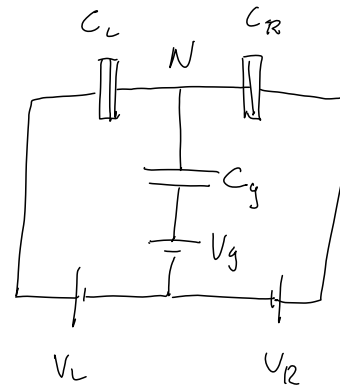
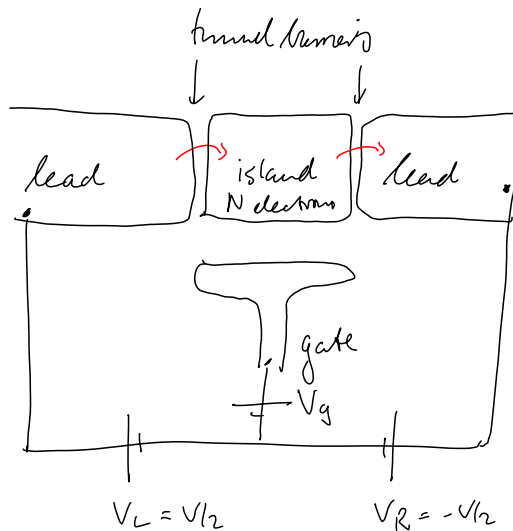


# Superconducting Charge Qubit

27.11.09

SQ1

## Single-Electron-Transistor



Charging energy :

Electrostatic work required to add  $N$  electrons, total charge

$Q = Ne$  ( $e < 0$ ), to initial random charge  $Q_0$ :

$$E(N) = E_C (N - N_g)^2,$$

$$N_g = -\frac{eV_g}{2E_C} \quad (\text{gate-tunable constant})$$

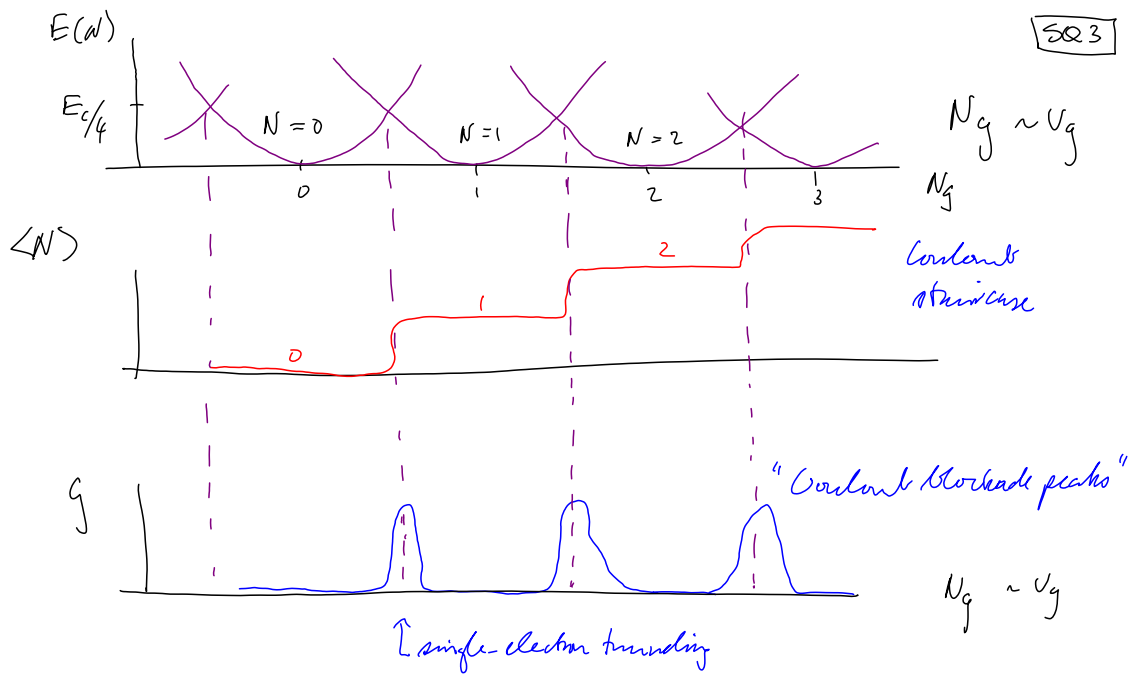
$$E_C = \frac{e^2}{C} = \text{charging energy} = \text{typically } 100 \text{ mK} \leftrightarrow 100 \text{ K}$$

