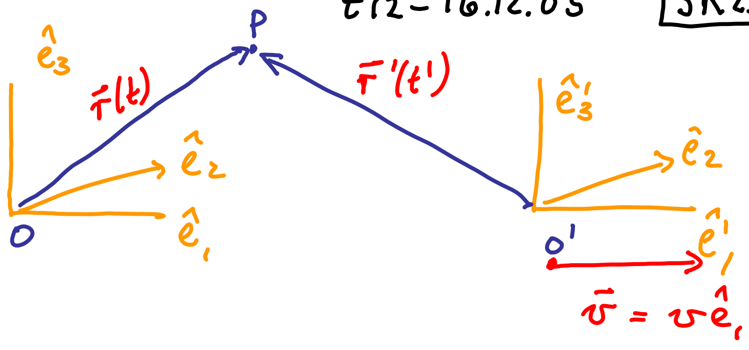
Lorentz-Transformation

Für $t = t' = 0$ sei $O = O'$,

also $x(0) = x'(0) = 0$ (1)



Lichtblitz starte bei $t = t' = 0$ in $O = O'$, und erreiche etwas später Punkt P.

O sagt: $r^2 = x^2 + y^2 + z^2 = (ct)^2$ (2)

O' sagt: $r'^2 = x'^2 + y'^2 + z'^2 = (ct')^2$ (3)

Gesucht: Beziehung zwischen Koordinaten von P laut O und O', die konsistent ist mit (2,3)

<u>Antwort:</u> <u>Lorentz-Transformation</u> <u>Rücktransformation</u>

$ct' = \gamma\left(ct - \frac{vx}{c}\right)$ (4a) $ct = \gamma\left(ct' + \frac{vx'}{c}\right)$ (4a)'

$x' = \gamma\left(x - \frac{v}{c}ct\right)$ (4b) $x = \gamma\left(x' + \frac{v}{c}ct'\right)$ (4b)'

$y' = y$ (4c) $y = y'$ (4c)'

$z' = z$ (4d) $z = z'$ (4d)'

mit

$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$ (5)

$= \frac{1}{\beta'} > 1$ (6)

$\rightarrow 1$ für $v/c \rightarrow 0$ (7)

$$(ct')^2 - (x')^2 = \left(\gamma\left(ct - \frac{v}{c}x\right)\right)^2 - \gamma^2(x - vt)^2$$

$$= \;?$$

$$= (ct)^2 - x^2$$

Ist (4)' inverse Transf. zu (4)? Benutze Matrix-Notation für x,t Komponenten:

$$\begin{pmatrix}ct'\\x'\end{pmatrix} \overset{(4a)}{=} \underbrace{\gamma\begin{pmatrix}1 & -v/c\\ -v/c & 1\end{pmatrix}}_{A(v)} \begin{pmatrix}ct\\x\end{pmatrix} \qquad (1a)$$

$$\begin{pmatrix}ct\\x\end{pmatrix} = \underbrace{\gamma\begin{pmatrix}1 & v/c\\ v/c & 1\end{pmatrix}}_{B = A^{-1}(v) = A(-v)} \begin{pmatrix}ct'\\x'\end{pmatrix} \qquad (1a')$$

(4') ist inverse Transf. zu (4) falls: $\begin{pmatrix}ct'\\x'\end{pmatrix} \overset{(1a)}{=} A\begin{pmatrix}ct\\x\end{pmatrix} \overset{(1a')}{=} A\,B\begin{pmatrix}ct'\\x'\end{pmatrix} \overset{?}{=} \begin{pmatrix}ct'\\x'\end{pmatrix}$ (2)

$\Rightarrow$ falls $1 \overset{?}{=} A\,B = \gamma^2 \begin{pmatrix}1&-v/c\\-v/c&1\end{pmatrix}\begin{pmatrix}1&v/c\\v/c&1\end{pmatrix} = \gamma^2\begin{pmatrix}1-v^2/c^2 & 0\\0&1-v^2/c^2\end{pmatrix} = \begin{pmatrix}1&0\\0&1\end{pmatrix}$ ✓ $\Longrightarrow B = A^{-1}$

