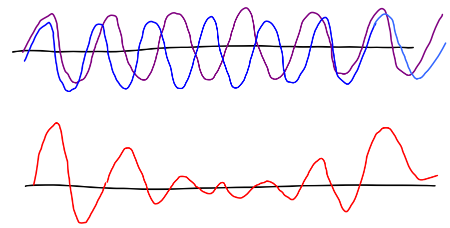
qm 01 **Solid State Interferometers** 20.10.09 SSI 1

General remarks

- Wave phenomena $\Rightarrow$ interference

$$\psi_1(x) + \psi_2(x) = \psi(x)$$



- QM matter waves (e.g. electrons/neutrons) $\Rightarrow$ also interference!

Consequences in solid state:

- band structure of crystals $\rightarrow$ metals, insulators, semiconductors
- localization in disordered metals ("Anderson localization")
- interferometers $\Rightarrow$ accurate probes of phase coherence!

Interference of Matter Waves with different potentials

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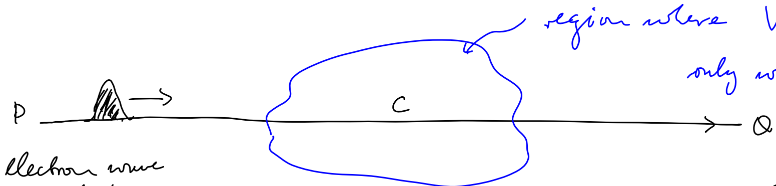
following Aharonov & Bohm, Phys. Rev., 115, 485 (1959)

region where $V(t) = \underset{\text{charge}}{q}\,\phi(t) \neq 0$, only when wave packet is near C.



$$\frac{\vec{p}^{\,2}}{2m} = -\frac{\hbar^2 \vec{\nabla}^2}{2m}$$

Solution of SE for $V = 0$: $\quad i\hbar\, \partial_t \psi_0 = H_0 \psi_0$ (1)

no need to consider $\vec{r}$-dependence

$V \neq 0$: $\quad i\hbar\, \partial_t \psi = \left(H_0 + V(t)\right)\psi$ (2)

Ansatz: $\quad \psi(t) = \psi_0(t)\, e^{iS(t)/\hbar}$ (3)

$\uparrow$ assumed $\vec{r}$-independent.

(1.3) into (1.2): $\quad i\hbar\partial_t\psi = \underbrace{i\hbar[\partial_t \psi_0]}_{(1.1)\; H_0\psi_0} e^{iS/\hbar} + [\partial_t S(t)]\psi_0 e^{iS/\hbar}$ (4)

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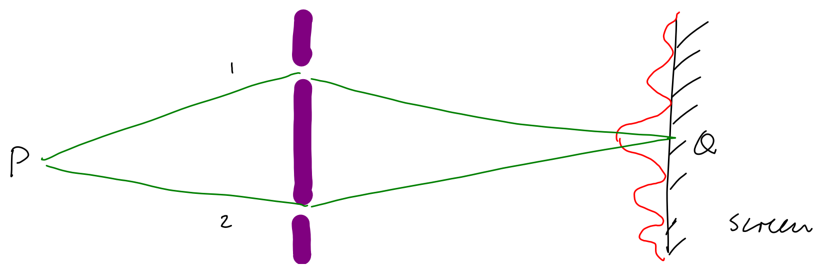
$$= \left[H_0 - \partial_t S(t) \right] \psi \quad (1)$$

(2.4) = 2.3) $\Longrightarrow$ $\partial_t S(t) = -V(t)$ (2)

$$\boxed{S(t) = -\int_{t_0}^{t} dt' \, V(t')} \quad \left[= V(t - t_0) \text{ if } V = \text{const.} \right] \quad (3)$$

phase ~ accumulated potential

Now, consider two possible paths.



