## Mathematics for Physicists: Introductory Concepts and Methods

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Please report typos via email to vondelft@lmu.de

Corrected versions of typos occurring in the book are indicated using magenta.

We thank the following readers for pointing out typos (brackets report number of typos found): Anonymous (4), Jannis Beck (1), Johannes Berg (2), Jérémy Berry (1), Michael Buchhold (2), Jack Collings (3), Tibor Cornelli (3), Sandra Conor (2), Bernhard Emmer (14), Andreas Gleis (1), Finn Göttlich (3), Fabian Hassler (1), Lena Heidenreich (2), Pauline Hofer (1), Patrick Haußmann (1), Martina Jung (11), Jheng-Wei Li (1), Jonas Märtens (1), Christoph Müller-Salditt (1), Carlos Navarro (1), Anna Neudert (1), Philipp Opitz (1), Sarah-Viviane Petzoldt (17), Elie Raphael (8), Nepomuk Ritz (22), Nicholas Todd (30), Marius Predel (13), Stefan Rinner (1), Thomas Risse (1), Alessandro Rubini (4), Pascal Schintzel (1), Oguzhan Sen (1), Chit Yan Toe (1), Elias Walter (9), Bogdan Wnętrzewski (2), Shuo Xu (1), Anton Zakrewski (1).

p.xvii. Line 8 should read: LMU Munich.

p. xviii. The following sentence should be added at the end of the second paragraph: "Check your results" hints are provided for almost all problems, both odd- and even-numbered.

p. 3. The first sentence of Section L1.1 should read: When we work with  $\frac{1}{2}$  complex systems of any kind ...

p. 12. Lines 6 and 7 should read: For example,  $\epsilon_{312} = 1$  because reordering (1, 2, 3) to (3, 1, 2) requires two pair permutations.

p. 12. In Section L1.3, first paragraph, the last sentence should contain a footnote: Addition and multiplication each define their own *abelian* group structure, <sup>Footnote</sup> the neutral elements ... The footnote should read: To be precise: the multiplicative group  $(A \setminus \{0\}, \cdot)$  excludes 0, because that number has no multiplicative inverse.

p. 14, Section L1.3. Remark for instructors: Eq. (L8) introduces the imaginary unit as  $i \equiv \sqrt{-1}$ . At this stage in the text, "i" is seen as a formal placeholder for  $\sqrt{-1}$ , a view that can become problematic if it is adopted blindly. The most apparent problem, mentioned in the INFO block on p. 16, reads  $1 = \sqrt{(-1)(-1)} = \sqrt{-1}\sqrt{-1} = i^2 = -1$ . Although the issues related to the ambiguity of i are all addressed later in the text, it may be preferable to work from the beginning with the more fault-proof "definition"  $i^2 \equiv -1$ . The quotes, because this equation,

too, implies an ambiguity (if i is a solution to the equation, so is -i) and further discussion is required to justify it as a proper definition.

While we are not aware of an introduction of i which does not create some pedagogical friction, perhaps the following strategy, differing from the text, is a satisfactory solution: (i) Introduce i as one (which one is arbitrary) solution to  $i^2 = -1$ . (ii) Use this as a prescription to replace all occurrences of  $i^2$  by -1. (iii, optional) State that the ambiguity is inconsequential in the sense that a replacement of i by  $j \equiv -i$ will not change the result of any calculation involving the prescription (ii).

p.23. For Eq. (L18), part (d), the text in brackets should read: (neutral element of multiplication of  $\mathbb{F}$  leaves vectors invariant).

p.24, INFO block. Line 2 should read: For example, *matrices*, which are... (delete '(L5)' before matrices).

p. 26. The second-to-last line should read:  $\mathbf{f} = (f^1, \dots, f^N)^T$ .

p. 27. In Fig. L7, the bottom right panel should be replaced by figure below, which is a more accurate depiction of function addition:



p. 32. The first sentence should conclude with: . . . in the figure (for half the atoms, the linear combination involves non-integer coefficients).

p. 34. In Fig. 9, the subspaces W should include the origin:





p. 34-35. In Eq. (L31), the notation  $\phi_{\hat{\mathbf{v}}}(\hat{\mathbf{v}})$  is potentially confusing: the subscript refers to the basis  $\{\hat{\mathbf{v}}_i\}$ , the argument to a vector  $\hat{\mathbf{v}}$  expanded in that basis, with

components  $v^j$ . To avoid confusion, rather call the argument vector  $\hat{\mathbf{u}}$ , with components  $u^j$ . Concretely, throughout the paragraph containing (L31), replace  $\hat{\mathbf{v}}$  by  $\hat{\mathbf{u}}$  and  $v^j$  by  $u^j$ , but leave the basis vectors  $\hat{\mathbf{v}}_j$  and mapping  $\phi_{\hat{\mathbf{v}}}$  unchanged. For example, (L31) should read:

$$\phi_{\hat{\mathbf{v}}}: V \to \mathbb{R}^{n}, \qquad \hat{\mathbf{u}} = \hat{\mathbf{v}}_{j} u^{j} \mapsto \phi_{\hat{\mathbf{v}}}(\hat{u}) \equiv \begin{pmatrix} u^{1} \\ \vdots \\ u^{j} \\ \vdots \\ u^{n} \end{pmatrix}, \qquad (L31)$$

p. 39. The equation in third to last line should read:  $\|\begin{pmatrix} a\\b \end{pmatrix}\| = \sqrt{a^2 + b^2}$ .