for large island      small island

$$C = C_L + C_R + C_g \quad (\sim \text{area of junctions})$$

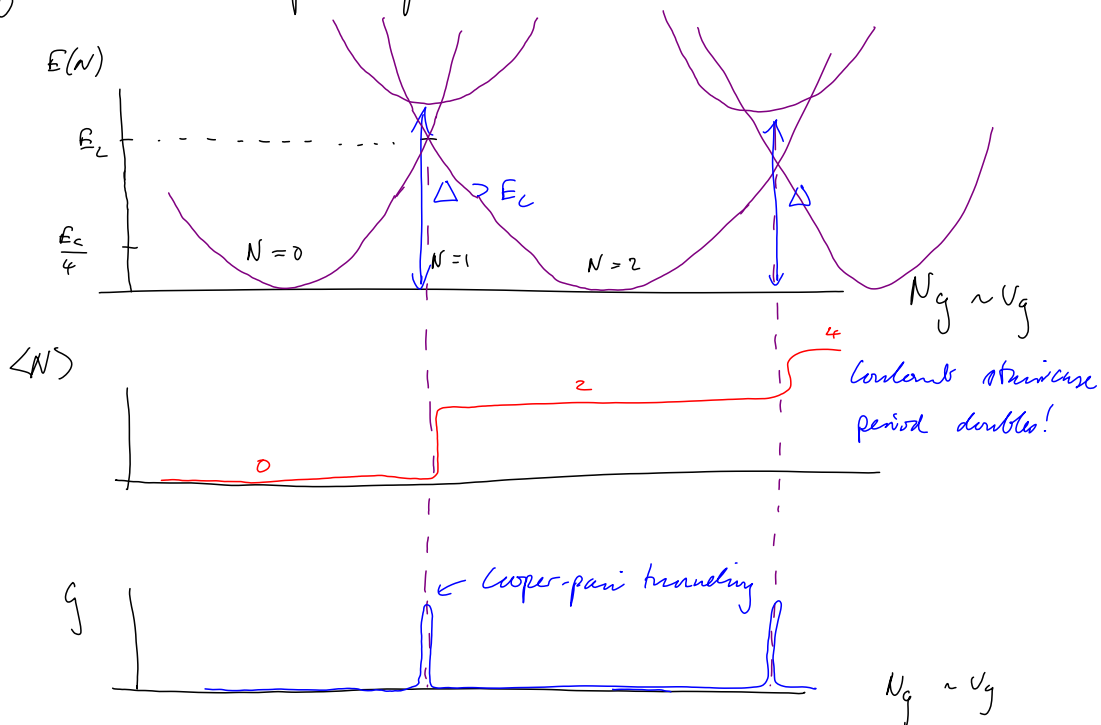
small for small junctions

SQ2



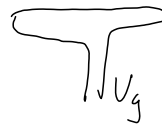
Transport possible only at intersections of parabolas,  
 where  $E(N) = E(N+1)$  "e-periodicity"

For superconducting islands: energy penalty  $\Delta$  for odd grains! SQ4  
 if  $\Delta > E_c$ : "2e-periodicity".