SSI3

Total amplitude to get from P to Q:

$$\psi_0^{P\to Q}(t) = \psi_0^{(1)} + \psi_0^{(2)} \quad \text{(sum over possibilities)} \quad (1)$$

↑ argument t will not be displayed below

with $\quad i\partial_t \psi_0^{(a)} = H_0 \psi_0^{(a)}(t) \qquad a = 1,2 \quad (2)$

write $\quad \psi_0^{(a)} = |\psi_0^{(a)}| e^{i\alpha_0^{(a)}} \quad (3)$

↑ amplitude ↑ phase

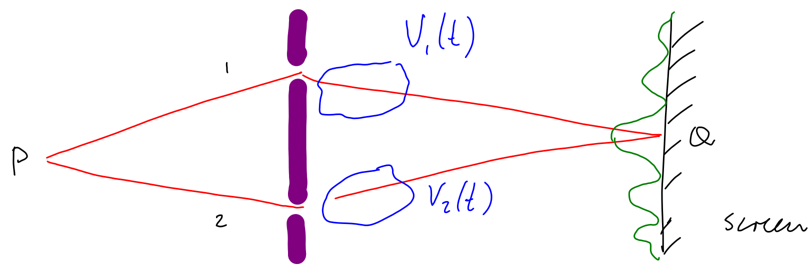
Probability to get from A → B:

$$|\psi_0^{P\to Q}|^2 = |\psi_0^{(1)}|^2 + |\psi_0^{(2)}|^2 + 2|\psi_0^{(1)}||\psi_0^{(2)}| \cos(\underbrace{\alpha_0^{(1)} - \alpha_0^{(2)}}_{\text{changes as we move along screen.}}) \quad (4)$$

Amplitude depends on phase difference between two paths!

That is why we see interference pattern in double-slit experiment.

Switch on potentials, with $V_1 \neq V_2$:



By same reasoning as above:

$$\psi^{P\to Q} = \psi^{(1)} + \psi^{(2)} , \qquad (1)$$

$$\psi^{(a)} = \psi_0^{(a)} e^{i S_{(a)}/\hbar} , \quad S^{(a)}(t) = -\int_{t_0}^{t} dt' \, V_a(t) \qquad (2)$$

$$\Rightarrow \quad = |\psi_0^{(a)}| e^{i\alpha^{(a)}} , \quad \boxed{\alpha^{(a)} = \alpha_0^{(a)} + S_{(a)}/\hbar} \qquad (3)$$

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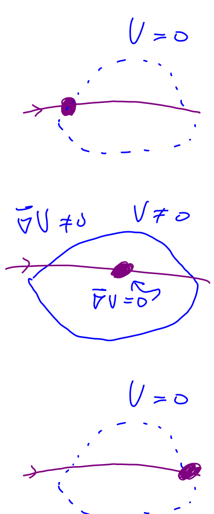
Effect on interference pattern: (4.3) into (3.4):

$$\Rightarrow \cos\left(\alpha_0^{(1)} - \alpha_0^{(2)} + \underbrace{[S_{(1)} - S_{(2)}]}_{\delta S = -\int dt' [V_1(t') - V_2(t')]} / \hbar \right) \qquad (4)$$

If $S_1 \neq S_2$, interference pattern shifts!

Note: $V_1(t)$, $V_2(t)$ can be switched on/off in such a way that electron never feels an <u>electric</u> field, by arranging for

- $\vec{\nabla} V \neq 0$ only at the edges of region where $V \neq 0$, and
- switching on V only when electron is deep inside this region, far from the edges.



So, here electromagnetic potential $\phi(t)$ has physical consequences, <u>independent</u> of those that occur via $E = \vec{\nabla}\phi$.

Analogous argument for magnetic field: Aharonov-Bohm effect

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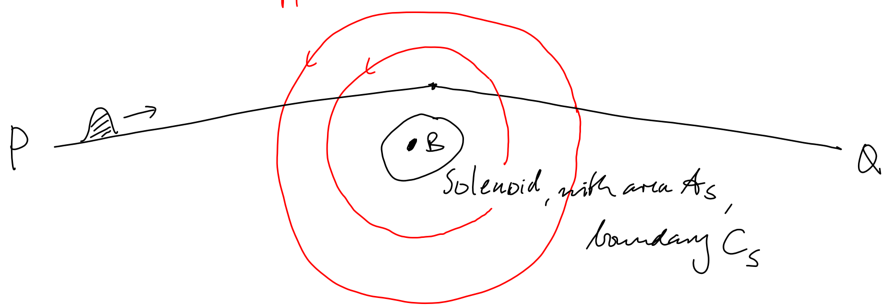
$\vec{B} \neq 0$ described by vector potential, $\qquad \vec{B} = \vec{\nabla} \times \vec{A} \qquad (1)$