p. 43. The statement before Eq. (L45) should read: (see footnote 24 of Chapter L2).

p. 47. In the 4th line from the top, "normed vector space" should be omitted so that the sentence reads: A vector space equipped with an inner product is called an **inner product space** or a *Euclidean vector space*.

p. 48. The 2nd line after Eq. (L57) should read: ... g is specified by n(n+1)/2 real numbers.

p. 49. In the in-line equation above Eq. (L64), the second-last expression should should read: ... =  $x^k \delta_k{}^i = ...$ 

- p. 49. Eq. (L64) should read:  $x_j y^j = x^i y_i$ .
- p. 53. Line 9 should read:  $\langle a \hat{\mathbf{v}}, \hat{\mathbf{w}} \rangle = \bar{a} \langle \hat{\mathbf{v}}, \hat{\mathbf{w}} \rangle$ .
- p. 64. The first line should read: for all  $a, b \in \mathbb{C}$  and  $\mathbf{v}, \mathbf{w} \in V$ .

p. 65. The left part of Eq. (L102) should read:

$$\mathbf{x} = \begin{pmatrix} x^1 \\ x^2 \\ \vdots \\ x^n \end{pmatrix} \longmapsto A\mathbf{x}$$

p. 76. The 8th line after Eq. (L129) should read:

- ... the largest number of zero matrix elements...
- p. 79. The definition in the first line should read:  $\hat{\mathbf{u}}_j \equiv \hat{A}\hat{\mathbf{v}}_j \in W$ .
- p. 79. In the EXAMPLE, the third equation should read:  $\hat{A}\mathbf{v}_3 = \hat{\mathbf{v}}_2 \sin\theta + \hat{\mathbf{v}}_3 \cos\theta$ .
- p. 79. The map in the third line above the EXERCISE should read  $A : \mathbf{e}_j \mapsto \mathbf{u}_j$ .

p. 81. In the diagram above Eq. (L135), the equation at the top should read:  $\hat{\mathbf{v}}_j x^j = \hat{\mathbf{x}} = \hat{\mathbf{v}}_{j'} x'^{j'}$ .

p. 91. The sentence after Eq. (L160) should read: The determinant of a diagonal matrix containing one or more zeros on the diagonal vanishes.

p. 92. The last sentence of point 6 should read: ... which in turn implies linearly dependent rows or columns.

p. 93. Eq. (L170) should read:  $det(AB) = \epsilon_{i_1,...i_n} (AB)^{i_1} \dots (AB)^{i_n} = \dots$ 

p. 94. Eq. (L175) should read  $A = (\mathbf{A}_1, \mathbf{A}_2, \mathbf{A}_3, \dots, \mathbf{A}_n), A' = (\mathbf{A}_2, \mathbf{A}_1, \mathbf{A}_3, \dots, \mathbf{A}_n).$ 

p. 95. Footnote 56 should read: The reason is that it is related by...

p. 99. Footnote 58 should read: ... with the "old" and "new" bases there, ...

p. 102. The right part of the second equation from the top should read  $\mathbf{v}_2 = c_2 \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ , to ensure consistency with Fig. 15.

p. 105. The third-last line before the INFO block should read:  $\mathbf{0} = (A - 1 \cdot \mathbb{1})\mathbf{v} = \begin{pmatrix} 0 & b \\ 0 & 0 \end{pmatrix} \mathbf{v}$ , has only one solution,  $\mathbf{v} = (c, 0)^{\mathrm{T}}$ , where ...

p. 109. At the end of the REMARK block, the following sentence should be added: Throughout this chapter, we consider maps from a vector space to itself,  $\hat{A} : V \to V$ . The sentence before Eq. (L198) should read

The defining feature of an inner-product-preserving map  $\hat{A}: V \to V$  is that<sup>footnote</sup> and the following footnote should be added: More generally, maps  $\hat{A}: V \to W$ between not necessarily identical vector spaces which preserve the respective inner products,  $\langle \hat{A}\hat{\mathbf{u}}, \hat{A}\hat{\mathbf{v}} \rangle_W = \langle \mathbf{u}, \mathbf{v} \rangle_V$ , are called **isometries**.

p. 112. The third line before Eq. (L209) should read ...  $\overline{U_k^m} \overline{u^k} g_{mj} U_l^j w^l = \overline{u^k} g_{kl} w^l$ .

p. 113. The fifth line from the bottom should read:  $\Lambda^{i}_{\ j} = -\delta^{i}_{\ j}$ 

p. 114. The fourth line below the box should read: ... the orthogonal matrix  $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ 

p. 115. In equation (L213), the specifications  $U^{\dagger} = U^{-1}$ , and  $O^T = O^{-1}$ , should be omitted. (They are correct, but redundant, since these properties already follow from the specifications  $U \in U(n)$  and  $O \in O(n)$ , respectively.)

p. 120. In the equation below (L219), the fourth term has one bracket too much. Instead of  $\sum_{m} \frac{(-iA)^m}{m!}$ , it should read  $\sum_{m} \frac{(-iA)^m}{m!}$ .

p. 120. The third-last line before Section L8.3 should read:  $T^{-1} = T^{\dagger}$  (orthogonal,  $T^{-1} = T^{T}$ ), ...