# Cooper pair box

SQ5



$$H_{\text{island}} = E_C (N - N_g)^2 + E_J \sum_{N=-\infty}^{\infty} |N\rangle \langle N+2| \quad (\text{for } \Delta > E_C)$$

charging energy      Cooper pair tunneling between island and lead

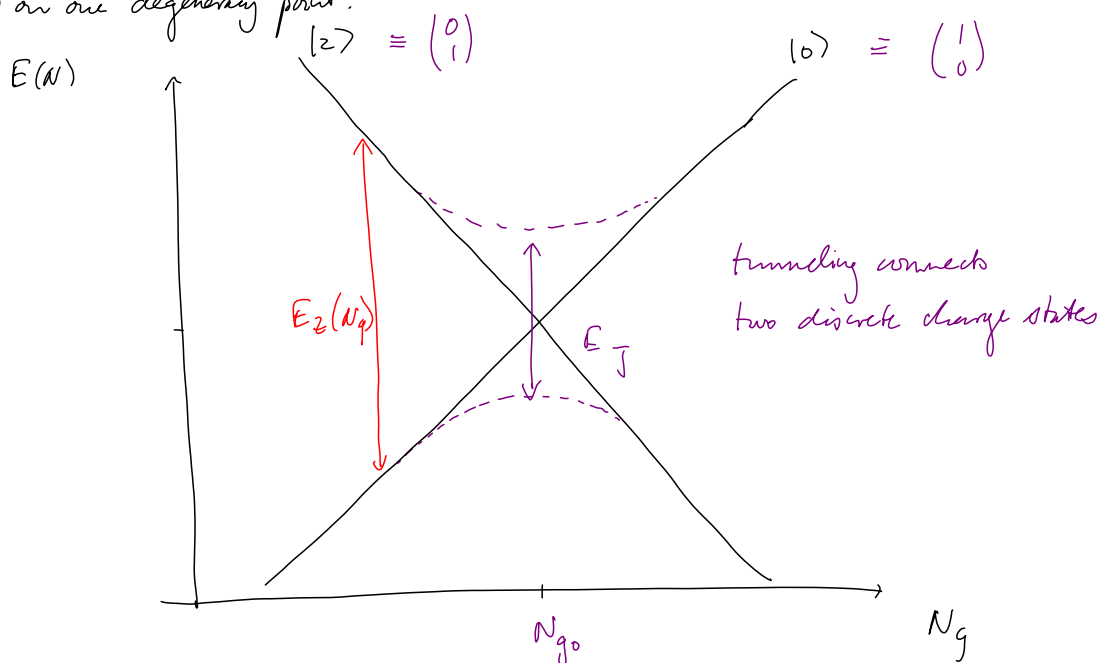
$E_J$  = "Josephson coupling" (describes tunneling strength)

$E_J$  = tunable by magnetic field if island is part of a loop, and flux through loop is tuned.



Focus on one degeneracy point:

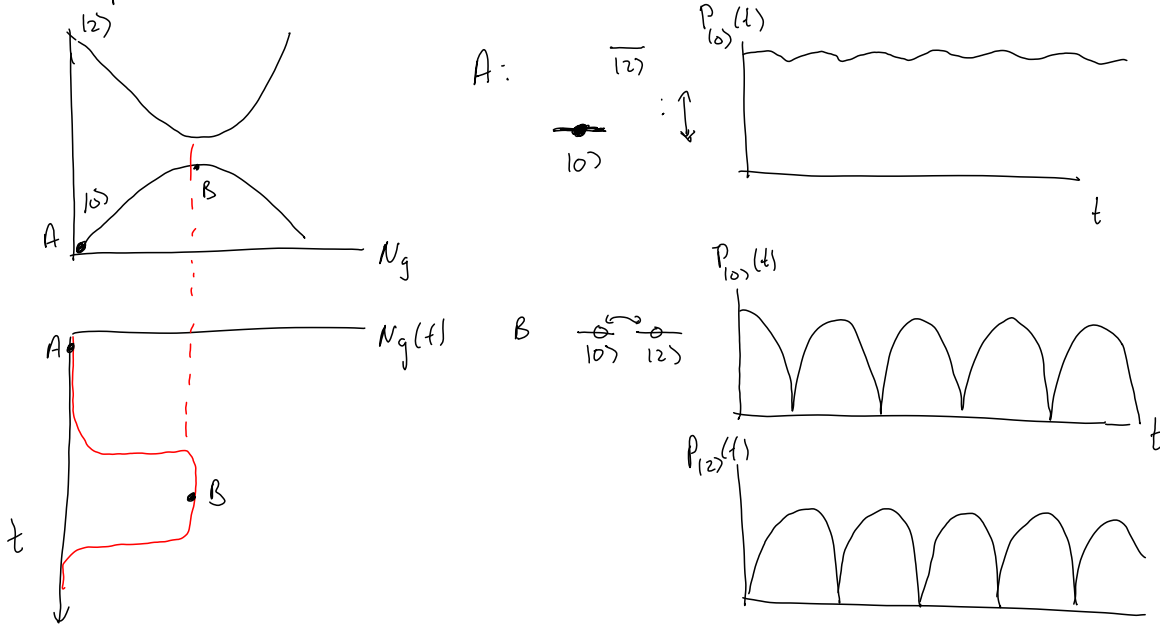
SQ6



$$H_{\text{eff}} = E_z(N_g) \sigma_z + E_J(B) \sigma_x = \text{"Dirac Hamiltonian"}$$

tunable !!

Pulse protocol to drive oscillations between  $|0\rangle$  and  $|2\rangle$  Sec 7



**letters to nature**, Vol 398, 786 (1999)

## Coherent control of macroscopic quantum states in a single-Cooper-pair box

Y. Nakamura<sup>\*</sup>, Yu. A. Pashkin<sup>†</sup> & J. S. Tsai<sup>\*</sup>

$$\Delta = 230 \mu\text{eV}$$

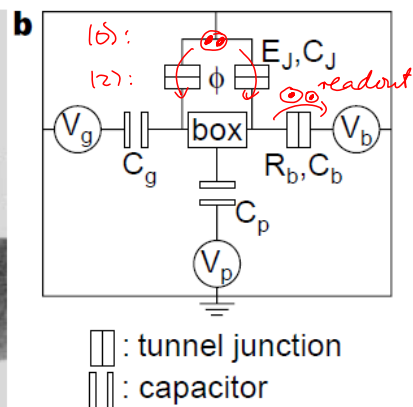
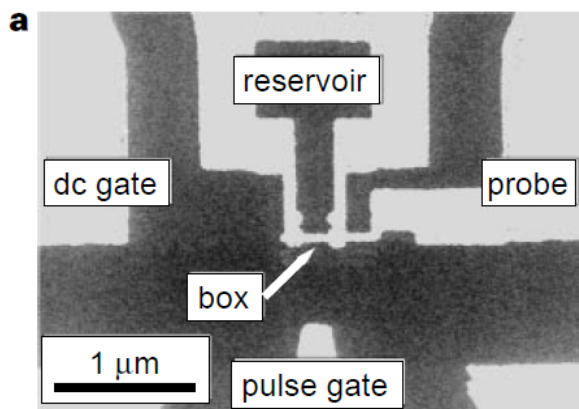
Sec 8

