

5 > 1 "supercyclic"

(weaker dissipation at $\omega \rightarrow 0$)

(stronger " " ")

$$1/T_1 = \frac{1}{2} g^2 J(\Delta) \coth(\Delta/2T) \quad (1)$$

$$(4) \equiv \alpha\pi$$

$$\tau_1 = \alpha \pi \Delta \text{ with } \Delta/2T \quad (3)$$

A hand-drawn graph showing the time response $P(t)$ versus time t . The graph illustrates two types of system behavior:

- Underdamped:** Represented by a red curve that oscillates with a decaying amplitude. It is labeled with $T < T_0$ and "underdamped" in red ink.
- Overdamped:** Represented by a purple curve that decays monotonically without oscillating. It is labeled with $T > T_0$ and "overdamped" in purple ink.

A box in the top right corner contains the text "DIS/10".

$T_0 \equiv$ crossover temperature where $\Delta T_i = 1$

A graph showing the normalized magnetization $\frac{M}{M_0}$ on the y-axis versus the normalized temperature $\frac{T}{T_0}$ on the x-axis. The curve starts at the origin (0,0) and increases monotonically, approaching 1 as temperature increases. A dashed blue line represents the high-temperature limit $\frac{M}{M_0} = \frac{T}{T_0}$. A horizontal dashed red line is drawn at $\frac{M}{M_0} = \frac{1}{2}$. The intersection of the curve and this line occurs at $\frac{T}{T_0} = 1$. Vertical dashed red lines are drawn from the origin and the intersection point to the x-axis, marking $\frac{T}{T_0} = 0$ and $\frac{T}{T_0} = 1$ respectively. The y-axis is labeled $\frac{M}{M_0}$ and the x-axis is labeled $\frac{T}{T_0}$. The text "funkt. $\Delta/2T$ " is written in the upper right corner. At the bottom right, there is a note $T = 0$ with an upward arrow.

$$\pi \alpha \approx \frac{\Delta}{2T_0} \Rightarrow T_0 = \frac{\Delta}{2\pi \alpha} \gg \Delta$$

At sufficiently large T_0 , oscillations get damped!

Adiabatic renormalization: effect of higher frequencies

DIS 11

Block Eq. (4.1) to (4.3) are not fully consistent with

Heisenberg equations for full system:

$$\text{For } H = -\Delta S_x + S_z \underbrace{\left(\sum_{\alpha} \lambda_{\alpha} (b_{\alpha} + b_{\alpha}^{\dagger}) \right)}_{B_z} + H_{\text{bath}}, \quad (1)$$

(form of 3.1),

$$= -\vec{S} \cdot \vec{B} + H_{\text{bath}}, \quad \text{with } \vec{B} = \hat{x} \Delta + \hat{z} B_z(t) \quad (2)$$

$$(3) \quad \partial_t \vec{S} = \vec{S} \times \vec{B} = \vec{S} \times \left[\Delta \hat{x} + B_z(t) \hat{z} \right] \quad (3)$$

so, we need also $\partial_t B_z(t) = -i[B_z(t), H] = \dots \quad (4)$

effective field has its own dynamics! High frequency modes can not be treated with golden rule perturbation theory!!

Consider single mode α for fixed σ_z :

$$\begin{aligned} H_{\alpha} &= \omega_{\alpha} (b_{\alpha}^{\dagger} b_{\alpha} + 1/2) + \frac{1}{2} \sigma_z \sum \lambda_{\alpha} (b_{\alpha} + b_{\alpha}^{\dagger}) \\ &= \frac{p_{\alpha}^2}{2m_{\alpha}} + \frac{1}{2} m_{\alpha} \omega_{\alpha}^2 x_{\alpha}^2 + \sigma_z C_{\alpha} x_{\alpha} \\ &= \frac{p_{\alpha}^2}{2m_{\alpha}} + \frac{1}{2} m_{\alpha} \omega_{\alpha}^2 \left[x_{\alpha} + \sigma_z \underbrace{\frac{C_{\alpha}}{m_{\alpha} \omega_{\alpha}^2}}_{\equiv \bar{x}_{\alpha}} \right]^2 \end{aligned} \quad (3)$$

(1)

DIS 12

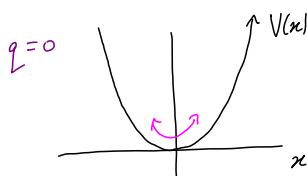
$$x_{\alpha} = \frac{1}{\sqrt{2m_{\alpha}\omega_{\alpha}}} (b_{\alpha} + b_{\alpha}^{\dagger})$$

$$p_{\alpha} = -i\sqrt{\frac{2m_{\alpha}\omega_{\alpha}}{2}} (b_{\alpha} - b_{\alpha}^{\dagger})$$