Ist (23.4) kompatibel mit Bedinungen (23.3) und (23.2)?

$$\underbrace{(ct')^2 - x'^2}_{(23.3)\ =0} = (ct', x')\begin{pmatrix}1&0\\0&-1\end{pmatrix}\begin{pmatrix}ct'\\x'\end{pmatrix} \overset{(1a)}{=} (ct, x)\overbrace{A^T\begin{pmatrix}1&0\\0&-1\end{pmatrix}A}^{\begin{pmatrix}1&0\\0&-1\end{pmatrix}}\begin{pmatrix}ct\\x\end{pmatrix} = \underbrace{(ct)^2 - x^2}_{=0\ (23.2)\ ✓} \qquad (3)$$

$$A^T\begin{pmatrix}1&0\\0&-1\end{pmatrix}A \overset{(1a)}{=} \gamma\begin{pmatrix}1&-v/c\\-v/c&1\end{pmatrix}\begin{pmatrix}1&0\\0&-1\end{pmatrix}\gamma\begin{pmatrix}1&-v/c\\-v/c&1\end{pmatrix} = \gamma^2\begin{pmatrix}1&-v/c\\-v/c&1\end{pmatrix}\begin{pmatrix}1&-v/c\\v/c&-1\end{pmatrix}$$

$$= \gamma^2\begin{pmatrix}1-v^2/c^2&0\\0&-1+v^2/c^2\end{pmatrix} = \begin{pmatrix}1&0\\0&-1\end{pmatrix} \qquad (4)$$

(3) bedeutet: falls (23.3) gilt, dann gilt auch (23.4), und umgekehrt! ☺

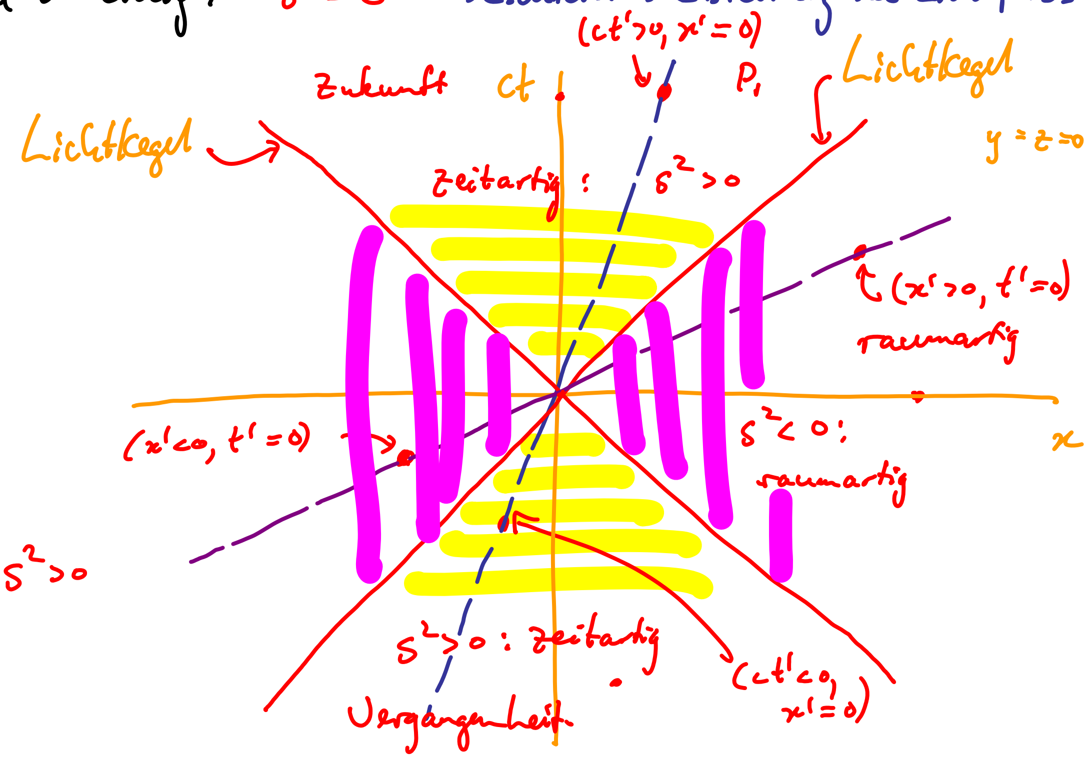
$\boxed{\text{Matrizen erleichtern das Rechnen!}}$