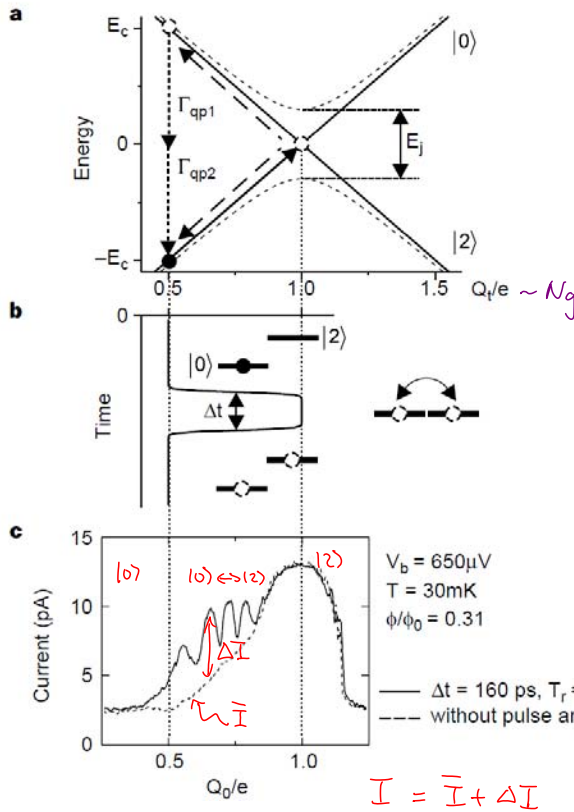
$$E_c = 117 \mu\text{eV}$$

$$E_J = 52 \mu\text{eV}$$

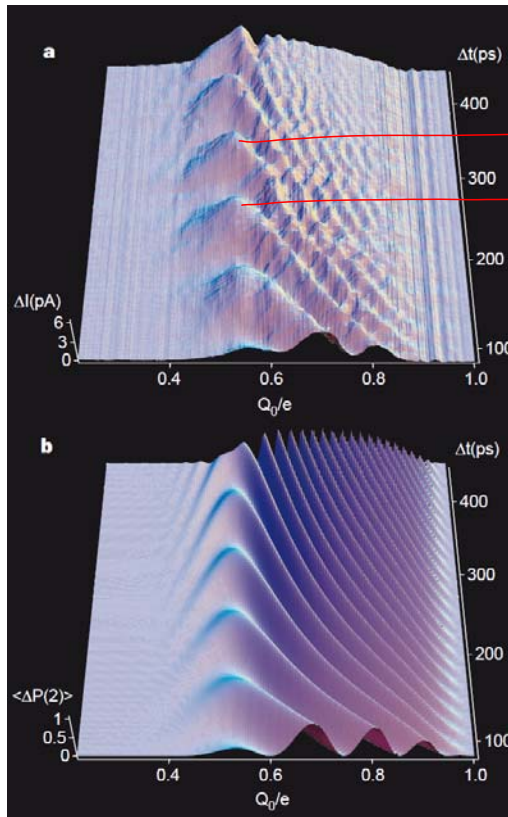
$$k_B T = 3 \mu\text{eV} \quad (T = 30 \text{ mK})$$

island has  $N \sim 10^8$





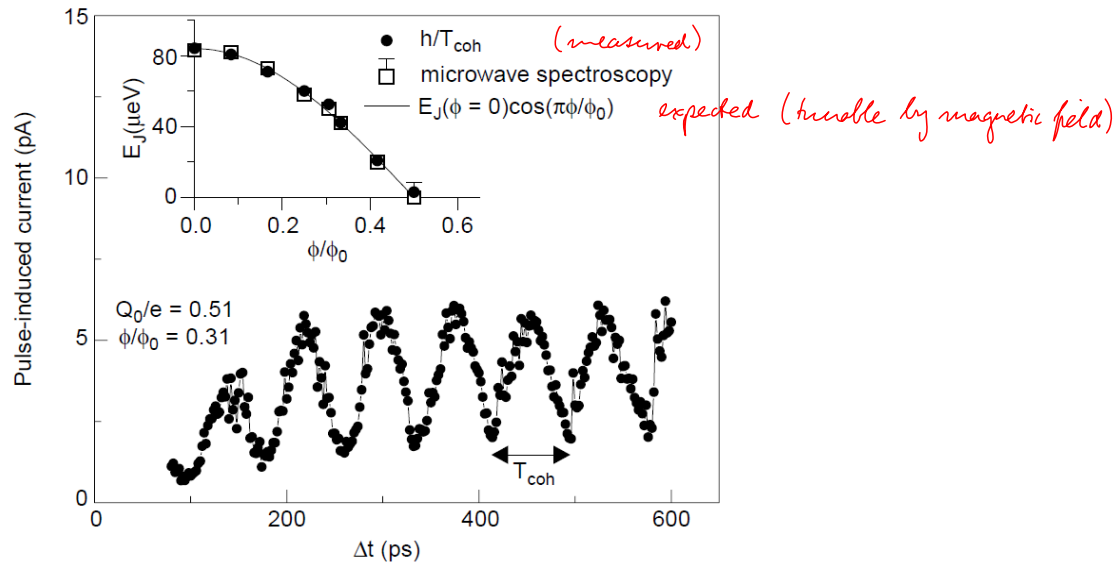
**Figure 2** Pulse modulation of quantum states. **a**, Energy diagram illustrating electrostatic energies (solid lines) of two charge states  $|0\rangle$  and  $|2\rangle$  (with the number of excess charges in the box  $n = 0$  and  $2$ ) as a function of the total gate-induced charge  $Q_t = Q_0 + C_g V_g(t)$ , where  $Q_0 = C_g V_g + C_b V_b$  is the d.c.-gate induced charge. The dashed curves show eigenenergies (in the absence of the quasiparticle tunnelling at the probe junction) as a function of  $Q_t$ . Suppose that before a pulse occurs,  $Q_t$  equals  $Q_0$ , which is far from the resonance point, and the system is approximately in the pure charge state  $|0\rangle$  (filled circle at lower left). Then, a voltage pulse of an appropriate height abruptly brings the system into resonance  $Q_t/e = 1$  (solid arrow), and the state starts to oscillate between the two charge states. At the end of the pulse, the system returns to  $Q_t = Q_0$  (dashed arrow) with a final state corresponding to the result of the time evolution. Finally, the  $|2\rangle$  state decays to  $|0\rangle$  with two quasiparticle tunnelling events through the probe junction with rates of  $\Gamma_{qp1}$  and  $\Gamma_{qp2}$  (dotted arrows). **b**, Schematic pulse shape with a nominal pulse length  $\Delta t$  (solid line). The rise/fall times of the actual voltage pulse was about 30–40 ps at the top of the cryostat. The voltage pulse was transmitted through a silver-plated Be-Cu coaxial cable (above 4.2 K), a Nb coaxial cable (below 4.2 K) and an on-chip coplanar line to the open-ended pulse gate shown in Fig. 1a. The insets illustrate situations of the energy levels before/during/after the pulse. **c**, Current through the probe junction versus  $Q_0$  with (solid line) and without (dashed line) the pulse array. The pulse length was  $\Delta t = 160$  ps and the repetition time was  $T_r = 16$  ns. The data were taken at  $V_b = 650 \mu V$  and  $\phi/\phi_0 = 0.31$ , where  $\phi_0 = h/2e$  is a flux quantum.



