Solid State Interferometers (cont.)

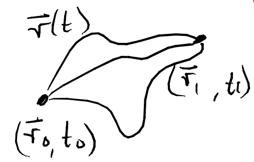
21.10.2010 $\boxed{SSI17}$

"Feynman path integral": $\psi(\vec{r}_1, t_1; \vec{r}_0, t_0) = \sum_\alpha e^{\frac{i}{\hbar} S[\vec{r}_\alpha]} \simeq \sum_\alpha^c A_\alpha e^{\frac{i}{\hbar} S[\vec{r}_\alpha]}$ (1)

↑ from fluctuations around class. path

$\sum_\alpha \equiv$ sum over all paths $\vec{r}_\alpha(t)$ with $\begin{cases} \vec{r}(t_0) = \vec{r}_0 \\ \vec{r}(t_1) = \vec{r}_1 \end{cases}$

$\sum_\alpha^c$ " " classical paths " " " " "



$$S[\vec{r}_\alpha] = \int_{t_0}^{t_1} dt\, L[\vec{r}_\alpha, \dot{\vec{r}}_\alpha] = \text{action along } \vec{r}_\alpha \qquad (2)$$

Classical limit: sum over many paths cancel due to fluctuating phases — except near a stationary point where

$$\left.\frac{\delta S}{\delta \vec{r}}\right|_{\vec{r} = \vec{r}_c} = 0 \quad \Rightarrow \quad \text{Hamilton's principle of least action for identifying classical paths.} \qquad (3)$$

so that neighboring paths have essentially the same phase.

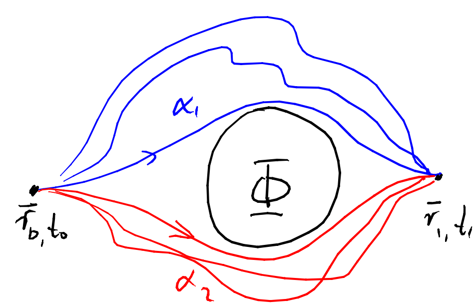
$\boxed{SSI18}$

Charged particle in magnetic field:

$$L = \underbrace{\tfrac{1}{2} m \dot{\vec{r}}^{\,2} - V(\vec{r}, t)}_{L^{(0)}} + \tfrac{q}{c} \vec{A} \cdot \dot{\vec{r}}$$

$$S[\vec{r}] = \int dt \left[L^{(0)} + q/c\, \vec{A} \cdot \frac{d\vec{r}}{dt} \right.$$

$$= S^{(0)} + q/c \int d\vec{A} \cdot d\vec{r}$$



For AB-effect: sum over paths splits into two parts:

$$\psi(\vec{r}_1, t_1; \vec{r}_0, t_0) = \underbrace{\sum_{\alpha_1} e^{i S_{\alpha_1}/\hbar}}_{\equiv \psi_1} + \underbrace{\sum_{\alpha_2} e^{i S_{\alpha_2}/\hbar}}_{\equiv \psi_2}$$

$\boxed{\text{SSI 19}}$

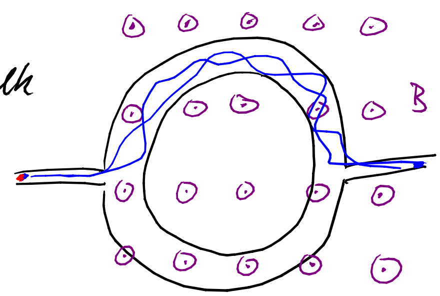
Probability: $P(\vec r_1, t_1; \vec r_0, t_0) = |\psi|^2$ (1)

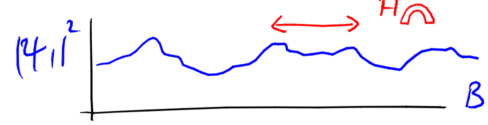
$= |\psi_1|^2 + |\psi_2|^2 + (\psi_1 \psi_2^* + c.c.)$ (2)

$|\psi_1|^2 = \left| \sum_\alpha e^{iS^{(0)}_{\alpha_1}/\hbar} \, e^{iq/c\hbar \int_{\alpha_1} d\vec r \cdot \vec A} \right|^2$ (similarly for ψ_2) (3)

Suppose there are many classical random walk paths, that add coherently. Each random walk α_1 has a random phase, $q/c \int_{\alpha_1} d\vec r \cdot \vec A$, that depends on B. So, $|\psi_1|^2$ is a random function of B: "universal conductance fluctuations".

for wide ring





$\boxed{\text{SSI 20}}$

When do AB-oscillations survive? (see Problem 1.6!)