Invariantes Intervall:

Falls $(ct')^2 - (x'^2 + y'^2 + z'^2) = s^2$

gilt auch $(ct)^2 - (x^2 + y^2 + z^2) \overset{(24.3)}{=} s^2$

Folglich sind sich O und O' einig: $s^2 = 0$ beschreibt ausbreitung des Lichtpulses



P_1

"zeitartiges" Intervall: $s^2 > 0$

"raumartiges" Intervall:

$S_{BA} = \sqrt{\qquad\qquad}$, $S'_{BA} = \sqrt{\qquad\qquad}$

$S_{BA}^2 = (\Delta t_{BA})^2 - (\Delta x_{BA})^2$

$S'^2_{BA} = \dots$

Längenkontraktion:

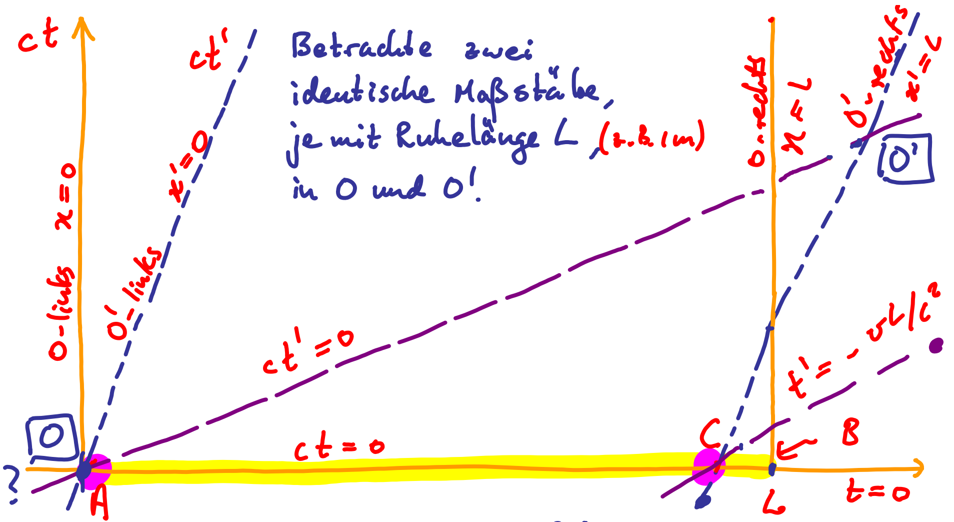
①: $ct' = \gamma\left(ct - \frac{vx}{c}\right)$

②: $x' = \gamma(x - vt)$

①': $ct = \gamma\left(ct' + \frac{vx'}{c}\right)$

②' $x = \gamma(x' + vt')$

$\gamma = 1/\sqrt{1 - v^2/c^2} > 1$



Betrachte zwei identische Maßstäbe, je mit Ruhelänge L (z.B. 1m) in O und O'.

Wie lang ist ein O'-Maßstab, laut O?

O macht Fotos von O'-links, O'-rechts neben O-Messlatte, zur selben Zeit, z.B. $t=0$

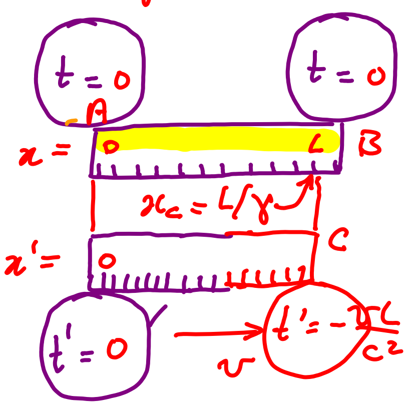
O-Maßstab: links: A: $x=0$, $t=0$, rechts: B: $x=L$, $t=0$;