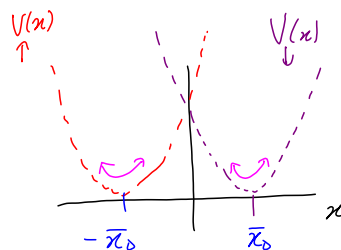
$$C_{\alpha} = \frac{g}{2} \lambda_{\alpha} \sqrt{2m_{\alpha}\omega_{\alpha}} \quad (5)$$

$$= \text{const} \quad (2)$$

$\pm \bar{x}_{\alpha}$ = shifted equilibrium positions of mode α



ground states: $|g_{\alpha}\rangle$



Shifted ground states:

(4)

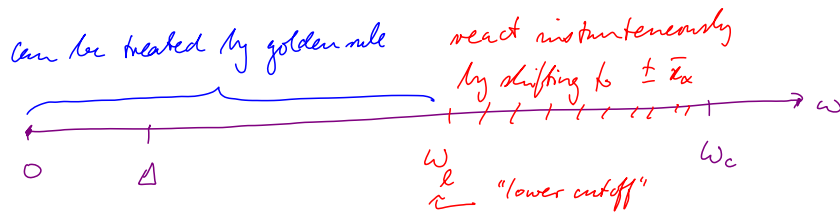
$$\left. \begin{aligned} |g_{\alpha\uparrow}\rangle \\ |g_{\alpha\downarrow}\rangle \end{aligned} \right\} = e^{\pm i \bar{x}_{\alpha} p_{\alpha}} |g_{\alpha}\rangle$$

shift operator
by $\pm \bar{x}_{\alpha}$

Adiabatic treatment of fast modes : assume modes with

D13

$\omega_\alpha > \omega_\ell(\Delta) = p\Delta$ (for $p \gg 1$) adjust instantaneously to state of spin: (1)



$$\sum_{\omega_\alpha} = \sum_{\omega_\ell < \omega_\alpha < \omega_c} + \sum_{\omega_\alpha < \omega_\ell}$$

react instantaneously *use in golden rule*

So, split up Hamiltonian: (prime denotes restriction to $\omega_\ell < \omega_\alpha < \omega_c$)

write $H = \underbrace{H_0 + H'}_{H_{\text{eff}}: \text{effective Hamiltonian}} + H^c$ *treat perturbatively*

$$\left. \begin{matrix} H' \\ H^c \end{matrix} \right\} = (H_{\text{int}} + H_{\text{bath}}) \text{ restricted to } \begin{cases} \omega_\ell < \omega_\alpha < \omega_c \\ \omega_\alpha < \omega_\ell \end{cases}$$

Base Hamiltonian : $H_0 = -\Delta \frac{1}{2} \sigma_x$,

D14

(1)

base tunneling rate: $\Delta = -2 \langle \downarrow | H_0 | \uparrow \rangle$

(2)

base basis states : $|\uparrow\rangle \pi'_\alpha |g_\alpha\rangle$ and $|\downarrow\rangle \pi'_\alpha |g_\alpha\rangle$

couple to high-frequency modes, $\downarrow$
switch on $H'_{\text{int}} + H'_{\text{bath}}$

$$|\uparrow\rangle \otimes \pi'_\alpha |g_\alpha\rangle$$

$$|\downarrow\rangle \otimes \pi'_\alpha |g_\alpha\rangle$$

Effective Hamiltonian for low-frequency modes :

$$H_{\text{eff}} = -\sigma_x \frac{1}{2} \tilde{\Delta}(\omega_\ell)$$

"Reduced tunneling rate" : $\tilde{\Delta}(\omega_\ell) = \underbrace{-2 \langle \downarrow | H_0 | \uparrow \rangle}_{\Delta} \pi'_\alpha \langle g_{\alpha\downarrow} | g_{\alpha\uparrow} \rangle$

$$\tilde{\Delta}(\omega_e) \stackrel{(12.4)}{=} \Delta \prod_{\alpha} \langle g_{\alpha} | e^{-i p_{\alpha}(\bar{x}_{\alpha})} e^{i p_{\alpha}(-\bar{x}_{\alpha})} | g_{\alpha} \rangle = \Delta e^{-A} \quad \boxed{D15} \quad (1)$$

$$e^{-\bar{x}_{\alpha}^2 \langle p_{\alpha}^2 \rangle}$$

$$\text{with } \frac{\langle p_{\alpha}^2 \rangle}{2m} = \frac{1}{2} \omega_{\alpha} \Rightarrow \langle p_{\alpha}^2 \rangle = m_{\alpha} \omega_{\alpha} \quad (2)$$

$$\Rightarrow A = \sum_{\alpha} \bar{x}_{\alpha}^2 \langle p_{\alpha}^2 \rangle \stackrel{(12.3)}{=} \sum_{\alpha} \frac{C_{\alpha}}{m_{\alpha} \omega_{\alpha}^2} \langle p_{\alpha}^2 \rangle = \sum_{\alpha} \frac{C_{\alpha}^2}{m_{\alpha} \omega_{\alpha}^3} \quad (3)$$

$$C_{\alpha} = \frac{g}{2} \lambda_{\alpha} \sqrt{2m_{\alpha} \omega_{\alpha}} \quad (12.5)$$

$$= \int_{\omega_e}^{\omega_c} d\omega \frac{g^2}{\pi^2} \sum_{\alpha} \frac{\lambda_{\alpha}^2}{\omega_{\alpha}^2} \delta(\omega - \omega_{\alpha}) \quad (4)$$

$$A = \frac{g^2}{2\pi} \int_{\omega_e}^{\omega_c} d\omega \frac{J(\omega)}{\omega^2} \quad (5)$$