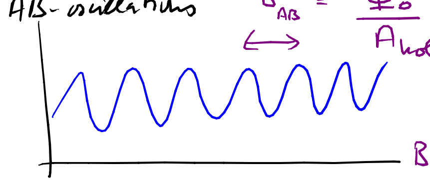
Interference term:
$$= \sum^c_{\alpha_1} \sum^c_{\alpha_2} \left[A_{\alpha_1} A_{\alpha_2} e^{i(S_{\alpha_1} - S_{\alpha_2})/\hbar} + c.c. \right]$$

$$= \sum_{\alpha_1} \sum_{\alpha_2} \left[A_{\alpha_1} A_{\alpha_2} e^{iS^{(0)}_{\alpha_1}/\hbar} e^{-iS^{(0)}_{\alpha_2}/\hbar} e^{\frac{iq}{c\hbar} \overbrace{\oint_{\underline{\alpha_1 - \alpha_2}} d\vec r \cdot \vec A}^{\Phi[\alpha_1 - \alpha_2]}} + c.c. \right]$$

closed loop formed by $\vec r_{\alpha_1}$ and $\vec r_{\alpha_2}$

$$= \sum_{\alpha_1} \sum_{\alpha_2} A_{\alpha_1} A_{\alpha_2} \cos\left(\varphi^{(0)}_{\alpha_1} - \varphi^{(0)}_{\alpha_2} + 2\pi \frac{\Phi(\alpha_1 - \alpha_2)}{\Phi_0} \right)$$

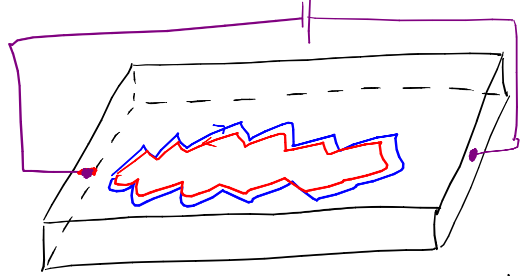
$\Phi(\alpha_1 - \alpha_2) \simeq B A_{hole}$



SSI21

<u>Weak localization:</u> enhanced resistivity in disordered metals due to coherent backscattering along time-reversed paths!

Return probability of electron diffusing through wire:



$$P_{\text{return}} \overset{(17.1)}{=} \Big| \sum_\alpha^{c} A_\alpha e^{iS_\alpha} \Big|^2 \quad (1)$$

all classical closed

$$= \sum_\alpha^{c} A_\alpha e^{iS_\alpha} \sum_{\alpha'} A_{\alpha'} e^{-iS_{\alpha'}} \quad (2)$$

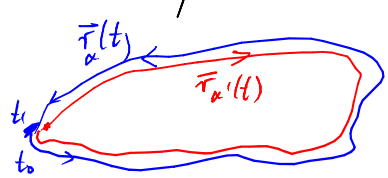
$$= \sum_\alpha^{c} A_\alpha^2 + \sum_{\alpha\neq\alpha'}^{c} A_\alpha A_{\alpha'} e^{i(S_\alpha - S_{\alpha'})} \quad (3)$$

$$= P_{\text{classical}} + P_{\text{quantum}} \quad (4)$$

SSI22

$$P_{\text{quantum}}(B=0) = \sum_{\alpha\neq\alpha'} A_\alpha A_{\alpha'} e^{i(S_\alpha^{(0)} - S_{\alpha'}^{(0)})} \quad (1)$$

Sum over random phases cancels, except for time-reversed paths:



If, for $t_0 \le t \le t_1$, we have

$$\vec r_{\alpha'}(t) = \vec r_\alpha(t_1 + t_0 - t) \quad (2)$$

[so that $\vec r_{\alpha'}(t_0) = \vec r_\alpha(t_1)$ and $\vec r_{\alpha'}(t_1) = \vec r_\alpha(t_0)$],

then

$$S_{\alpha'}^{(0)} = \int_{t_0}^{t_1} dt \left[\tfrac{1}{2} m \left(\frac{d\vec r_{\alpha'}(t)}{dt} \right) - V(\vec r_{\alpha'}(t), t) \right] \quad (3)$$

if $V(\vec r, t) = V(\vec r)$

$t_1 + t_0 - t = \bar t$

$$= \int_{t_0}^{t_1} dt \left[\tfrac{1}{2} m \left(\frac{d\vec r_\alpha(t_1+t_0-t)}{dt} \right)^2 - V(\vec r_\alpha(t_1+t_0-t), t) \right] \quad (4)$$

$\int_{t_0}^{t_1} dt = -\int_{t_1}^{t_0} d\bar t$

$$= \int_{t_0}^{t_1} d\bar t \left[\tfrac{1}{2} m \left(\frac{d\vec r_\alpha(\bar t)}{d\bar t} \right)^2 - V(\vec r_\alpha(t), t_1+t_0-\bar t) \right] = S_\alpha^{(0)} \quad (5)$$

So, if α = time-reversed (α')

then $S^{(0)}_{\alpha} = S^{(0)}_{\alpha'}$

and $A_\alpha = A_{\alpha'}$ (can be shown)

$$\Rightarrow P_{\text{quantum}} \overset{(22.1)}{=} \sum_\alpha^c A_\alpha^2 \overset{(21.4)}{=} P_{\text{classical}} \; !$$

So, probability for backscattering is enhanced by factor 2 relative to classical case!

$\Rightarrow$ quantum correction to resistivity!

called "weak localization"

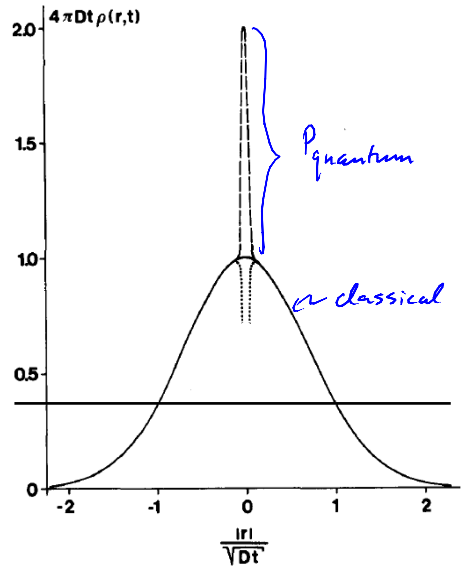


Fig. 2.6. The probability distribution of a diffusing electron which starts at $r = 0$ at the time $t = 0$. In quantum diffusion (dashed peak) the probability to return to the origin is twice as great as in classical diffusion (full curve). Large spin–orbit scattering reduces the probability by a factor of two (dotted peak) and yields a weak anti-localization.

from Bergmann, Phys. Rep. 107, 1 (1984)

To detect weak localization, measure magnetoconductance $(B \neq 0)$

From (21.3), we now have

$$P_{\text{quantum}}(B \neq 0) = \sum_{\alpha \neq \alpha'}^c A_\alpha A_{\alpha'} \, e^{i(S_\alpha - S_{\alpha'})}$$

consider only $\alpha' = \text{time-reversed}(\alpha) \equiv \bar{\alpha}$

$$= \sum_\alpha^c A_\alpha A_{\bar{\alpha}} \, e^{i(S^{(0)}_\alpha - S^{(0)}_{\bar{\alpha}})} \, \underbrace{e^{i\frac{q}{c\hbar}\left[\oint_\alpha d\bar{r}\cdot\vec{A} - \oint_{\bar{\alpha}} d\bar{r}\cdot\vec{A}\right]}}$$

since time-reversed paths enclose "opposite fluxes", their phase difference adds!

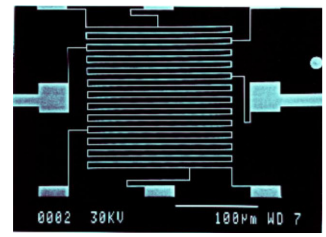
$$e^{i2\pi\left(\frac{2\Phi_\alpha}{\Phi_0}\right)} = e^{2i\, q/c\hbar \oint_\alpha d\bar{r}\cdot\vec{A}}$$

$$= \sum_\alpha^c A_\alpha^2 \, e^{i2\pi(2\Phi_\alpha/\Phi_0)} \simeq 0 \text{ if } (\Phi_\alpha)_{\text{typical}} \gtrsim \Phi_0$$

B-field produces random phases, which kills $P_{\text{quantum}} \to 0$.