O'-Maßstab: links: A: $x'=0$, $t'=0$, rechts: C: $x'=L$, $t=0$, $x \overset{②}{=} \frac{L}{\gamma}$, $t' \overset{①'}{=} -\frac{vL}{c^2}$

Laut O, bei $t=0$: obwohl A: O'-Links = O-Links, ist C: O'-rechts < O-rechts

Bewegte Maßstäbe schrumpfen:

②: $x_C = \frac{x'_C}{\gamma} = \frac{L}{\gamma} < L \Rightarrow$ [O' ist kürzer als O]



Das ist nicht paradox, denn laut O', hat O die L- und R-Messung nicht gleichzeitig gemacht! (R vor L)

[laut O: Bewegte O'-Uhren sind asynchron: SR15, R1, mit $E_1 \xrightarrow{O}$ $E_2 \xrightarrow{O'}$]

Längenkontraktion:

① : $ct' = \gamma\left(ct - \frac{vx}{c}\right)$

② : $x' = \gamma(x - vt)$

①' : $ct = \gamma\left(ct' + \frac{vx'}{c}\right)$

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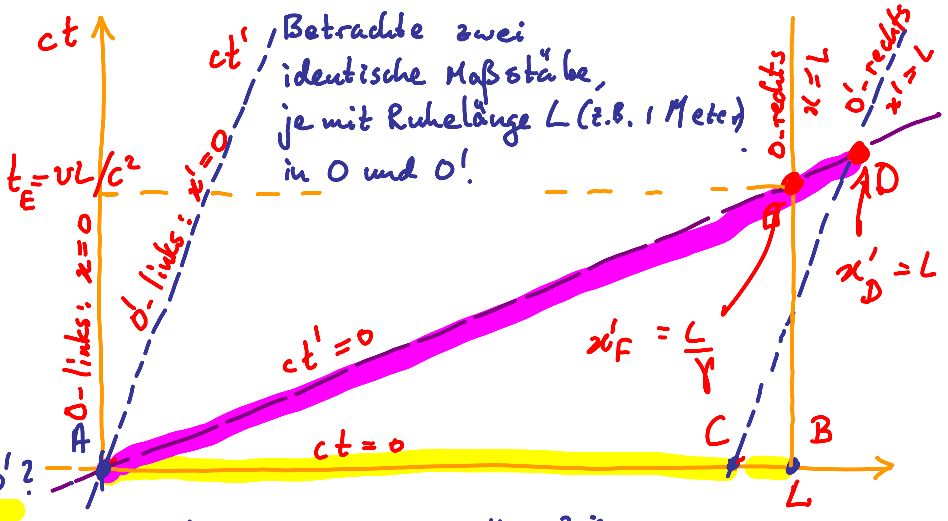
$\gamma = 1/\sqrt{1 - v^2/c^2} > 1$

Betrachte zwei identische Maßstäbe, je mit Ruhelänge L (z.B. 1 Meter) in O und O'.



Wie lang ist ein O-Maßstab, laut O'?

O' macht Fotos von O-links, O-rechts neben O'-Messlatte, zur selben Zeit, z.B.

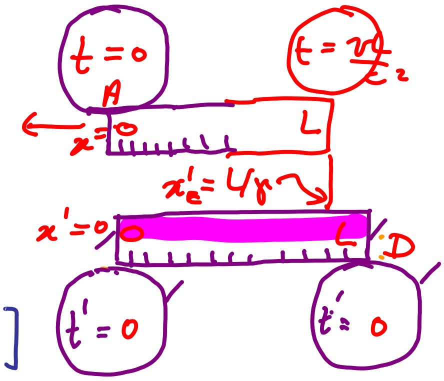
O'-Maßstab: links: A: $x' = 0$, $t' = 0$, rechts: D: $x' = L$, $t' = 0$;

O-Maßstab: links: A: $x = 0$, $t = 0$, rechts: E: $x = L$, $t' = 0$, $x' \overset{②'}{=} \frac{L}{\gamma}$, $t \overset{①}{=} \frac{vL}{c^2}$