p. 130. The sentence after Eq. (L222) should read: ... of the index  $i \in \{1, \ldots, n\}$ .

p. 132. The second line above Eq. (L229) should read:  $\langle \mathbf{e}^i, \mathbf{v} \rangle = \sum_j (\mathbf{e}^i)_j v^j = v^i$ .

p. 133. The end of footnote 72 should read: ... with different indices y and k.

p. 134. The second line after Eq. (L235) should read: Taking inner products  $\langle \delta_x, \rangle$ ,

p. 136. Footnote 73: The solution functions should contain  $\sqrt{\lambda}$  in the exponents, i.e.  $e^{\pm\sqrt{\lambda}x}$  and  $c_+e^{+\sqrt{\lambda}x} + c_-e^{-\sqrt{\lambda}x}$ .

p. 151. In the Example near the bottom of the page, the last sentence should read: In the subsection after the next one we will see...

p. 153. The first boxed statement should read: The canonical isomorphism J maps vectors. . .

p. 166. The right side of Eq. (L286) should read:  $*\phi = \frac{1}{2}\sqrt{|g|}\phi_i g^{il}\epsilon_{ljk}e^j \wedge e^k$ .

p. 171, Problem L1.2.2(a). The last line should read: For a given  $n \in Z_5$ , which element is the inverse of n?

p. 173, Problem L1.2.6. The second line should read: ... i.e. permutations which...

p. 176, Problem L2.3.4. In the title, the third line, and the line before Eq. (1), 'of degree n' should be changed to 'of degree at most n'.

p. 178, Problem L3.3.1: The second sentence should read: on a finite interval  $I \subset \mathbb{R}$ ,

p. 179, Problem L3.3.3(a): The second vector should read:  $\mathbf{e}'_2 = \frac{1}{\sqrt{2}}(1,-1)^{\mathrm{T}}$ .

p. 184, Problem L5.3.6(a): "and m = 2" should be deleted.

p. 184, Problem L5.3.6(b): "and  $1 \le m \le N$ " should be deleted.

p. 186, Problem L5.4.3: The following specification should be added at the end of the first line: and  $m \in \mathbb{R}$ ,  $m \notin \{0, -1\}$ .

p. 186, Problem L5.4.4: The following specification should be added at the end of the first line: and  $m \in \mathbb{R}$ ,  $m \neq 0$ .

p. 199, Problem L10.2.6: The vector should read:  $e^{3} = (1, 0, 1)$ .

p. 199, Problem L10.2.7: In (a), the vector should read:  $u = e_1 + a e_2$ . In (b), the dual vector should read:  $J^{-1}(w)$ .

p. 200, Problem L10.2.8(d): 'Check your result' should read:  $x^{+} - x^{-} = \frac{3}{2}(1, -1)$ .

p. 200, Problem L10.3.1(a,b): 'Check your result' should read:  $t'_{\pm} = \pm t_{\pm}$ .

p. 201, Problem L10.4.1(b) should read: Evaluate  $\phi \equiv \omega(., v, w)$  and... For (c), 'Check your results' should read: If a = 2 then  $\phi = e^1 + e^2 - 3e^3$  and  $\omega(u, v, w) = -4$ .

p. 202, Problem L10.6.3: The canonical form for  $\phi$  should read:  $\phi = \sum_{i_1 < \ldots < i_4} \phi_{i_1,\ldots,i_4} e^{i_1} \wedge \ldots \wedge e^{i_4}.$  p. 203, Problem L10.7.2: The inner derivates should read: (a)  $i_u \omega$ , (b)  $i_{u,v} \omega$ . 'Check your result' for (b) should read:  $-e^1 - e^2 + 2e^3$ .

p. 204, Problem L10.9.4: The prefactor before the product should read:  $\frac{\sqrt{|g|}}{(n-p)!}$ 

p. 209. In Fig. C1, the topmost label in the right part of the figure should read  $f(x) + f'(x) \delta$ , as shown below.



Fig. C1 Differentiation of a function. Discussion, see text.

p. 221. The second paragraph is an INFO block, not an EXERCISE.

p. 225. In the figure next to Eq. (C30), the step height associated with  $y_{\ell+1} - y_{\ell}$  should read  $\delta \frac{dy(x_{\ell})}{dx}$ , as indicated in the lower part of the figure on the right.

p. 225. In the first line of Eq. (C31), the expression in square brackets on the right side should read:  $[y(x_{\ell} + \delta) - y(x_{\ell})].$ 

p. 227. Line 6 of the INFO block should read: . . . if  $(x + i)^2$  is decomposed . . .

p. 234, EXAMPLE. The formula should read  $\frac{\mathrm{d}T(\mathbf{r}(t))}{\mathrm{d}t} = \sum_{j=1}^{3} \frac{\partial T(\mathbf{r}(t))}{\partial r^{j}} \frac{\mathrm{d}r^{j}(t)}{\mathrm{d}t}.$ 

p.234, Eq. (C40). The second formula should read:

$$rac{\partial {f f}}{\partial y^j} \equiv \left(rac{\partial f^1}{\partial y^j},\ldots,rac{\partial f^m}{\partial y^j}
ight)^T$$

p. 247. The sixth line above Eq. (C57) should read:



basis vectors as  $\mathbf{v}_j(\mathbf{y}) = \mathbf{e}_a J^a{}_j(\mathbf{y})$ , with components  $J^a{}_j(\mathbf{y}) = \frac{\partial x^a}{\partial y^j}$ , see Eq. (V42).

