Optical Lattices: Superfluid-Mott Transition

Quantum phase transition from a superfluid to a Mott insulator in a gas of ultracold atoms

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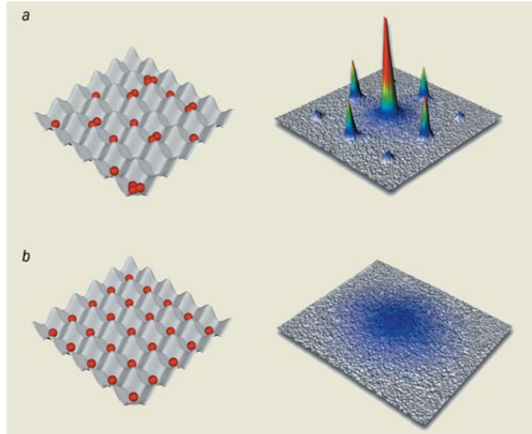
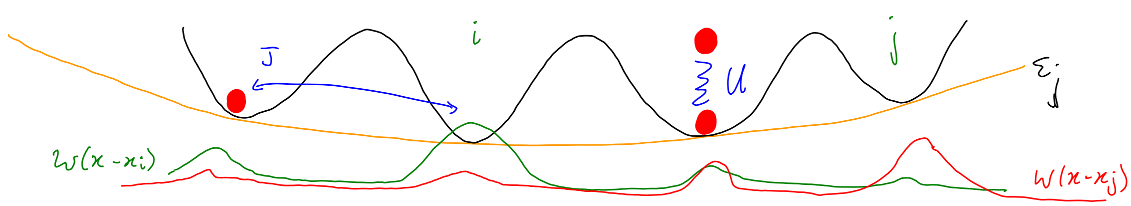
150 000 lattice sites

2.5 atoms per site at center of trap



For a system at a temperature of absolute zero, all thermal fluctuations are frozen out, while quantum fluctuations prevail. These microscopic quantum fluctuations can induce a macroscopic phase transition in the ground state of a many-body system when the relative strength of two competing energy terms is varied across a critical value. Here we observe such a quantum phase transition in a Bose–Einstein condensate with repulsive interactions, held in a three-dimensional optical lattice potential. As the potential depth of the lattice is increased, a transition is observed from a superfluid to a Mott insulator phase. In the superfluid phase, each atom is spread out over the entire lattice, with long-range phase coherence. But in the insulating phase, exact numbers of atoms are localized at individual lattice sites, with no phase coherence across the lattice; this phase is characterized by a gap in the excitation spectrum. We can induce reversible changes between the two ground states of the system.



$$H = -J\sum_{\langle i,j\rangle}\hat{a}_i^\dagger\hat{a}_j + \sum_i \epsilon_i\hat{n}_i + \frac{1}{2}U\sum_i \hat{n}_i(\hat{n}_i - 1)$$

$$J = -\int d^3x\, w(\mathbf{x}-\mathbf{x}_i)(-\hbar^2\nabla^2/2m + V_{\mathrm{lat}}(\mathbf{x}))w(\mathbf{x}-\mathbf{x}_j) \qquad U = (4\pi\hbar^2 a/m)\int |w(\mathbf{x})|^4 d^3x$$

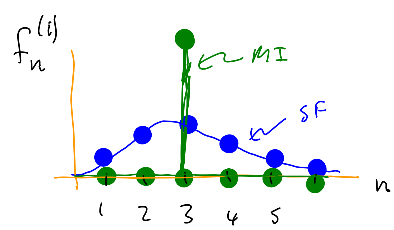
$J \gg U$: $$|\Psi_{\mathrm{SF}}\rangle_{U=0} \propto \left(\sum_{i=1}^{M}\hat{a}_i^\dagger\right)^N|0\rangle$$ Superfluid

$J \ll U$: $$|\Psi_{\mathrm{MI}}\rangle_{J=0} \propto \prod_{i=1}^{M}(\hat{a}_i^\dagger)^n|0\rangle$$ M = lattice sites; Mott-Insulator

Interpolate with Gutzwiller-Ansatz:

$$|\Psi_{GW}\rangle = \prod_{i=1}^{M}\underbrace{\sum_{n=0}^{\infty} f_n^{(i)}|n\rangle_i}_{|\phi_i\rangle}$$

Fock state with n atoms on site i



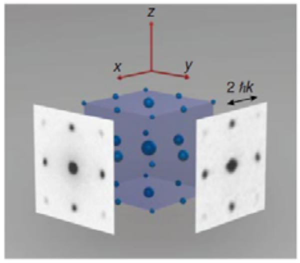


Figure 1 Schematic three-dimensional interference pattern with measured absorption images taken along two orthogonal directions. The absorption images were obtained after ballistic expansion from a lattice with a potential depth of $V_0 = 10E_r$ and a time of flight of 15 ms.