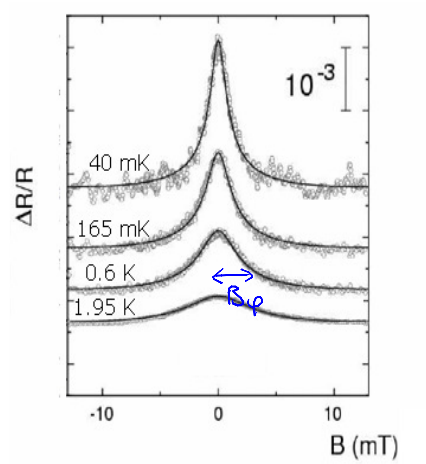
$\boxed{SSI\ 25}$

$(\Phi_\alpha)_{\text{typical}}$ = flux of path of duration $(t_1 - t_0) \simeq \tau_\varphi$ = "dephasing time"



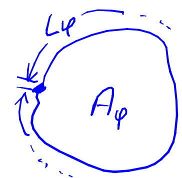
Distance covered diffusively in time τ_φ :

$$L_\varphi = \sqrt{D \tau_\varphi} \qquad D = \text{diffusion constant}$$



resistance of disordered gold wire

Area covered by loop of length L_φ : $A_\varphi \sim L_\varphi^2$



B-field that produces flux $\Phi_\alpha \simeq \Phi_0$ for loop of area A_φ :

$$B_\varphi = \frac{\Phi_0}{A_\varphi} \sim \frac{\Phi_0}{L_\varphi^2} = \frac{\Phi_0}{D\tau_\varphi}$$

$\boxed{SSI\ 26}$

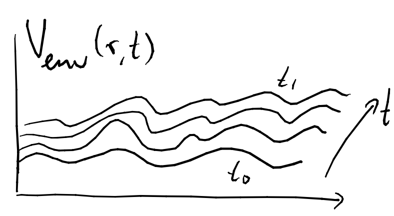
The field scale at which weak localization disappears can be used to study the dephasing time!

Effect of temperature:

"dephasing" = loss of phase coherence due to environment, which produces random phase changes : $\psi_\alpha \to \psi_\alpha\, e^{\,i\int_{t_0}^{t_1} dt\, V_{env}(\vec{r}_\alpha(t), t)}$

Experiment indicates: with decreasing T,

$B_\varphi \sim \dfrac{\Phi_0}{D\tau_\varphi}$ decreases, so



τ_φ increases! (environment fluctuates less strongly, produces less dephasing)

SSI 27

Rough phenomenological estimate:

$$\tau_\varphi \sim \frac{\hbar}{k_B T} \qquad \text{(use in Problem 1.5)}$$

Detailed form of temperature dependence is topic of current research:

For example, for quasi-1D wire of length L:

$$\tau_\varphi^{-1} \sim \begin{cases} T^{2/3} & \text{for } \frac{1}{k_B T} \ll \frac{\tau_\varphi}{\hbar} \ll \frac{L^2}{D} \\ T & \text{for } \frac{1}{k_B T} \ll \frac{L^2}{D} \ll \frac{\tau_\varphi}{\hbar} \\ T^2 & \text{for } \frac{L^2}{D} \ll \frac{1}{k_B T} \ll \frac{\tau_\varphi}{\hbar} \end{cases}$$

observed 1980's

observed 2008

predicted [Diploma Thesis, Max Treiber, 2008]

SSI 28

General challenge in transport in disordered metals:

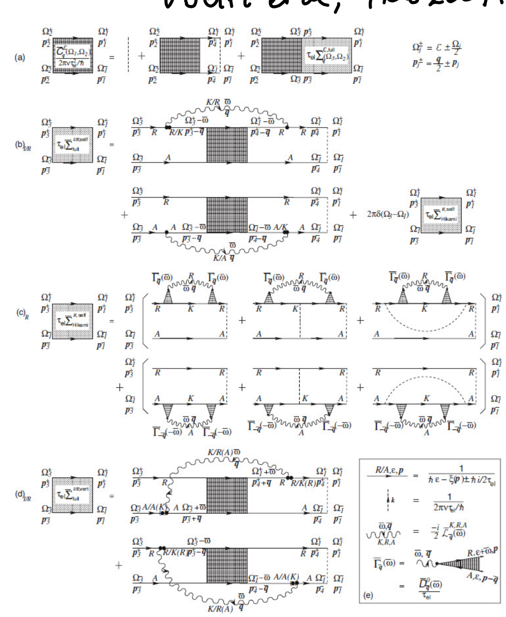
Understand interplay between

- disorder → diffusion, localization
- coherence → quantum effects
- interactions → produce dephasing