p. 247. The sentence after Eq. (C57) should read: where  $\mathbf{x} = (x^1, x^2)^{\mathrm{T}}$  is the component vector...

p. 260. Before Eq. (C81), the text should read: ... represents the function at x as

p. 263. Eq. (C86) should read:  $\ln(1-x) = -\sum_{n=1}^{\infty} \frac{1}{n} x^n$ .

p. 276. The equation three lines above Eq. (C117) should end with = -f'(0). [Equation (C117) is correct as is.]

p. 282. The fourth line of the first INFO block should read  $\frac{1}{L}\sum_k \exp(ik(x_1-x_2)) = \delta(x_1-x_2).$ 

p. 291. The right-hand side of Eq. (C156) should read:  $\frac{1}{L} \sum_{p} \tilde{f}_{k-p} \tilde{g}_{p}$ .

p. 293. The left part of the third equation should read:  $\tilde{f}_s(k)\tilde{g}(k) \stackrel{k\sim 1/\ell}{\simeq} \tilde{f}_s(k)$ .

**p. 303.** Footnote 40 should read: We assumed here that  $d_t f$  is positive. If it is negative, then the left side of (C169) yields  $\int dt \frac{df}{dt} \frac{1}{h(f(t))} = -\int df \frac{1}{h(f)}$  (see Eq. (C28)). Alternatively, definite integrals can be used, as done in Eq. (C172) below (cf. (C27)). If h(f) has a zero, the sectors with h > 0 and h < 0 need to be treated separately.

p. 311. Above Eq. (C188), the initial condition should read  $(q(0), v(0))^{T} = (q_0, v_0)^{T}$ .

p. 325. In the figure and three further cases on this page, the argument and subscript of  $\Phi$  were inadvertently exchanged. The corrected figure is shown at the right, and the text should read as follows:

Three lines above Eq. (C212):

... the trajectories  $\Phi_t(\mathbf{x}_0)$  and  $\Phi_t(\mathbf{x}_0 + \delta)$  starting... On the left side of Eq. (C212):  $\|\Phi_t(\mathbf{x} + \delta) - \Phi_t(\mathbf{x})\| =$ In the second line of the last paragraph on p. 325: ... the curve  $\Phi_t(\mathbf{x}_0)$  plotted ...

p. 332. In the unnamed figure, the initial points should be named  $\mathbf{r}_0$  and  $\mathbf{r}_1$ , respectively.

p. 332. The in-text equation near the bottom should read:  $dF_{\mathbf{r}}[c_1\mathbf{u}_1 + c_2\mathbf{u}_2] = c_1 dF_{\mathbf{r}}[\mathbf{u}_1] + c_2 dF_{\mathbf{r}}[\mathbf{u}_2] (c_{1,2} \in \mathbb{R}) \dots$ 

p. 339. The equation just before (C232) should begin as follows:  $f'(z) = \lim_{x \to 0} \frac{1}{z} \left[ f(z + iz + i\delta) - f(z + iz) \right] = 0$ 







p. 358, Problem C1.2.1(b): the 'check your results' hint should read:  $\left[\frac{1}{2}, -\pi\sqrt{2}\right]$ .

p. 361. Problem C1.3.8(c): the 'check your results' hint should read:  $\left[2, \frac{1}{4}\right]$ .

p. 361. Problem C2.2.2(a) should be stated as  $I(x) = \int_0^x \mathrm{d}y \frac{1}{2y+4}$ .

p. 381. Since Problems P.C6.1.5-6 are first cited in Section C6.2, they arguably could have been assigned to section P.C6.2 rather than P.C6.1. That said, since they address properties of the  $\delta$  function introduced in Section C6.1, and knowledge of that sections suffices to solve these problems, they were assigned to Section P.C6.1.

p. 387, Problem V7.3.2(b): The second line should read "... variation of constants. What is the general solution, x(t), with  $x(0) = x_0$ ?"

Problem V7.3.2(c): The lines just before and after the equation for x(t) should read: "... inhomogeneous), the general solution x(t), with  $x(0) = x_0$ , can be expressed as

$$x(t) = x_h(t) + x_p(t) = x_h(t) + c(t)x_h(t) = (1 + c(t))x_h(t) = \tilde{c}(t)x_h(t),$$

while imposing on  $x_h(t)$  and  $\tilde{c}(t)$  the initial conditions  $x_h(0) = 1$  and  $\tilde{c}(0) = x_0$ ."

p. 388, Problem V7.4.1: The fourth line should read: ... the eigenvalue  $\lambda_{n-1} = \lambda_n$  is a two-fold zero of the characteristic polynomial ...

p. 389, Problem V7.4.4: The ansatz after Eq. (2) should read:  $\mathbf{x}(t) = \mathbf{v}e^{\lambda t}$ .

p. 390, Problem V7.4.4(d): The first line should read: . . . can be thought of as the limit  $\gamma \to \Omega$  of both . . .

p. 393, Problem V7.6.1: The second displayed equation should read:

$$\frac{\mathrm{d}y(x)}{\mathrm{d}x} = \frac{u_y(\mathbf{r})}{u_x(\mathbf{r})}.$$

p. 392. Problem C7.5.1(c): 'Check your result' should read: for a = 1 and  $\omega = 1$ ,  $|\tilde{G}(\omega)| = \frac{1}{\sqrt{2}}$ .