**Figure 3** Effect of applying pulses as a function of d.c.-induced charge  $Q_0$  and pulse length  $\Delta t$ . **a**, Three-dimensional plot of pulse-induced current  $\Delta I$  which is the difference between currents measured with and without a pulse array.  $\Delta t = 80$  ps was the shortest pulse length available with our pulse-pattern generator (Anritsu MP1758A). **b**, Calculated average increase in probability density at  $|2\rangle$  after a single-pulse operation,  $\langle \Delta P(2) \rangle$ . The averaged probability density after the pulse was calculated by numerically solving a time-dependent Schrödinger equation and by averaging out small residual oscillations in the time domain. The effect of decoherence was not included. As the initial condition of the Schrödinger equation, we used a mixture of two eigenstates at  $Q_t = Q_0$  with weights obtained from a steady-state solution of density-matrix equations that describe charge transport through the device in the absence of a pulse array. The initial probability density was also calculated from the steady-state solution. In the calculations, Josephson energy  $E_J = 51.8 \mu eV$  and an effective pulse height  $\Delta Q_0/e = 0.49$  were used. The solid line in Fig. 2b shows an example (at  $\Delta t = 300$  ps) of the pulse shape used in this calculation.

$$T_{coh} = \frac{t_h}{E_J} \quad \text{or} \quad N_g = 1$$

$$= \frac{1}{[E_J^2 + E_2^2(N_g)]^{1/2}}$$



**Figure 4** Pulse-induced current as a function of the pulse length  $\Delta t$ . The data correspond to the cross-section of Fig. 3a at  $Q_0/e = 0.51$ . Inset, Josephson energy  $E_J$  versus the magnetic flux  $\phi$  penetrating through the loop.  $E_J$  was estimated by two independent methods. One was from the period of the coherent oscillation  $T_{\text{coh}}$  as  $h/T_{\text{coh}}$ . The other was from the gap energy observed in microwave spectroscopy<sup>4</sup>. The solid line shows a fitting curve with  $E_J(\phi = 0) = 84 \mu\text{eV}$  assuming cosine  $\phi$ -dependence of  $E_J$ .