Laut O', bei $t' = 0$: obwohl A: O-Links = O'-Links, ist E: O-rechts < O'-rechts

Bewegte Maßstäbe schrumpfen:

②': $x'_E = \frac{x_E}{\gamma} = \frac{L}{\gamma} < L \Rightarrow$ [O ist kürzer als O']



Das ist nicht paradox, denn laut O, hat O' die L- und R-Messung nicht gleichzeitig gemacht! (L vor R)

[Laut O': Bewegte O-Uhren sind asynchron: SR15, R1, mit $E_1 \to$ O →, $E_2 \to$ O' →]

Zeitdilatation:

① : $ct' = \gamma\left(ct - \frac{vx}{c}\right)$

② : $x' = \gamma(x - vt)$

①' : $ct = \gamma\left(ct' + \frac{vx'}{c}\right)$

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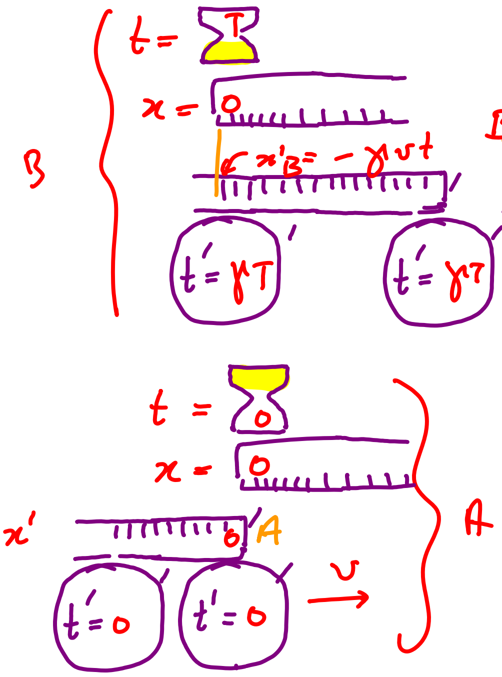
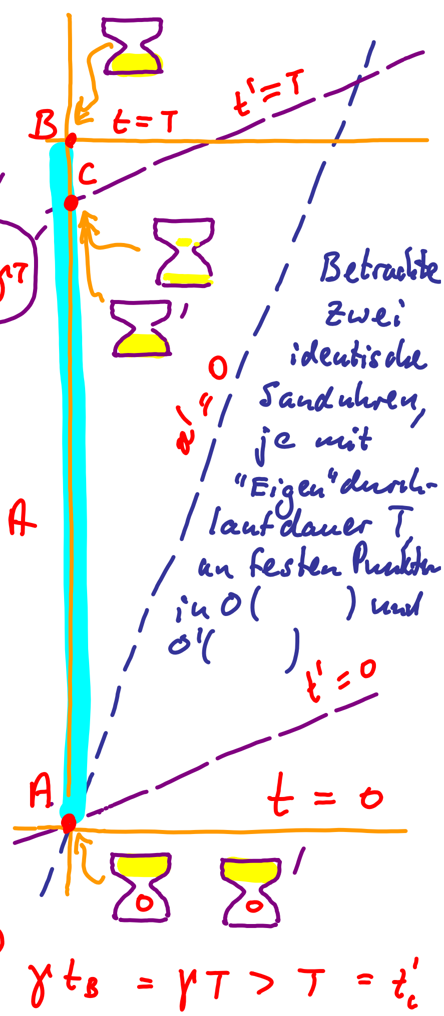




Betrachte zwei identische Sanduhren, je mit "Eigen"durchlaufdauer T an festen Punkten in O ( ) und O' ( )

Wie lange läuft O-Sanduhr laut O'?

O' macht, neben O'-Stopuhren, Fotos von O-voll, O-leer, an festem O-Ort (z.B. $x = 0$):

O-Sanduhr voll: A: $x = 0$, $t = 0$, leer: B: $x = 0$, $t = T$; ①: $t' = \gamma T$ ②: $x' = -\gamma v t$

O'-Sanduhr voll: A: $x' = 0$, $t' = 0$, leer: C: $x = 0$, $t' = T$, ①': $t = T/\gamma$ ②': $x' = -vT$

laut O', bei $x = 0$: obwohl A: O'-voll wenn O-voll

bewegte Uhren gehen langsamer

ist C: O'-leer bevor B: O-leer, bei $t'_B \overset{①}{=} \gamma t_B = \gamma T > T = t'_C$

$\Rightarrow$ O-Uhren gehen langsamer als O'-Uhren!