p. 397, Problem C8.3.3: The last part of subproblem (e) should be split off as a separate subproblem (f): Show that both these equations are satisfied ...

p. 400. Problem C9.4.4. The first line should read: (with  $a, b \in \mathbb{R}$  and  $a > 0, b \neq 0$ )

p. 419. Line 2 should read:  $\dots$  lack of orthonormality  $\dots$ 

p. 424. In Eqs. (V37), the top right equation should read:  $\mathbf{e}_r = \mathbf{v}_r$ .

p. 428. Line 3 should read: ... the right-hand side does not depend on ...

p. 433. In the line after Eq. (V55), the last equality should read:  $\delta df_{\mathbf{r}}(\mathbf{u}) \stackrel{\text{(V55)}}{=} df_{\mathbf{r}}(\delta \mathbf{u}).$ 

- p. 434. The second line before (V57) should read: ... given by  $df_{\boldsymbol{x}}(\boldsymbol{u}) = \dots$
- p. 442. Two lines above Eq. (V77), the in-line equation should read:  $u_i = \partial_i \varphi$ .
- p. 444. The last part of Eq. (V82) should read:  $i, j = 1, \ldots, d$ ,
- p. 445. The first arrow in Eq. (V83) should not have a foot:  $\mathbb{R}^2 \setminus \{0, 0\} \to \mathbb{R}^2$ ,
- p. 446. In the third-last line of the second paragraph, the integral should read:  $\int_{\hat{z}} d\mathbf{r} \cdot \mathbf{B} = 2\pi$ .
- p. 448. The last sentence should read: ... throughout this text.
- p. 467. The last sentence before the INFO block should end with: ... circle in  $\mathbb{R}^2$ .

p. 470. In the paragraph above Eq . (V126), the second sentence should read: It associates tangent vectors with "tufts" of curves...

p. 513. The second-last sentence before Eq. (V226) should read: Assume that this velocity is much smaller than the speed of light, c, i.e.  $\tau_A g/c \ll 1$ . After Eq. (V226), the last sentence of the *Hints*, Neglect velocities ... compared to c. should be replaced by: Retain only terms to leading order in  $\mathcal{O}(t_1, t_2, \tau_A, \tau_B)g/c \ll 1$ .

p. 514. The last sentence of paragraph A1 should read: Using the equation for  $t_1$  and retaining only terms to leading order in  $\mathcal{O}(t_1, t_2, \tau_{\rm A}, \tau_{\rm B})g/c \ll 1$ , we obtain Eq. (V226).

p. 515. The first sentence of paragraph Q3 should read: ... Minkowski metric introduced on p. 52, where ...

p. 533, Problem V1.2.1: The expression for S(t) in the second line should read:  $S(t) = \sin[\pi(1 - \cos \omega t)].$ 

p. 533, Problem V1.2.2: The curve should be defined as  $\gamma = {\mathbf{r}(t) | t \in (0, \infty)}, \dots$ 

p. 533, Problem V1.3.1: 'Check your result' should read:  $L[\gamma] = \frac{1}{12}(5^{3/2} - 1)$ .

p. 550, Problem V4.1.1: The definition of set  $S^1$  should read  $S^1 = \{(x^1, x^2)^T | (x^1)^2 + (x^2)^2 = 1\} \subset \mathbb{R}^2$ , and likewise  $S_{i,\pm} \equiv \{(x^1, x^2)^T \in S^1 | x^i \ge 0\} \subset S^1$ .

p. 550, Problem V4.1.2: The definition of set  $S^2$  should read  $S^2 = \{(x^1, x^2, x^3)^T | (x^1)^2 + (x^2)^2 + (x^3)^2 = 1\} \subset \mathbb{R}$ , and likewise  $S_{i,\pm} \equiv \{(x^1, x^2, x^3)^T \in S^2 | x^i \ge 0\} \subset S^2$ .

p. 552, Problem V4.2.2: The expression for y should read:  $y = (\theta, \phi)^{\mathrm{T}} \equiv \theta e_{\theta} + \phi e_{\phi}$ .

p. 552, Problem V4.2.3: In 'Check your results', some superscripts are missing: (a) if  $(x^1, x^2)^T = (4, 1)^T$  then ..., (b) if  $(\rho, \alpha)^T = (2, \ln 2)^T$ , then...

p. 553, Problem V5.1.2: 'Check your results' for (c,d) should read: (c)  $(f_r, f_\theta, f_\phi) = (1, 4\sqrt{3}, 0)$ ; (d)  $df(\partial_u) = 1 - 4\sqrt{3}$ . (a):  $\lambda \wedge \eta = -(12 + 13e^6) dx^1 \wedge dx^2$ , and the third (c):  $\lambda \wedge \eta = -8(1 + e^2) d\rho \wedge d\phi$ .

p. 555, Problem V5.3.3: The third line should read: (see figure on p. 492).

p. 557, Problem V5.3.7: The definition of  $\kappa$  should read:  $\kappa = \frac{1}{2}\rho^2 d\phi$ .

p. 559, Problem V5.4.3: The expression for  $\lambda$  should read:  $\lambda = j_0 x^1 dx^2$ .