Theoretical methods:

- Path integrals
- Diagrammatics
- Field theory
- Random matrix theory

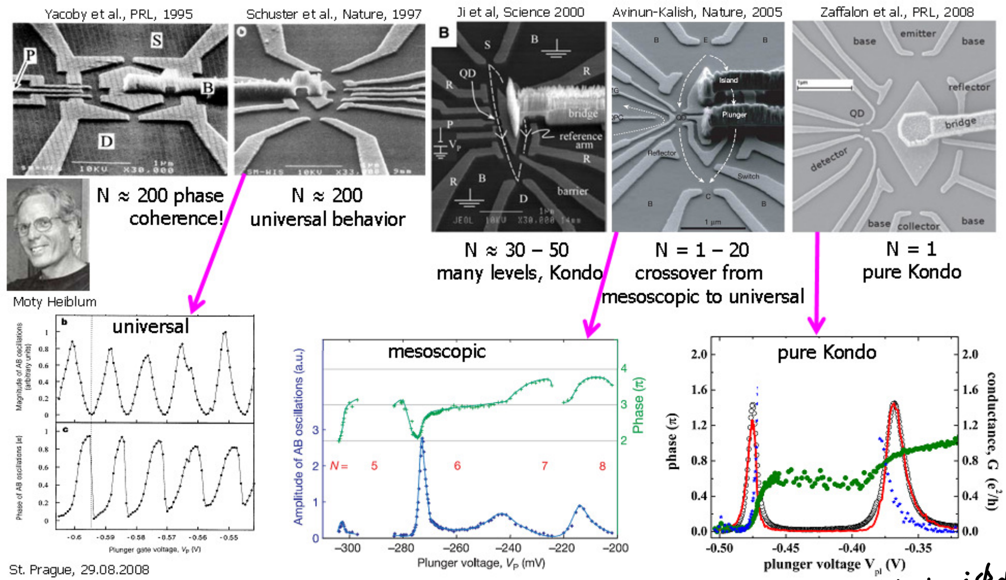
vDelft et al, PRB 2007.



Phase-Coherent Transport through Quantum Dots

$\boxed{SSI\ 29}$

Jan von Delft



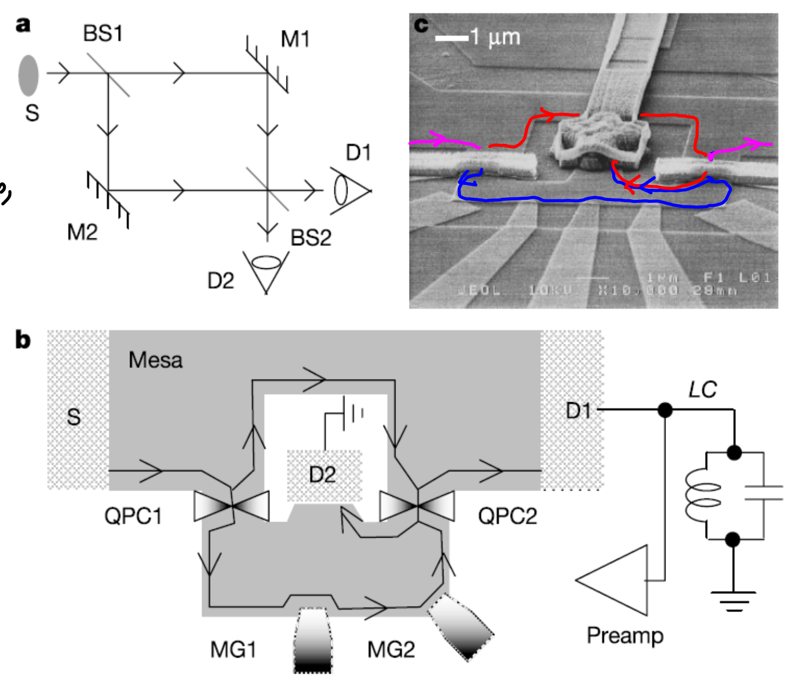
St. Prague, 29.08.2008

Use AB-ring to measure the transmission amplitude $t_d = |t_d| e^{i\phi_d}$ through QD

[Theory: PhD thesis, Theresa Hecht, 2008]

Mach-Zehnder interferometer, Heiblum group, Nature 2003, PRL 2006 $\boxed{SSI\ 30}$

Allows to study influence of interactions, finite voltage on decoherence.