Zeitdilatation:

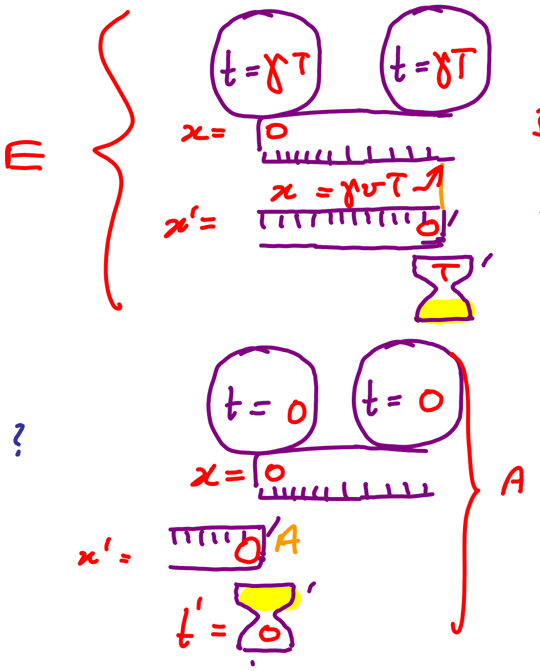
①: $ct' = \gamma\left(ct - \frac{vx}{c}\right)$

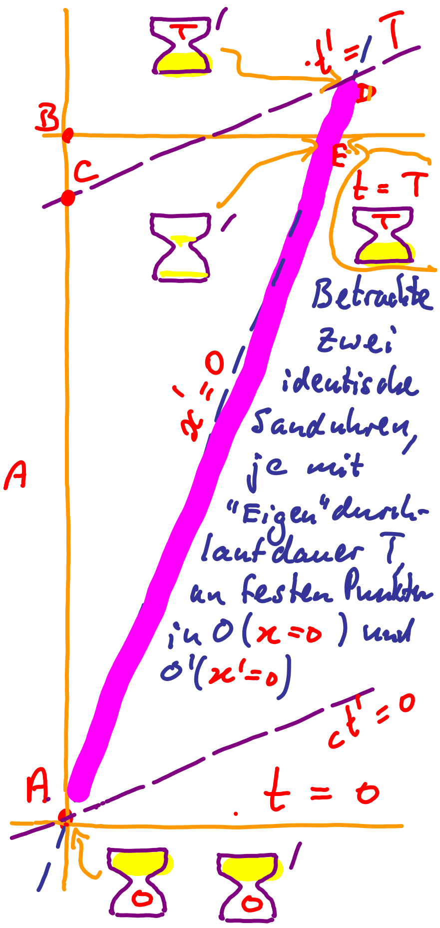
②: $x' = \gamma(x - vt)$

①': $ct = \gamma\left(ct' + \frac{vx'}{c}\right)$

②': $x = \gamma(x' + vt')$

$\gamma = 1/\sqrt{1 - v^2/c^2} > 1$





Betrachte zwei identische Sanduhren, je mit "Eigen"durchlaufdauer T, an festen Punkten in O ($x=0$) und O' ($x'=0$)

Wie lange läuft O'-Sanduhr laut O?

O macht, neben O-Stopuhren, Fotos von O'-voll, O'-leer, an festem O'-Ort (z.B. $x'=0$):

O'-Sanduhr voll: A: $x'=0$, $t'=0$; leer: D: $x'=0$, $t'=T$;

①': $t = \gamma T$

②': $x = \gamma v T$

O-Sanduhr voll: A: $x=0$, $t=0$, leer: E: $x'=0$, $t=T$,

①': $t' = T/\gamma$

②: $x = vt$

laut O, bei $x'=0$: obwohl A: O-voll wenn O'-voll,

ist E: O-leer bevor D: O'-leer, bei $t_D = \gamma t'_D = \gamma T > T = t_E$

bewegte Uhren gehen langsamer $\Rightarrow$ O-Uhren gehen langsamer als O'-Uhren!