p. 560, Problem V6.1.3: 'Check your results' should read: At  $(z^1, z^2)^T = (1, 3)^T \dots$ 

p. 560, Problems V6.2.1 and V6.2.2: The metric should read:  $g_{rr} = 1$ ,  $g_{\theta\theta} = r^2$ ,  $g_{\phi\phi} = r^2 \sin^2 \theta$ 

p. 562, Problem V7.4.4(c): The equation for  $F^{\alpha\beta}$  should read:  $F^{\alpha\beta} = -\frac{1}{2} \epsilon^{\alpha\beta\mu\nu} G_{\mu\nu}$ .

p. 577, Problem L4.3.1(c): The final result for  $\mathbf{c} \times (\mathbf{a} \times \mathbf{b})$  should read:  $\begin{pmatrix} -7 \\ -2 \\ 16 \end{pmatrix}$ 

p. 594, Problem L8.4.2(c): The fourth line should read: Check:  $\lambda_1 + \lambda_2 + \lambda_3 = 3$  = TrA = 1 + 1 + 1

p. 599, Problem L10.3.1(c): The second sentence should read: The tensor will remain invariant if its components remain unchanged,  $t^{kl} = t'^{kl}$ , i.e.  $t^{kl} = A^k_{\ i}A^l_{\ j}t^{ij} = A^k_{\ i}t^{ij}(A^T)_j^{\ l}$  or  $t = AtA^T$ , in matrix notation.

p. 611, Problem C3.3.1(a): The first line should read:  $f(\mathbf{g}(\mathbf{x})) = \|\mathbf{g}(\mathbf{x})\|^2 = \dots$ 

p. 616, Problem C4.1.7(c): The last line for A(a, b) should start with  $\stackrel{(b)}{=} \frac{1}{4}\pi ab - \dots$ 

p. 619, Problem C4.4.1: The first line in (b) should read: We use  $\mathbf{r}(\theta, \phi) = R(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)^{\mathrm{T}} \dots$  In the subsequent displayed equation the first equality should read:  $\partial_{\theta}\mathbf{r} = R(\cos\theta\cos\phi, \cos\theta\sin\phi, -\sin\theta)^{\mathrm{T}}, \dots$ 

The cross products in (a) and (b) should read, respectively:

$$\begin{aligned} \|\partial_x \mathbf{r} \times \partial_y \mathbf{r}\| &= \left[ \|\partial_x \mathbf{r}\|^2 \|\partial_y \mathbf{r}\|^2 - \|\partial_x \mathbf{r} \cdot \partial_y \mathbf{r}\|^2 \right]^{1/2} \\ \|\partial_\theta \mathbf{r} \times \partial_\phi \mathbf{r}\| &= \left[ \|\partial_\theta \mathbf{r}\|^2 \|\partial_\phi \mathbf{r}\|^2 - \|\partial_\theta \mathbf{r} \cdot \partial_\phi \mathbf{r}\|^2 \right]^{1/2} = \left[ R^4 \sin^2 \theta - 0 \right]^{1/2} = R^2 \sin \theta \end{aligned}$$

The inequality just before the equation for  $A_S$  should read:  $|y| \leq \sqrt{R^2 - x^2}$ 

p. 623, Problem C5.3.1(a): The first line of the equation for f(x) should read:

$$f(x) = \frac{1}{1 - \sin x} = \frac{1}{1 - [x - \frac{1}{6}x^3 + \mathcal{O}(x^5)]}$$

p. 625, Problem C5.4.1(a): The expression in the second line should read:  $e^c \left[1 + z + \frac{1}{2!}z^2\right] + O(z^3)$ 

p. 627. Problem C5.4.3(b): Method 1: expansion of Eq. (3), using  $\ln(1+z) =$ 

 $z - \frac{1}{2}z^2 + \mathcal{O}(z^3)$ , should read:

$$0 = \ln \left(y_0 + y_1 x + \frac{1}{2} y_2 x^2 + \mathcal{O}(x^3)\right) - x \ln (2)$$
  
=  $\ln (y_0) + \ln \left(1 + \frac{y_1}{y_0} x + \frac{1}{2} \frac{y_2}{y_0} x^2 + \mathcal{O}(x^3)\right) - x \ln (2)$   
=  $\ln (y_0) + \left(\frac{y_1}{y_0} x + \frac{1}{2} \frac{y_2}{y_0} x^2 + \mathcal{O}(x^3)\right) - \frac{1}{2} \left(\frac{y_1}{y_0} x + \frac{1}{2} \frac{y_2}{y_0} x^2 + \mathcal{O}(x^3)\right)^2 + \mathcal{O}(x^3) - x \ln (2)$   
=  $\ln (y_0) + \left[\frac{y_1}{y_0} - \ln (2)\right] x + \frac{1}{2} \left[\frac{y_2}{y_0} - \frac{y_1^2}{y_0^2}\right] x^2 + \mathcal{O}(x^3).$ 

The second sentence should read: Solving ... for  $x^2$ :  $0 = \frac{1}{2} \left[ \frac{y_2}{y_0} - \frac{y_1^2}{y_0^2} \right]$ , thus  $y_2 = \left[ \ln^2(2) \right]$ .