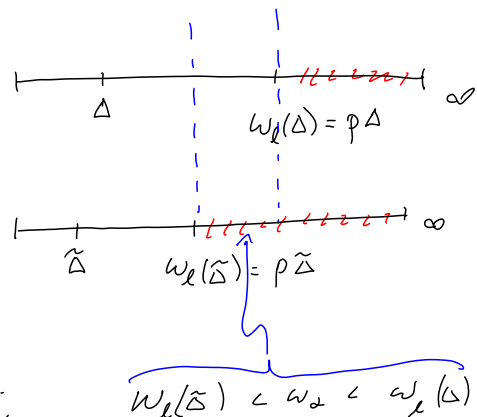
$$\text{So: } \tilde{\Delta}(\omega_e) = \Delta e^{-A} < \Delta \quad (1) \quad \boxed{D16}$$

$\Rightarrow$ bath reduces flipping rate

Repeat this procedure, with reduced cutoff $\omega_e(\tilde{\Delta}) = p\tilde{\Delta}$,

to incorporate effect of frequency regime

Renormalization stops, if $\omega_e(\tilde{\Delta})$ is reduced more quickly than $\tilde{\Delta}$, so that it "catches up". If this does not happen, $\tilde{\Delta}$ is "renormalized" down to $\tilde{\Delta} \rightarrow 0$.



Which case wins? Consider $\omega_e \rightarrow \omega_e - \delta\omega_e$

Dis 17

$$\Delta \rightarrow \Delta - \delta\Delta$$

If $\frac{\delta\Delta}{\Delta} < \frac{\delta\omega_e}{\omega_e}$, then renormalization stops. (1)

$$1 > \frac{\omega_e}{\Delta} \frac{d\Delta(\omega_e)}{d\omega_e} = \omega_e \frac{d \ln [\Delta(\omega_e)]}{d\omega_e} \quad (2)$$

$$= \omega_e \frac{d}{d\omega_e} \left[-\frac{q^2}{2\pi} \int_{\omega_e}^{\omega_c} d\omega \frac{J(\omega)}{\omega^2} \right] \quad (3)$$

$$1 > \frac{q^2}{2\pi} \frac{J(\omega_e)}{\omega_e}$$

Renormalization stops if an ω_e exists where this holds. (4)

Chiral bath:

$$J(\omega) \stackrel{(9.1)}{=} A \omega e^{-\omega/\omega_c}$$

Dis 18

$$1 > \frac{q^2}{2\pi} A e^{-\omega_e/\omega_c} \quad (17.4)$$

$$1 > \underbrace{\quad} = \alpha \quad (\text{see 12.4})$$

So: for $\alpha < 1$, renormalization stops at

a finite splitting, say Δ_* .

for $\alpha > 1$, renormalization continues until $\Delta_* = 0$

$\Rightarrow$ both cases open to stop flipping: " $\alpha > 1 \Rightarrow$ localization transition"

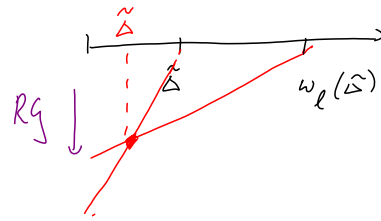
For $\alpha < 1$, determine finite value of Δ^* :

DIS19

Renormalization stops when

$$\tilde{\Delta}[\omega_c(\Delta_*)] = \Delta_*$$

(13.1)



$$\text{But: } \tilde{\Delta}(\omega_c) \stackrel{(15.5)}{=} \Delta e^{-\alpha \int_{\omega_l}^{\omega_c} \frac{1}{\omega}} = \Delta e^{-\alpha \ln(\omega_c/\omega_l)} \quad (2)$$

$$= \Delta (\omega_l/\omega_c)^\alpha \quad (3)$$

$$\text{According to (1): set } \Delta_* = \Delta (P\Delta_*/\omega_c)^\alpha \quad (4)$$

$$\text{solve: } \Delta_*^{1-\alpha} = \Delta (P/\omega_c)^\alpha \Rightarrow \Delta_* = \Delta \left(\frac{P\Delta}{\omega_c} \right)^{\frac{\alpha}{1-\alpha}} \quad (5)$$