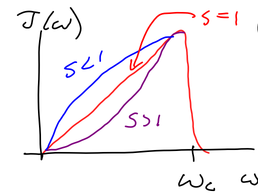
Popular Ansatz: $J(\omega) = A\,\omega_c \left(\omega/\omega_c\right)^s e^{-\omega/\omega_c}$ (1)



⑨

$s = 1$ "ohmic bath"

$s > 1$ "superohmic" (weaker dissipation at $\omega \to 0$)

$s < 1$ "sub-ohmic" (stronger " " ")

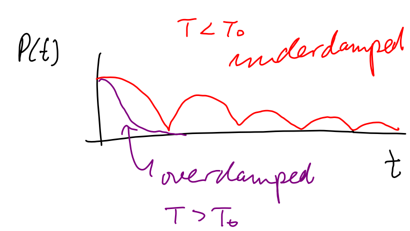
Henceforth: $s = 1$:

$$1/T_1 = \tfrac{1}{2} q^2 J(\Delta) \coth(\Delta/2T) \quad (1)$$

$$= \underbrace{\tfrac{1}{2} q^2 A}_{(4)\ \equiv \alpha\pi} \;\Delta\; \underbrace{e^{-\Delta/\omega_c}}_{\simeq 1} \coth(\Delta/2T) \quad (2)$$

$$\boxed{1/T_1 = \alpha\pi\,\Delta\,\coth \Delta/2T} \quad (3)$$

$\left.\begin{array}{l}\text{Underdamped}\\ \text{overdamped}\end{array}\right\}$ oscillations for

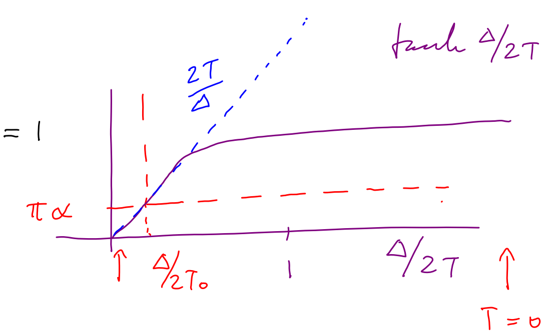


DIS/10

$$\frac{\tanh \frac{\Delta}{2T}}{\pi\alpha} = \Delta T_1 \begin{array}{c} \ll \\ \gg \end{array} 1$$

$T_0 \equiv$ crossover temperature where $\Delta T_1 = 1$

$$\pi\alpha = \tanh \frac{\Delta}{2T_0}$$



For $\pi\alpha \ll 1$ ("weak damping") we can use $\tanh x \simeq x$

$$\pi\alpha \simeq \frac{\Delta}{2T_0} \implies T_0 = \frac{\Delta}{2\pi\alpha} \gg \Delta$$

At sufficiently large T_0, oscillations get <u>damped</u>!

Adiabatic renormalization: effect of high frequencies

DIS 11

Bloch Eqs. (4.1) to (4.3) are not fully consistent with Heisenberg equations for full system:

$$\text{For} \quad H = -\Delta S_x + S_z \underbrace{q \overbrace{\sum_\alpha \lambda_\alpha (b_\alpha^\dagger + b_\alpha)}^{\sim \hat{x}_\alpha}}_{B_z \;\longrightarrow\; \text{operator!!}} + H_{bath}, \qquad (1)$$

(form of 3.1),

$$= -\vec{S}\cdot\vec{B} + H_{bath}, \quad \text{with } \vec{B} = \hat{x}\Delta + \hat{z}\, B_z(t) \qquad (2)$$

$$(3.3) \qquad \partial_t \vec{S} = \vec{S} \times \vec{B} = \vec{S} \times \left[\Delta \hat{x} + B_z(t)\, \hat{z}\right] \qquad (3)$$

so, we need also
$$\partial_t B_z(t) = -i\left[B_z(t), H\right] = \ldots \qquad (4)$$

effective field has its own dynamics! High frequencies modes can not be treated with golden rule perturbation theory!!