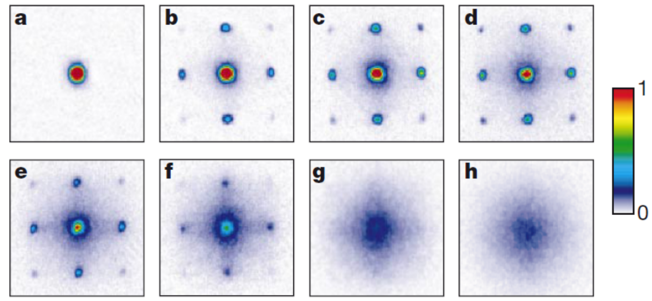


Figure 2 Absorption images of multiple matter wave interference patterns. These were obtained after suddenly releasing the atoms from an optical lattice potential with different potential depths V_0 after a time of flight of 15 ms. Values of V_0 were: **a**, 0 E_r; **b**, 3 E_r; **c**, 7 E_r; **d**, 10 E_r; **e**, 13 E_r; **f**, 14 E_r; **g**, 16 E_r; and **h**, 20 E_r.

Interference depends on: (1D argument)

$$\rho(\vec{k}) = \sum_{ij} \langle a_i^\dagger a_j \rangle e^{i\vec{k}\cdot(\vec{r}_i - \vec{r}_j)}$$

$$\langle a_i^\dagger a_j \rangle_{SF} \simeq \text{const.} \Rightarrow \rho_{SF}(\vec{k}) \simeq \sum_{ij} e^{i\vec{k}\cdot(\vec{r}_i - \vec{r}_j)} = \begin{cases} \text{large if } \vec{k} \in \text{reciprocal lattice} \\ 0 \quad \text{otherwise} \end{cases}$$

$$\langle a_i^\dagger a_j \rangle_{MI} = \delta_{ij} \Rightarrow \rho_{MI}(\vec{k}) \simeq \sum_i e^{i\vec{k}\cdot 0} \simeq \text{no structure in } \vec{k}\text{-space.}$$

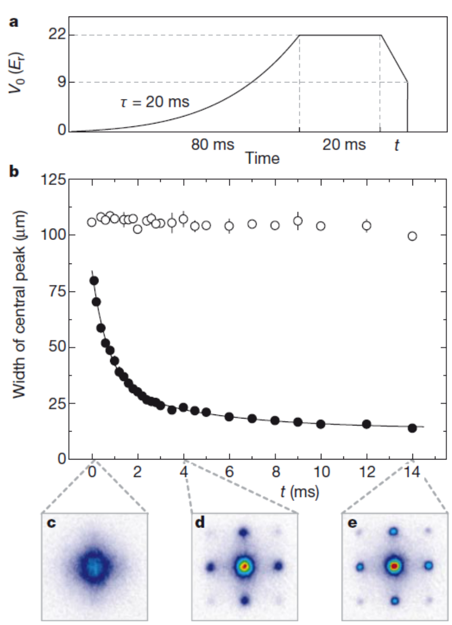


Figure 3 Restoring coherence. **a**, Experimental sequence used to measure the restoration of coherence after bringing the system into the Mott insulator phase at $V_0 = 22E_r$ and lowering the potential afterwards to $V_0 = 9E_r$, where the system is superfluid again. The atoms are first held at the maximum potential depth V_0 for 20 ms, and then the lattice potential is decreased to a potential depth of 9 E_r in a time t after which the interference pattern of the atoms is measured by suddenly releasing them from the trapping potential. **b**, Width of the central interference peak for different ramp-down times t, based on a lorentzian fit. In case of a Mott insulator state (filled circles) coherence is rapidly restored already after 4 ms. The solid line is a fit using a double exponential decay ($\tau_1 = 0.94(7)$ ms, $\tau_2 = 10(5)$ ms). For a phase incoherent state (open circles) using the same experimental sequence, no interference pattern reappears again, even for ramp-down times t of up to 400 ms. We find that phase incoherent states are formed by applying a magnetic field gradient over a time of 10 ms during the ramp-up period, when the system is still superfluid. This leads to a dephasing of the condensate wavefunction due to the nonlinear interactions in the system. **c–e**, Absorption images of the interference patterns coming from a Mott insulator phase after ramp-down times t of 0.1 ms (**c**), 4 ms (**d**), and 14 ms (**e**).

$$|\psi_{PI}\rangle = \prod_{i=1}^{M} \left(e^{i\phi_i} a_i^\dagger\right)^n$$

Phase-incoherent

random local phases