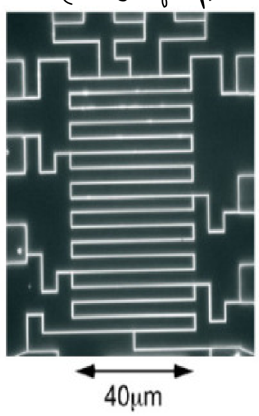
Mesoscopic Physics : Quantum Transport   SoSe 2010

Intro 1

23.4.2010

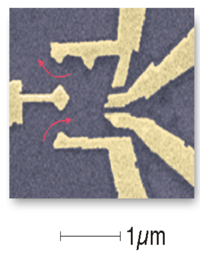
Textbook: Nazarov & Blanter (NB), "Quantum Transport", Cambridge University Press, 2009.

① (Birge group)



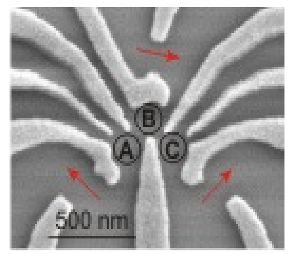
quasi-1D metallic wire

② (Marcus group)



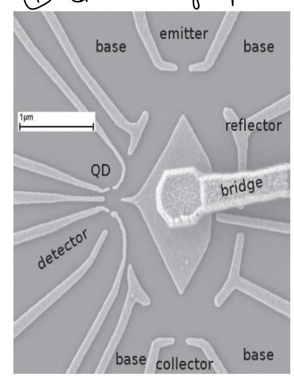
large quantum dot

③ (Ludwig/Kotthaus)



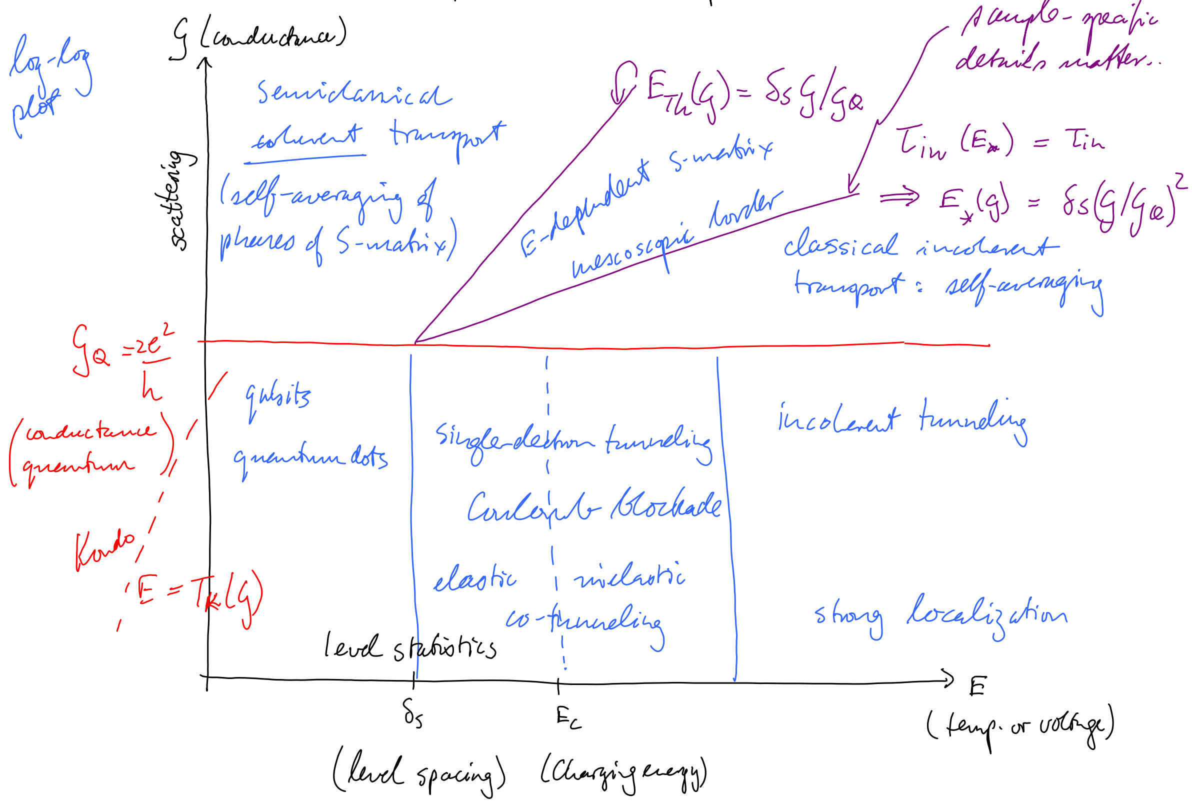
triple quantum dot

④ (Heiblum group)



few-electron dot in arm of Aharonov-Bohm ring ③

"Map of Quantum Transport"   ②



③

Conductance $= G = \frac{\sigma A}{L} = \text{conductivity} \cdot \frac{\text{Area}}{\text{Length}} = \frac{1}{R} = \frac{1}{\text{resistance}}$