DIS 12

Consider single mode α for fixed σ_z: (1)

$$H_\alpha = \omega_\alpha (b_\alpha^\dagger b_\alpha + 1/2) + \tfrac{1}{2}\sigma_z\, q\, \lambda_\alpha (b_\alpha + b_\alpha^\dagger)$$

$$= \frac{p_\alpha^2}{2m_\alpha} + \tfrac{1}{2} m_\alpha \omega_\alpha^2 x_\alpha^2 + \sigma_z C_\alpha x_\alpha$$

$$= \frac{p_\alpha^2}{2m_\alpha} + \underbrace{\tfrac{1}{2} m_\alpha \omega_\alpha^2 \Big[x + \sigma_z \underbrace{\frac{C_\alpha}{m_\alpha \omega_\alpha^2}}_{\equiv\, \bar{x}_\alpha \;(3)}\Big]^2}_{\equiv\, V_{\sigma_z}(x)} - \text{const} \qquad (2)$$

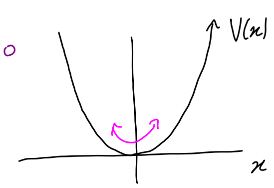
$$x_\alpha = \frac{1}{\sqrt{2m_\alpha\omega_\alpha}}(b_\alpha + b_\alpha^\dagger)$$

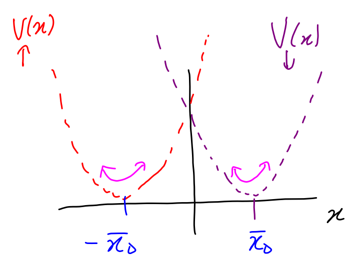
$$p_\alpha = -i\sqrt{\frac{2\omega_\alpha}{m_\alpha}}\,(b_\alpha - b_\alpha^\dagger)$$

$$C_\alpha = \frac{q}{2}\lambda_\alpha \sqrt{2m_\alpha\omega_\alpha} \qquad (5)$$

$\pm \bar{x}_\alpha$ = shifted equilibrium positions of mode α

$q = 0$





Shifted ground states: (4)

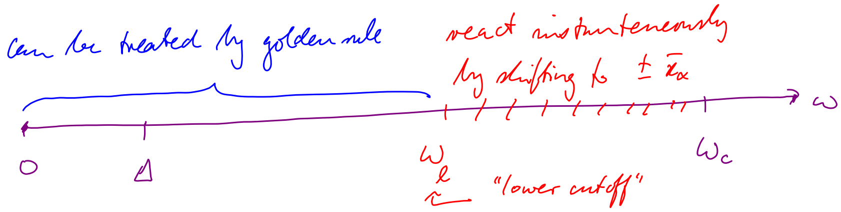
$$\left.\begin{array}{l} |g_{\alpha\uparrow}\rangle \\ |g_{\alpha\downarrow}\rangle \end{array}\right\} = \underbrace{e^{\mp i \bar{x}_\alpha p_\alpha}}_{\text{shift operator by } \mp \bar{x}_\alpha} |g_\alpha\rangle$$

ground states: $|g_\alpha\rangle$

D13

Adiabatic treatment of fast modes: assume modes with

$\omega_\alpha > \omega_\ell(\Delta) = p\Delta$ (for $p \gg 1$) adjust instantaneously to state of spin: (1)



$$\sum_{\omega_\alpha} = \sum_{\omega_\ell < \omega_\alpha < \omega_c} + \sum_{\omega_\alpha < \omega_\ell}$$

react instantaneously — use in golden rule

So, split up Hamiltonian: (prime denotes restriction to $\omega_\ell < \omega_\alpha < \omega_c$)

write $H = \underbrace{H_0 + H'}_{H_{eff}\text{: effective Hamiltonian}} + H^<$ ← treat perturbatively

$$\left.\begin{matrix} H' \\ H^< \end{matrix}\right\} = \left(H_{int} + H_{bath}\right) \text{restricted to} \left\{\begin{matrix} \omega_\ell < \omega_\alpha < \omega_c \\ \omega_\alpha < \omega_\ell \end{matrix}\right.$$