Mermin, Chapter 10: Length contraction

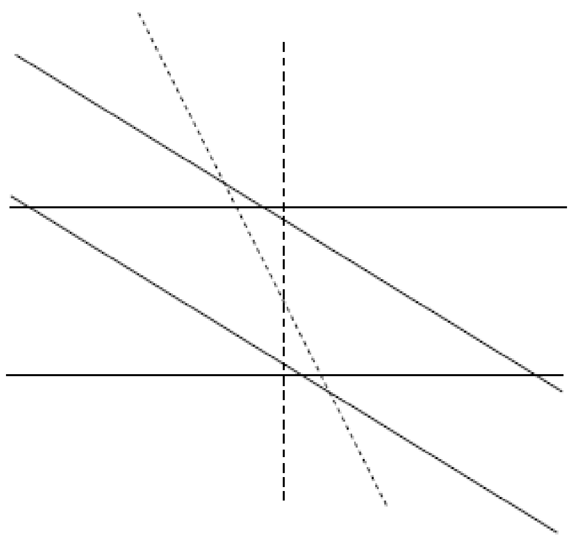


Figure 9. The two vertical solid lines are the left and right ends of a stick. The two solid lines that tilt upward to the right are the left and right ends of a second stick that moves to the right past the first stick. The horizontal dashed line is a line of constant time in the frame in which the first stick is stationary. Note that both ends of the first stick extend beyond both ends of the second along that line of constant time, thereby establishing that the first stick is longer than the second in its proper frame. The dashed line that tilts upward to the right is a line of constant time in the frame in which the second stick is stationary. (Note that it tilts away from the horizontal by the same amount that the lines representing the ends of the second stick tilt away from the vertical.) Along this tilted line of constant time both ends of the second stick extend beyond both ends of the first stick, thereby establishing that the second stick is longer than the first in its proper frame.

The figure vividly demonstrates that what one means by *a stick at a given moment of time* depends on the frame of reference in which the stick is described, and that it is this that makes it possible for people using the proper frame of either stick to maintain that the other stick is shorter.

Mermin, Chapter 10: Twin paradox

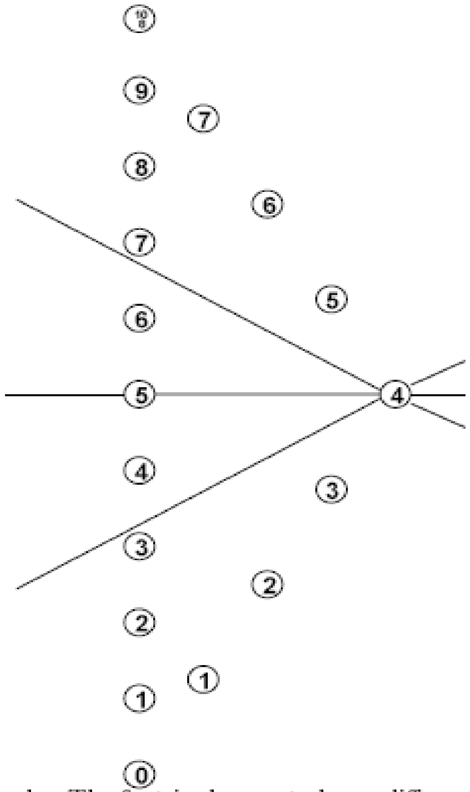


Figure 11. Two identical clocks. The first is shown at eleven different moments along the vertical line, as its reading advances from 0 to 10. The second moves away from the first as it advances from 0 to 4; it then moves back to the first, as its reading advances from 4 to 8. At the bottom and top of the figure both clocks are at the same place at the same time and are represented by a single circle. The first clock is stationary in a single inertial frame of reference. Since lines of constant position are vertical in that frame, lines of constant time are horizontal. Consequently it is evident from the figure that in the proper frame of the first clock, the outward and inward journeys of the second clock each take 5 seconds, during each of which the second clock only advances by 4 seconds.