p. 641, Problem C7.4.3(b): The first line at the top of the page should start with: Since  $\det(A - \lambda_1 \mathbb{1}) = 0, \ldots$  Similarly, the first line of the paragraph in the middle of the page should start with: Since  $\det(A - \lambda_2 \mathbb{1}) = 0, \ldots$ 

p. 644, Problem C7.4.7(b): Eq. (5) should read:

$$(\mathbf{d}_t^2 + 2\gamma \mathbf{d}_t + \Omega^2) \mathbf{e}^{\lambda_j t} = \left(\lambda_j^2 + 2\gamma \lambda_j + \Omega^2\right) \mathbf{e}^{\lambda_j t} \stackrel{(3)}{=} 0 \,. \quad \checkmark$$

p. 645, Problem C7.4.7(b): The equation before Eq. (7) should read:

$$(\mathrm{d}_t^2 + 2\gamma \mathrm{d}_t + \Omega^2) x_h = \frac{1}{2\gamma_r} \left[ \left( \lambda_+^2 + 2\gamma \lambda_+ + \Omega^2 \right) \mathrm{e}^{\lambda_+ t} - \left( \lambda_-^2 + 2\gamma \lambda_- + \Omega^2 \right) \mathrm{e}^{\lambda_- t} \right] \stackrel{(3)}{=} 0.\checkmark$$

p. 648. Problem C.7.7.1(c): The last line should read: (ii)  $\lambda > 0 : x_{-}^{*}$  is stable (see I, IV);  $x_{0}^{*}$  semistable (see II, III) ...

p. 651, Problem C8.3.3(c): In the first line, the second equality should read:  $\phi(\rho) = \phi_0 + \arccos(\rho_0/\rho)$ . The right side of the following equation should read:

$$\frac{\rho_0}{\rho\sqrt{\rho^2-\rho_0^2}}$$

p. 653, Problem C9.4.1(b): The equation for g(z) should read:

$$g(z) = \frac{e^{iaz}}{z^2 + 1} = \frac{e^{iaz}}{(z - i)(z + i)} = \frac{e^{iaz}}{(z - z_+)(z - z_-)}.$$

In the equation for  $I_3(a < 4)$ , the third expression should read:

$$I_3(a < 4) = \dots = \lim_{R \to \infty} \oint_{|z|=R} \frac{\mathrm{d}z}{z^2} =$$

p. 654, Problem C9.1.4(c): The equation for 
$$\text{Res}(f, z_3)$$
 should start as follows:

$$\operatorname{Res}(f, z_3) = \lim_{z \to z_3} \frac{\mathrm{d}}{\mathrm{d}z} \left[ (z - z_3)^2 f(z) \right] = \dots$$

In the equation for  $I_3(a < 4)$ , the third term should read:

$$I_3(a < 4) = \dots = \lim_{R \to \infty} \oint_{|z|=R} \frac{\mathrm{d}z}{z^2} = \dots$$

In the last line of the *Note*, the limit should read  $\lim_{y\to-\infty} |(iy)f(iy)| = \infty$ . p. 655, Problem C9.4.5(a): In the last equation, for  $\operatorname{Res}(f, z_a^{\pm})$ , the second-last expression should read  $\frac{-a^2}{(-a^2+4)2(\pm ai)}$ .

p. 656, Problem C9.4.7(a): The third equation should read:

$$\int_{\Gamma_{\pm}} \mathrm{d}z f(z) = \int_{0}^{\pm \pi} \mathrm{d}\phi \, \frac{\mathrm{d}z(\phi)}{\mathrm{d}\phi} f(z(\phi)) = \int_{0}^{\pm \pi} \mathrm{d}\phi \left(\mathrm{i}R\mathrm{e}^{\mathrm{i}\phi}\right) \frac{1}{2\pi} \frac{\mathrm{e}^{-\mathrm{i}tR(\cos\phi + \mathrm{i}\sin\phi)}}{a - \mathrm{i}R\mathrm{e}^{\mathrm{i}\phi}}$$

p. 658, Problem V1.2.1(c): The first box should contain:  $\pi\omega\sin(\omega t)CS(1-a^2)$ 

p. 664, Problem V3.2.1(f): The last sentence should end: ... consistent with (e).

p. 667, Problem V3.4.1(b): On the far right, it would be more logical to replace -2x + 2x by 2x - 2x (top line) and  $-3z^2 + 3z^2$  by  $3z^2 - 3z^2$  (bottom line).

p. 670, Problem V3.5.5(a): In the last system of three equations, the first equality for bottom, side wall and top should read:

$$\Phi_B = \frac{1}{3} \int_B d\mathbf{S} \cdot \mathbf{r} = \dots \qquad \Phi_W = \frac{1}{3} \int_W d\mathbf{S} \cdot \mathbf{r} = \dots \qquad \Phi_T = \frac{1}{3} \int_T d\mathbf{S} \cdot \mathbf{r} = \dots$$

p. 673, Problem V3.6.5(b): The equation for the flux should read as follows,

Flux: 
$$\Phi = \oint_{\gamma} d\mathbf{r} \cdot \mathbf{A} = \int_{0}^{2\pi} d\phi \, \frac{d\mathbf{r}}{d\phi} \cdot \mathbf{A} = \frac{m}{cR} \int_{0}^{2\pi} d\phi = \boxed{\frac{2\pi m}{cR}}.$$

i.e. the first  $\int d\phi$  integral should not have a prefactor  $\frac{m}{cR}$ .