D14

Bare Hamiltonian: $H_0 = -\Delta \frac{1}{2} \sigma_x$, (1)

bare tunneling rate: $\Delta = -2 \langle \downarrow | H_0 | \uparrow \rangle$ (2)

bare basis states: $|\uparrow\rangle \prod'_\alpha |q_\alpha\rangle$ and $|\downarrow\rangle \prod'_\alpha |q_\alpha\rangle$

couple to high-frequency modes, switch on $H'_{int} + H'_{bath}$ ↓ ↓

$|\uparrow\rangle \otimes \prod'_\alpha |q_{\alpha\uparrow}\rangle$ $\quad$ $|\downarrow\rangle \otimes \prod'_\alpha |q_{\alpha\downarrow}\rangle$

Effective Hamiltonian for low-frequency modes: $H_{eff} = -\sigma_x \frac{1}{2} \tilde{\Delta}(\omega_\ell)$

"Reduced tunneling rate": $\tilde{\Delta}(\omega_\ell) = \underbrace{-2 \langle \downarrow | H_0 | \uparrow \rangle}_{\Delta} \prod'_\alpha \langle q_{\alpha\downarrow} | q_{\alpha\uparrow} \rangle$

D15

$$\tilde{\Delta}(\omega_\ell) \overset{(12.4)}{=} \Delta \prod_\alpha{}' \underbrace{\langle q_\alpha | e^{-i p_\alpha (+\bar{x}_\alpha)} e^{i p_\alpha(-\bar{x}_\alpha)} | q_\alpha \rangle}_{e^{-\bar{x}_\alpha^2 \langle p_\alpha^2 \rangle}} \equiv \Delta e^{-A} \qquad (1)$$

with $$\frac{\langle p_\alpha^2 \rangle}{2m} = \frac{1}{2}\omega_\alpha \implies \langle p_\alpha^2 \rangle = m_\alpha \omega_\alpha \qquad (2)$$

$$\implies A = \sum_\alpha \bar{x}_\alpha^2 \langle p_\alpha^2 \rangle \overset{(12.3):\ \bar{x}_\alpha = \frac{C_\alpha}{m_\alpha \omega_\alpha^2}}{=} \sum_\alpha \frac{C_\alpha^2}{m_\alpha \omega_\alpha^3} \qquad (3)$$

(12.5) $C_\alpha = \frac{q}{2} \lambda_\alpha \sqrt{2 m_\alpha \omega_\alpha}$

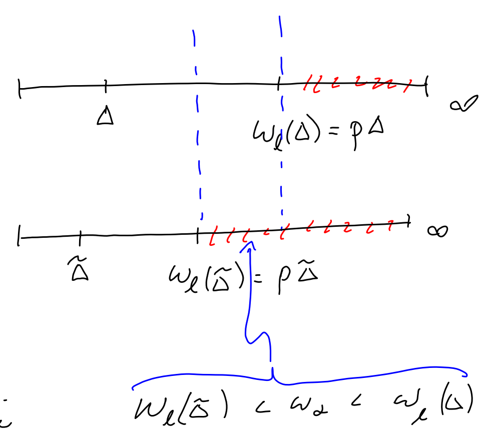
$$= \int_{\omega_\ell}^{\omega_c} d\omega \, \frac{q^2}{\pi 2} \, \pi \underbrace{\sum_\alpha \frac{\lambda_\alpha^2}{\omega_\alpha^2} \delta(\omega - \omega_\alpha)}_{J(\omega)} \qquad (4)$$

$$\boxed{A = \frac{q^2}{2\pi} \int_{\omega_\ell}^{\omega_c} d\omega \, \frac{J(\omega)}{\omega^2}} \qquad (5)$$