Two other lines of constant time are shown passing through the point at which the second clock begins its return journey. One line (going downward to the left) is appropriate to the proper frame of the second clock during its outward journey; the other (going upward to the left) is appropriate to the proper frame of the second clock during its inward journey. Note that just before the second clock changes frames, the time on earth in the outward-going frame is about 3.2 seconds. Just after the second clock has changed frames, the time on earth in the inward-going frame is about 6.8 seconds.

Mermin, Chapter 10: Time dilation

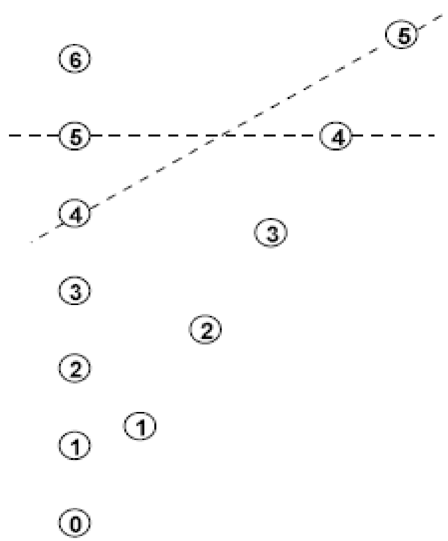


Figure 10. Several moments in the histories of two uniformly moving clocks (represented by circles with numbers inside giving their readings.) Both clocks read 0 at the same place and time and are represented by just a single circle in the figure. Subsequent readings of the first clock (1-6) are shown on the set of circles uniformly spaced along a vertical line; subsequent readings of the second clock (1-5) are shown on the set of circles that lie on a line sloping upward to the right. The horizontal dashed line is a line of constant time in the frame of the first clock. In that frame the second clock has advanced from 0 to 4 in the time it took the first to advance from 0 to 5, so the second clock is running slowly by a factor $s = 4/5$. The slanting dashed line is a line of constant time in the frame of the second clock (and tilts away from the horizontal by the same amount that the line along which the pictures of the second clock lie tilts away from the vertical.) In that frame the first clock has advanced from 0 to 4 in the time it took the second to advance from 0 to 5.

The figure makes clear that how one compares the rates of two clocks in relative motion depends on how one judges whether two events in different places are simultaneous. This is what makes it possible for people using the proper frame of either clock to maintain that the other clock is running slowly.

Mermin, Chapter 10: Twin paradox continued

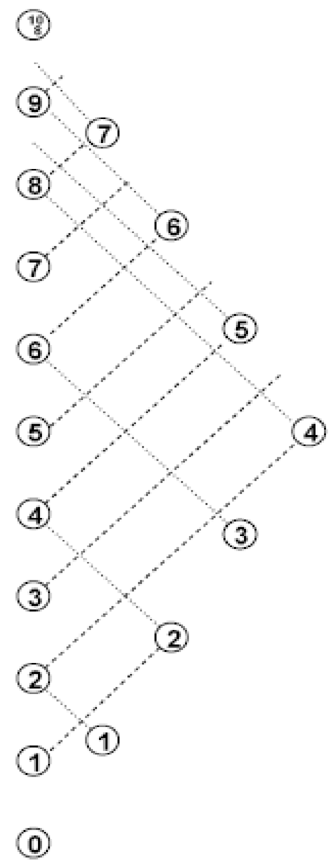


Figure 12. This repeats figure 9, but without the lines of constant time, and with many photon trajectories indicating what somebody moving with either clock *sees* the other clock doing. Each clock emits a flash of light each time its reading changes, and those flashes are seen by people moving with the other clock. People at the position of the first clock (vertical line of constant position) see the second clock advancing at *half* its normal rate during the 8 seconds they are watching the flashes emitted on the outward journey, and they see the second clock advancing at *twice* its normal rate during the 2 seconds they are watching flashes emitted by the second clock on its inward journey.

People moving with the second clock, on the other hand, see the first clock running at *half* its normal rate during the four seconds of their outward journey and at *twice* its normal rate during the four seconds of the inward journey.