$\vec{A}$



Consider solenoid with flux

$$\Phi_0 = \int_{\text{Area } A_s} d\vec{a}\cdot\vec{B} \overset{(1)}{=} \int_{A_s} d\vec{x}\cdot(\vec{\nabla}\times\vec{A}) \overset{\text{Stokes}}{=} \oint_{C_s} d\vec{x}\cdot\vec{A} \qquad (2)$$

Argument holds for
larger Area A', with boundary C':

$$\boxed{\Phi_0 = \oint_{C'} d\vec{x}\cdot\vec{A}} \qquad (3)$$

$\Rightarrow \quad \vec{A} \neq 0$ outside of solenoid!

Reminder: charged particle in electromagnetic field:

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Lagrange: $\qquad L = \frac{1}{2} m \dot{\vec{r}}^{\,2} - q\,\phi(\vec{r}) + q/c\, \vec{A}\cdot\dot{\vec{r}} \qquad (1)$

Canonical momentum:

$$\vec{p} = \frac{\partial L}{\partial \dot{\vec{r}}} = m\dot{\vec{r}} + q/c\,\vec{A} \quad \Rightarrow \quad \dot{\vec{r}} = (\vec{p} - q/c\,\vec{A})/m \qquad (2)$$

Hamiltonian: $\qquad H = \vec{p}\cdot\dot{\vec{r}} - L \qquad (3)$

$$= \vec{p}\cdot(\vec{p} - q/c\,\vec{A})/m - \frac{m}{2}(\vec{p} - q/c\,\vec{A})^2/m^2 + q\,\phi(\vec{r}) - q/c\,\vec{A}\cdot(\vec{p} - q/c\,\vec{A})/m$$

$$\boxed{H = \frac{(\vec{p} - q/c\,\vec{A})^2}{2m} + q\,\phi(\vec{r})} \qquad (4)$$

(5)

"minimal coupling

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Solution of SE for $\vec{A} = 0$: $\quad i\hbar\,\partial_t \psi_0 = \frac{-\hbar^2 \vec{\nabla}^2}{2m}\psi_0$ (1)

$\vec{A} \neq 0$: $\quad i\hbar\,\partial_t \psi = \frac{1}{2m}\left[(-i\hbar\vec{\nabla}) - e/c\,\vec{A}\right]^2 \psi$ (2)

Ansatz: $\quad \psi = \psi_0\, e^{iS(\vec{r})/\hbar}$ (3)

assume t-independent

so $\partial_t \psi = (\partial_t \psi_0)\, e^{iS/\hbar}$

(3) into (2):

$$(-i\hbar\vec{\nabla} - e/c\,\vec{A})\psi = \left([-i\hbar\vec{\nabla}\psi_0] + \underbrace{[\vec{\nabla}S - e/c\,\vec{A}]}_{\equiv 0}\psi_0\right)e^{iS/\hbar} \qquad (4)$$

$$(-i\hbar\vec{\nabla} - e/c\,\vec{A})^2\psi = \left([-\hbar^2\vec{\nabla}^2\psi_0] + (\vec{\nabla}S - e/c\,\vec{A})\psi_0\right)e^{iS/\hbar} = -\hbar^2\vec{\nabla}^2\psi \qquad (5)$$

$\Longrightarrow$ (3) satisfies (2) provided that $\vec{\nabla}S = e/c\,\vec{A}$ (6)

or

$$\boxed{S(\vec{r}) = e/c\int_{\vec{r}_0}^{\vec{r}} d\vec{r}\,'\cdot\vec{A}(\vec{r}\,')} \qquad (7)$$

line integral along trajectory

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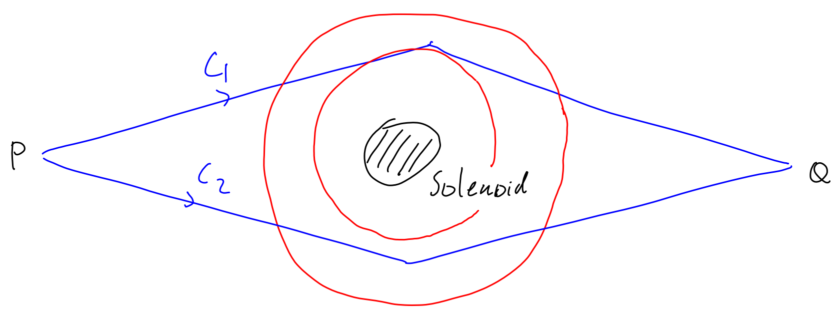
Now consider two paths:



Total amplitude to get from P to Q:

$$\psi^{P\to Q} \overset{(5.1,\,5.2)}{=} \psi_0^{(1)}e^{iS_{(1)}} + \psi_0^{(2)}e^{iS_{(2)}} \qquad \text{(sum over possibilities)} \qquad (1)$$

$$\text{Interference term} \overset{(6.1)}{=} \cos\left(\underbrace{\alpha_0^{(1)} - \alpha_0^{(2)}}_{\equiv\varphi} + [S_{(1)} - S_{(2)}]/\hbar\right) \qquad (2)$$

$\boxed{\text{SSI 11}}$

$$(S_{(1)} - S_{(2)})/\hbar = \frac{e}{c\hbar}\left[\int_{C_1} d\vec{r}\cdot\vec{A}(\vec{r}) \;-\; \int_{C_2} d\vec{r}\cdot\vec{A}(\vec{r})\right] \qquad (1)$$

since $C_1 - C_2$ form a closed path:

$$= e/c\hbar \oint_{C_1 - C_2 = C} d\vec{r}\cdot\vec{A}(\vec{r}) \qquad (2)$$

Stokes

$$= e/c\hbar \int d\vec{a}\cdot\underbrace{(\vec{\nabla}\times\vec{A})}_{\vec{B}} = \frac{e}{\hbar c}\,\Phi \qquad (3)$$

$$= 2\pi\,\Phi/\Phi_0\,, \qquad \text{where } \Phi_0 = \frac{hc}{e} = \text{flux quantum} \qquad (4)$$

$$= 4.1358\times 10^{-15}\ \mathrm{T.m^2}$$

$\Rightarrow$ $|\psi^{PQ}(B)|^2 = |\psi^{PQ}(0)|^2 + \text{const.}\cos(\varphi + 2\pi\,\Phi/\Phi_0)$

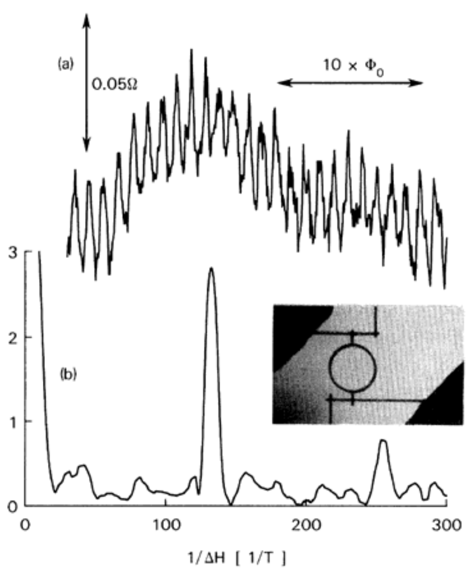
Aharonov-Bohm oscillations: depend on flux enclosed by path!

$\boxed{\text{SSI 12}}$

Observation of h/e Aharonov-Bohm Oscillations in Normal-Metal Rings

R. A. Webb, S. Washburn, C. P. Umbach, and R. B. Laibowitz   Phys. Rev. Lett., 54, 2696 (1985)

[see also: Physics Today, 39, 17 (1986)]



Gold rings: thickness: 38 nm

inner diameter: $D \simeq 784$ nm

width: 41 nm

resistivity: $\rho = 5\ \mu\Omega.\mathrm{cm}$

temperature: $T = 0.01$ K

(a) $G(B)$ shows oscillations!!