p. 676, Problem V3.7.5(g): The last part of the second line for  $\nabla \times \mathbf{u}$  should read: + $\mathbf{e}_z \frac{1}{\rho} \Big[ \partial_\rho \left( \rho \cdot 0 \right) - \partial_\phi(\rho) \Big] = \boxed{\mathbf{0}}$ . (Note that the **0** should be bold.)

p. 685, Problem V5.3.1(c): The last two equations lack an overall minus sign:

$$\lambda \wedge \eta = -\left[2x^1x^2 + ((x^1)^2 + (x^2)^2)e^{x^1x^2}\right]_{x=x(y)}(\rho\cos^2\phi + \rho\sin^2\phi)\,\mathrm{d}\rho \wedge \mathrm{d}\phi$$
$$= \boxed{-\rho^3\left[2\cos\phi\sin\phi + e^{\rho^2\cos\phi\sin\phi}\right]\,\mathrm{d}\rho \wedge \mathrm{d}\phi}.$$

## **Even-numbered solution manual:**

p. 9, Problem L2.3.4: In the title, in (b)(i), in the third sentence of (b)(ii), and in the last sentence of (c), 'of degree n' should be changed to 'of degree at most n'.

p. 22, Problem L5.5.2: The last equation of part (c) should read:

$$B\mathbf{v} = \frac{1}{2} \begin{pmatrix} 1 & -\sqrt{2} & -1\\ \sqrt{2} & 0 & \sqrt{2}\\ -1 & -\sqrt{2} & 1 \end{pmatrix} \begin{pmatrix} 1\\ 0\\ 1 \end{pmatrix} = \begin{bmatrix} 0\\ \sqrt{2} \begin{pmatrix} 0\\ 1\\ 0 \end{bmatrix}$$

p. 24, Problem L5.6.2(e): The second equation should read:  $\hat{\mathbf{v}}_2 \mapsto -\frac{1}{3}(4\hat{\mathbf{v}}_1 + \hat{\mathbf{v}}_2) \equiv \hat{\mathbf{v}}_1 A^1_2 + \hat{\mathbf{v}}_2 A^2_2.$ 

p. 28, Problem L6.2.2(a)(ii): The second sentence should read: The columns 1 and 2 are the same when c = 1 and d = 2 [case (i)]; and the rows 2 and 3 of the matrix are identical when e = 3 and d = 2 [case (ii)].

p. 30, Problem L7.3.2(b): In the second line from the top, the expression for  $v_{11}^2$ should read:  $v_1^2 = -\frac{1-i}{1+i}v_1^1 = iv_1^1$ .

p. 45, Problem L10.3.4(a): in the first line, superscripts should be subscripts and vice versa:  $t(w, w; u, v) = 2w_1w_2u^1v^3 - w_2w_1u^3v^2$ . In the second line, the result in the box should read -96c - 8abc.

p. 46, Problem L10.7.2(b): the box should contain  $-abe^1 - be^2 + 2e^3$ .

p. 50, Problem C1.3.4(c): Two intermediate expressions for  $\operatorname{arctanh}' x$  should read:  $\frac{1}{\operatorname{sech}^2(\operatorname{arc} \tanh x)} = \frac{1}{1 - \tanh^2(\operatorname{arc} \tanh x)}$ 

p. 51, Problem C1.3.8(c). The solution should read:

(c) 
$$\lim_{x \to 0} \frac{e^{x^2} - 1}{(e^{ax} - 1)^2} = \lim_{x \to 0} \frac{2xe^{x^2}}{2a(e^{ax} - 1)e^{ax}} = \lim_{x \to 0} \frac{2(1 + 2x)e^{x^2}}{2a^2(2e^{2ax} - e^{ax})} = \left| \begin{array}{c} \frac{1}{a^2} \\ \frac{1}{a^2} \end{array} \right|.$$

p. 58, Problem C2.3.6(b). The 5th line from the bottom should read  $\tilde{I}(b) = \cdots =$  $\frac{1}{2} \left[ b + \sinh b \sqrt{1 + \sinh^2 b} \right].$ 

p. 69, Problem C4.5.4. The second and third terms in the equation for the Jacobian J should read:

$$\left| \det \begin{pmatrix} d/a & 1/a & 0 \\ -1/b & 1/b & 1/b \\ 0 & 1/c & -1/c \end{pmatrix} \right|^{-1} = \left| \frac{-2d-1}{abc} \right|^{-1}$$

p. 72, Problem C5.4.2(c). The second equation after (3) should read:

$$h(x) = h(1) + h^{(1)}(1)(x-1) + \frac{1}{2}h^{(2)}(1)(x-1)^2 + \frac{1}{6}h^{(3)}(1)(x-1)^3 + \mathcal{O}\left((x-1)^4\right)$$

p. 84, Problem C6.3.6(c). Case (I) should read  $f_{-\gamma}(t') = f_{-\gamma}(0) e^{-\gamma t'}$ . In the figure, the curve segments showing this function should tend upwards, since  $f_{-\gamma}(0) < 0$ , hence  $f_{-\gamma}(t')$  increases with increasing t'.



p. 131, Problem PV3.7.6(g): In the third

line of the set of equations for  $\nabla \times \mathbf{u}$ , the zero at the end should be bold,  $\begin{vmatrix} \mathbf{0} \end{vmatrix}$