Quantum Matter | Winter Semester 2009/10
LMU München | on 19.2.2010, 10:00
Prof. Jan von Delft, Dr. Andreas Weichselbaum | Hints for exam

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Important Issues

Aharonov-Bohm effect
• What is the AB-effect? Describe in words?
• Show that the conductance through a mesoscopic ring penetrated by a flux Φ oscillates as $\cos(\phi + 2\pi\Phi/\Phi_0$, and *derive* value of flux quantum $\Phi_0 = hc/e$.
• What is weak localization? Describe in words.

Quantum Information
• State and prove no-cloning theorem.
• Quantum teleportation: state the problem. Given a logical circuit diagram for teleportation (no need to memorize it!), and a specific input state for Alice, show how this state is processed by the protocol and that it is indeed teleported to Bob.
• What is meant by quantum parallelism?

Second Quantization (very important)

• Why is second quantization useful?
• Given quantum state $|\Psi\rangle$ describing n fermions with Slater-determinant wave-function

$$\langle r_1, \ldots, r_n|\Psi\rangle = \psi(r_1, \ldots, r_n) = \mathcal{A}[\psi_1(r_1) \ldots \psi_n(r_n)],$$

where $\mathcal{A}$ denotes antisymmetrization. Show how $|\Psi\rangle$ can be represented in terms of creation operators acting on a vacuum state $|\mathrm{Vac}\rangle$.
• Given a many-body system with n particles. Write down the following operators in first- and second-quantized language: density operator, kinetic energy, electrostatic energy in a potential $V(r)$, interaction energy.
• Given a set of bosonic or fermionic creation and annihilation operators $a_i^\dagger$ and a_i, you should know how to evaluate expectation values of the kind

$$\sum_{ij} B_{ij}\langle 0|a_i^\dagger a_j|0\rangle, \quad \sum_{ijkl} B_{ijkl}\langle 0|a_i^\dagger a_j a_k^\dagger a_l|0\rangle$$

Superconductivity
• Write down reduced BCS-Hamiltonian. Explain and interpret each term.

• Show how reduced BCS-Hamiltonian can be represented in terms of hard-core bosons.
• Write down BCS-Ansatz for ground state wave-function. Interpret the meaning of the coefficients u_k and v_k; explain how they can be determined.
• How does the gap equation arise? Given the gap equation and form of coefficients u_k and v_k (no need to memorize this), solve it to find Δ as function of interaction constant.

Spin-Boson model
• Write down the model. Explain each term. Give examples of systems for which it can be applied.
• What is meaning of spectral function $J(\omega)$? What is meant by "ohmic", "subohmic", "superohmic" spin-boson models?
• Explain in words what is meant by "adiabatic renormalization" of tunnel rates, and why it is needed.

Superconducting Qubits
• Describe a physical example of a superconducting qubit (scetch & describe in words how it works).
• How are the logical states $|0\rangle$ and $|1\rangle$ encoded?
• With which external controls can this qubit be manipulated?
• How can coherent oscillations between $|0\rangle$ and $|1\rangle$ be detected?

Noisy Damped Harmonic Oscillator
• Why do experimentalists like to give measurement results in units of "something *per root Hertz*"?
• What is meant by the "fluctuation spectrum" of a noise quantity?
• Define the "spectral density" $S_{xx}(\omega)$. How can the the mean squared value $\langle x^2 \rangle$ be extracted from the spectral density?
• Write down a Langevin equation for describing a damped harmonic oscillator under the influence of *classical* noise. Interpret the noise term. Explain a strategy for fixing its parameters in order to describe thermal equilibrium at a given temperature T? (You should be able to carry out this strategy, when provided with some hints; no need to memorize results, though.)
• Scetch qualitatively the spectral function $S_{xx}(\omega)$ produced by classical noise. How does it depend on the strength of damping, and on temperature?
• Write down Caldeira-Leggett-model for describing damped harmonic oscillator with *quantum* noise. Interpret each term.
• What is main complication relative to case of classical noise?
• Scetch qualitatively the spectral function $S_{xx}(\omega)$ produced by quantum noise? How does it differ from case of classical noise?

• In which limit are the results of classical noise recovered?

Bose-Einstein Condensation

• Describe in words what is meant by BEC, and why it happens. Illustrate your arguments using a scetch of the Bose occupation function, and explain how the chemical potential changes with temperature, and how that affects the occupancy of states of various energies.
• How is the critical temperature for BEC related to density?
• Write down the ground state wave function for a BEC in first- and second-quantized form.
• What is meant by the "order parameter" of a BEC? How is it related to the ground state wave function?
• What is the superfluid velocity of a BEC? Derive a quantization condition for superfluid flow around a closed loop.
• What is meant by a "time-of-flight measurement" of a cloud of cold atoms? What is the quantity measured by the detector? How can this be related to a quantum-mechanical wave-function describing the initial atomic system?
• Write down the Gross-Pitaevsky equation. Under what conditions is it needed? Derive it using a Hartree-Fock Ansatz for the wave function.
• Why is it possible to observe an interference pattern when allowing two completely independent condensates (with uncorrelated phases) to overlap? Why are the interference fringes not washed out? How can one show experimentally that the relative phase between the two condensates is a random quantity?

Optical lattices

• What is an optical lattice? How is it created?
• Write down the Bose-Hubbard model for bosons in an optical lattice. Interpret all terms. Describe how they can be tuned experimentally.
• What types of ground states are observed in the two extreme cases of a very shallow or a very deep lattice? Characterize these states in words, and give a simple Ansatz for each (in second-quantized language).
• Describe in words & with scetches how the crossover (as a function of adiabatically increasing lattice depth) between these states can be observed experimentally.
• Describe what happens if the lattice depth is increased suddenly (collapse and revival). Explain with formulas why a "revival" of the wave-function occurs. Derive the revival time.

Numerical Many-Body Methods

• Consider a 1-dimensional quantum chain. Show with formulas how a matrix product state arises if the chain is diagonalized iteratively by adding one site at a time.

Bell's inequality

• Describe in words & with a scetch the Einstein-Rosen-Podolsky Gedankenexperiment.
• What are EPR's criteria for "a complete theory" and for "physical reality". Explain why, according to EPR, quantum mechanics can not be complete.
• Write down a "Bell inequality", defining the correlators $E(\hat{l}, \hat{n})$ that it contains. Show how these correlators are calculated using hidden variables, or using quantum mechanics.
• Explain how/why Bell inequalities allow experimental tests of hidden variable theories.

Single-Spin Dectection and Manipulation

• Explain in words & with scetches how a single-spin detection can be achieved in a transport experiment using pulsed gates.
• What is a quantum quench?
• Explain in words & with scetch how a quantum quench can be implemented by photon absorption and exciton creation in a quantum dot.

Raman effect

• Explain in words what the Raman effect is. Why can it be used to gain information about molecular excitations?
• Give a classical explanation for the frequency shift.
• Write down and derive Fermi's golden rule to *second* order in perturbation theory.
• Starting from minimal coupling of light and matter, use a gauge transformation to show that the Raman effect is governed by *dipole* matrix elements.
• Explain with some formulas how a quantum-mechanical treatment of radiation, suitable for describing the Raman effect, can be achieved.