(b) Fourier transform: $\int dB\, e^{i2\pi B\omega}\, G(B)$

Peaks at $\omega = 1/B_0,\ 2/B_0$, with $B_0 = \frac{\Phi_0}{\text{Area}}$

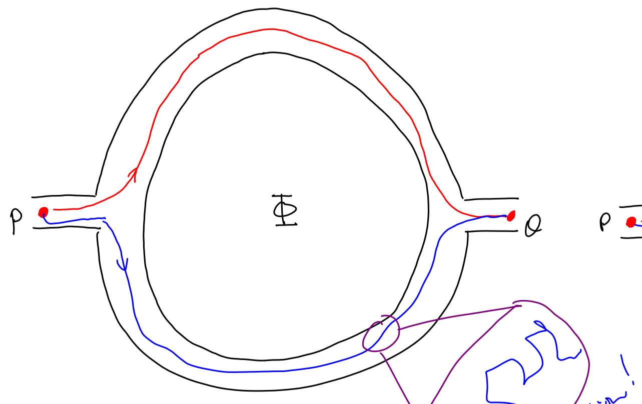
Observation of Aharonov-Bohm oscillations in normal metals with disorder:

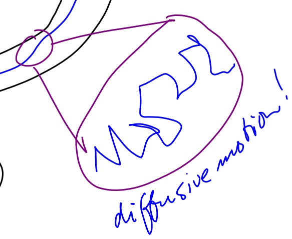
$\Rightarrow$ phase coherence, $L_\varphi \gtrsim D$ $\Rightarrow$ only elastic scattering

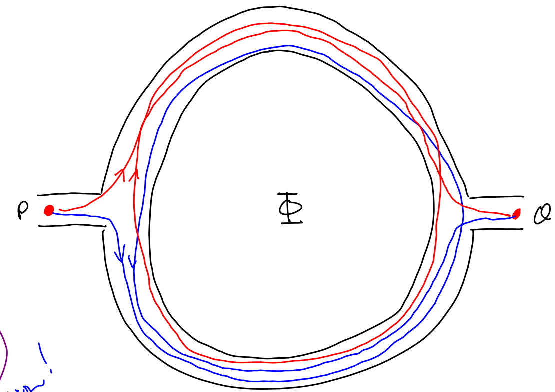
Electrons diffuse phase-coherently (bounce elastically off impurities) $\boxed{\text{§SI 13}}$

Conditions: $L_{in} > L_{Ring}$ (but we may have $L_{elastic} \ll L_{Ring}$!!)

where L_{in} = typical distance travelled (diffusively) between inelastic collisions, which cause changes in phase ("dephasing")







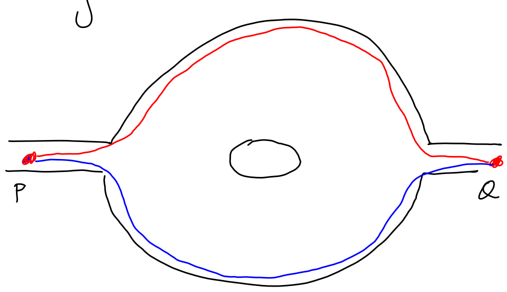
$\cos(2\pi B \cdot A/\Phi_0)$ $\qquad$ $\cos(2\pi B\, 2A/\Phi_0)$

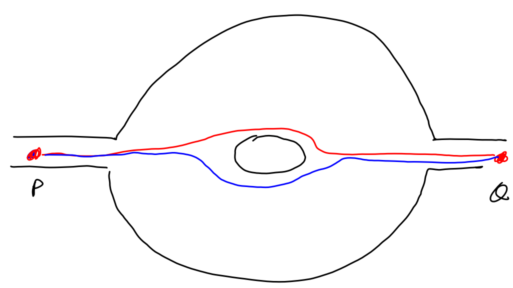
Period: $\Phi = \Phi_0$ $\qquad$ Period: $\Phi = \Phi_0/2$

$\boxed{\text{IIS14}}$

What does not work:

thick ring:





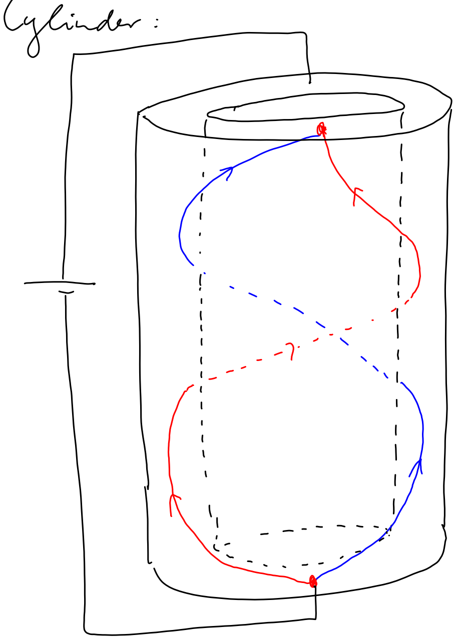
Paths near outer edge: Φ_{max} $\qquad$ Paths near inner edge: Φ_{min}

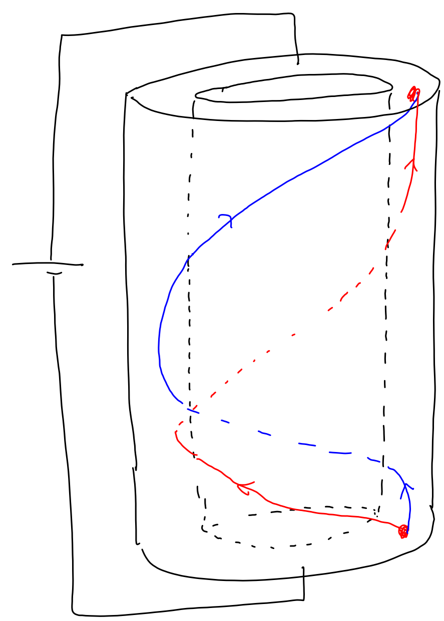
Coherent sum of paths with different fluxes:

$$\int_{\Phi_{min}}^{\Phi_{max}} d\Phi \, \cos(\varphi + 2\pi \Phi/\Phi_0) \simeq 0$$

if $\Phi_{max} - \Phi_{min} > \Phi_0$

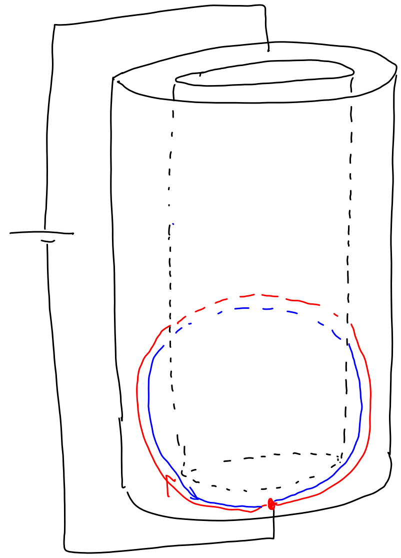
Cylinder: $\boxed{\text{II S 15}}$

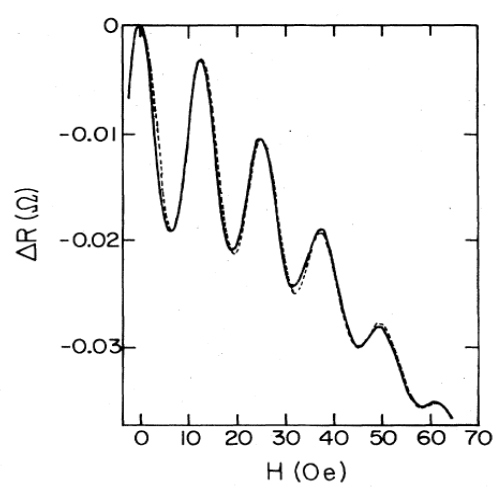




· different starting points contribute with different phases,
randomizing signals $\quad \int d\varphi_0 \cos(\varphi_0 + 2\pi \Phi/\Phi_0) \simeq 0.$

Cylinders do show oscillations: due to coherent backscattering of time-reversed paths ! $\boxed{\text{II S 16}}$





weak localization + cylinder:
Altshuler-Aronov-Spivak oscillations
JETP Lett. 33, 94 (1981)
(theory)

Sharvin & Sharvin, JETP Lett. 34, 272 (1981) }
Sharvin, Physica B 126, 288 (1984) } exp.