coordinates of space-time no longer commute and that geometry becomes non-commutative. This non-commutativity also comes out very naturally of D-brane physics and is well incorporated in this framework. In the above example, we saw that classical space-time coordinates correspond to commuting diagonal matrices that have to be generalized to full, non-commuting matrices for very nearby objects. More generally, the D-brane view on space-time is very similar to Connes' version of non-commutative geometry and many interesting consequences in this direction can be expected. For example, the celebrated string dualities might have an non-commutative analog in terms of Morita equivalence and can be generalized from this point of view. We can also expect interesting findings on the connection between metric properties and Dirac operators on the one hand and super-symmetry on the other from the D-brane version of space-time non-commutativity.