Theory: Marquardt group

SSI31

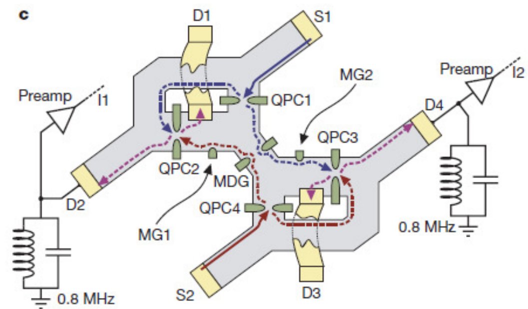
Interference between two indistinguishable electrons from independent sources

I. Neder[1], N. Ofek[1], Y. Chung[2], M. Heiblum[1], D. Mahalu[1] & V. Umansky[1]

Nature, 2007

Interference is due to indistinguishability of whether electrons from sources S_1 and S_2 end up in detectors D_1 and D_2, or vice versa!

c



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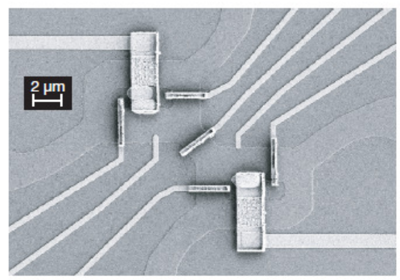
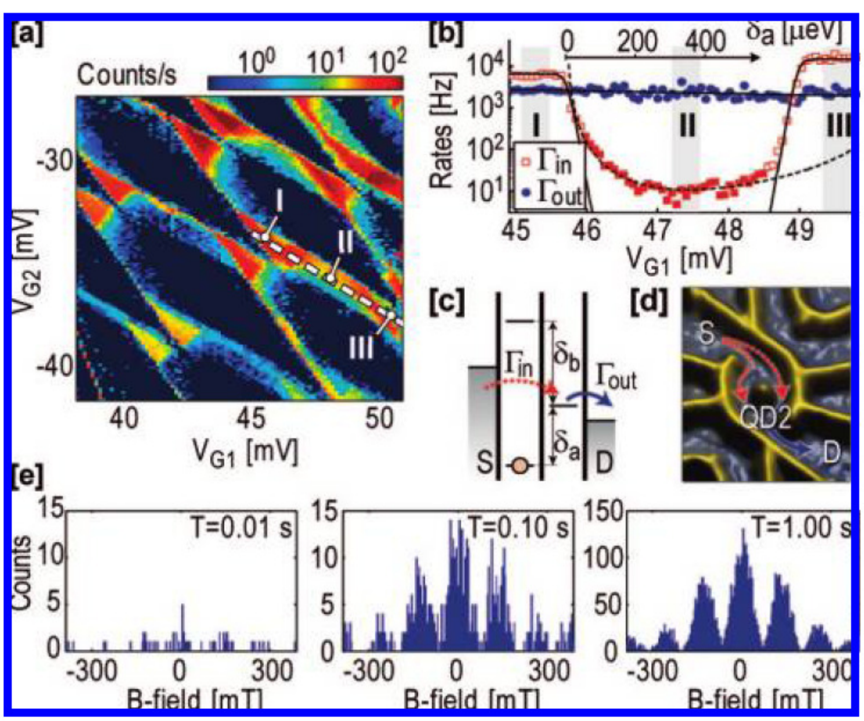


SSI16

Time-Resolved Detection of Single-Electron Interference

S. Gustavsson,* R. Leturcq, M. Studer, T. Ihn, and K. Ensslin
D. C. Driscoll and A. C. Gossard

Nanoletters, 2008



Double-slit geometry, interference pattern builds up one electron at a time!