D16

So: $$\tilde{\Delta}(\omega_\ell) = \Delta e^{-A} < \Delta \qquad (1)$$

$\implies$ bath reduces flipping rate



Repeat this procedure, with reduced cutoff $\omega_\ell(\tilde{\Delta}) = p\tilde{\Delta}$,

to incorporate effect of frequency regime

Renormalization stops, if $\omega_\ell(\tilde{\Delta})$ is reduced more quickly than $\tilde{\Delta}$, so that it "catches up". If this does not happen, $\tilde{\Delta}$ is "renormalized" down to $\tilde{\Delta} \to 0$.

Which case wins? Consider $\omega_\ell \to \omega_\ell - \delta\omega_\ell$ DIS 17

$$\Delta \to \Delta - \delta\Delta$$

If $\frac{\delta\Delta}{\Delta} < \frac{\delta\omega_\ell}{\omega_\ell}$, then renormalization stop. (1)

$$1 \overset{?}{>} \frac{\omega_\ell}{\Delta}\frac{d\Delta(\omega_\ell)}{d\omega_\ell} = \omega_\ell \frac{d \ln[\Delta(\omega_\ell)]}{d\omega_\ell} \qquad (2)$$

$$= \omega_\ell \frac{d}{d\omega_\ell}\left[-\frac{q^2}{2\pi}\int_{\omega_\ell}^{\omega_c} d\omega \frac{J(\omega)}{\omega^2}\right] \qquad (3)$$

$$\boxed{1 \overset{?}{>} \frac{q^2}{2\pi}\frac{J(\omega_\ell)}{\omega_\ell}} \qquad (4)$$

Renormalization stops if an ω_ℓ exists where this holds.

<u>Ohmic bath:</u> DIS 18

$$J(\omega) \overset{(9.1)}{=} A\,\omega\, e^{-\omega/\omega_c}$$

$$1 \overset{(17.4)}{>} \underbrace{\frac{q^2}{2\pi} A}\, e^{-\omega_\ell/\omega_c}$$

$1 >$ $= \alpha$ (see 12.4)

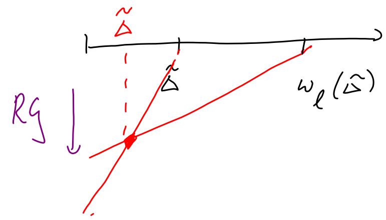
So: for $\alpha < 1$, renormalization stops at a finite splitting, say Δ_*.

for $\alpha > 1$, renormalization continues until $\Delta_* = 0$

$\Rightarrow$ both causes spin to stop flipping: "$\alpha > 1 \Rightarrow$ localization transition"

For $\alpha < 1$, determine finite value of Δ^*: $\boxed{\text{D1519}}$

Renormalization stops when



$$\tilde{\Delta}[\omega_\ell(\Delta_*)] = \Delta_* \qquad (13.1)$$

$$\text{But:} \quad \tilde{\Delta}(\omega_\ell) \overset{(15.5)}{=} \Delta\, e^{-\alpha \int_{\omega_\ell}^{\omega_c} \frac{1}{\omega}} = \Delta\, e^{-\alpha \ln(\omega_c/\omega_\ell)} \qquad (2)$$

$$= \Delta \left(\omega_\ell/\omega_c\right)^{\alpha} \qquad (3)$$

According to (1): set

$$\Delta_* = \Delta\left(p\Delta_*/\omega_c\right)^{\alpha} \qquad (4)$$

solve:

$$\Delta_*^{1-\alpha} = \Delta\left(p/\omega_c\right)^{\alpha} \Longrightarrow \boxed{\Delta_* = \Delta\left(\frac{p\Delta}{\omega_c}\right)^{\frac{\alpha}{1-\alpha}}} \qquad (5)$$