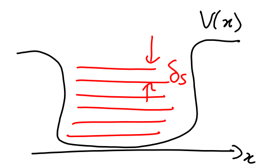
quantum conductance: $G_Q = \frac{e^2}{\hbar}$ = max. conductance of single quantum channel

$G \gg G_Q$: many "transport channels" (page 1, pictures ①, ②)

$G \ll G_Q$: rare tunneling events (page 1, pictures ③, ④)

δ_s = mean level spacing (of single-particle-in-a-box-levels)

$= 1/\nu$ (ν = s.p. density of states)



E_c = charging energy (cost of adding one electron)

for atoms: $\delta_s \simeq 1\,eV$, $E_c \simeq 10\,eV$

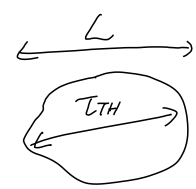
system size ↑ : δ_s and E_c ↓, with $\delta_s \ll E_c$

④

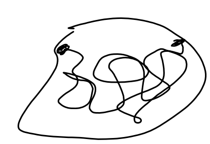
For $G \gg G_Q$: electrons don't stay in nanostructure long enough to feel δ_s or E_c. Relevant energies instead:

<u>Thouless Energy</u>: $E_{Th} = \frac{\hbar}{\tau_{Th}}$ where τ_{TH} = time to cross the nanostructure (1)

ballistic systems: $\tau_{TH} = L/v_F = \frac{\text{length}}{\text{Fermi velocity}}$ (2)



diffusive systems: $\tau_{TH} = L^2/D$ (3)



D = diffusion constant $= \frac{v_F \ell}{3}$ (ℓ = scattering length) (4)

Can be shown: $E_{Th} \simeq \delta_s \, G/G_Q$ (5)

⑤

Inelastic decay rate $= \gamma_{in}(E) = \dfrac{1}{\text{time between inelastic collisions}} = \dfrac{1}{\tau_{in}(E)}$ (1)

Classical transport happens if $\tau_{Th} \gg \tau_{in}(E)$ (2)

(system looses phase coherence before traversing sample). (3)

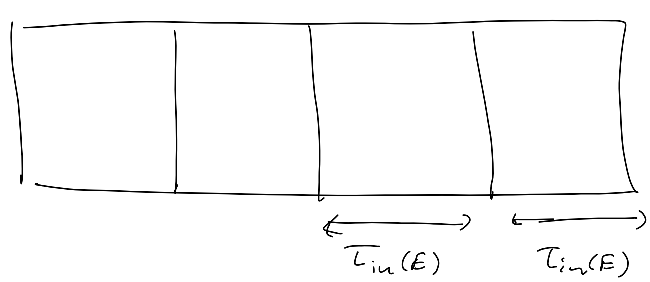
Boundary of classical region is where $\tau_{in}(E_*) = \tau_{Th}$

Can be shown that $E_* = \delta_s (g/g_Q)^2$ (4)

$= E_{TH} (g/g_Q) \geq E_{TH}$

⑥

Thouless: Think of big conductor divided in many small ones, whose traversal times are of order the inelastic time, $\tau_{in}(E)$



contributions of these boxes "self-average"; i.e. sample-specific properties don't matter.

When entire system becomes as small as one such box, i.e. when $\tau_{TH} = \tau_{in}(E)$ ("mesoscopic boundary") (1)

then there is no self-averaging, hence "mesoscopic" (sample-specific) effects begin to show up.

Chapter 1 : scattering approach ($G \gg G_Q$, $E < E_*$)

Conductor is a large scatterer, described by scattering matrix S.

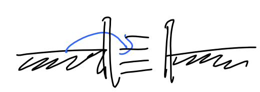
For $E < E_{Th}$, S-matrix is E-independent.

For $E_{Th} < E < E_*$, S-matrix becomes E-dependent.

Chapter 2: semi-classical coherent regime ($G \gg G_Q$)

more efficient description than scattering approach is available: self-averaging over phases of S-matrix can be done, to arrive at "quantum circuit theory" (similar in spirit to classical circuit theory)

Chapter 3: Coulomb blockade ($G \ll G_Q$)



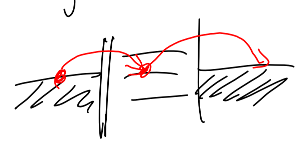
single-electron tunneling, charging energy E_c becomes important!

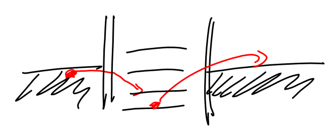
quantum-corrections to SET: cotunneling.

$E < \sqrt{\delta_S E_c}$: elastic cotunneling

(through same level)

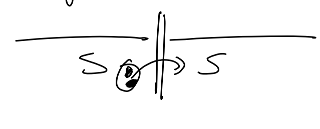
(scattering approach can be used!)





$\sqrt{\delta_S E_c} < E < E_c$: inelastic cotunneling

tunneling + superconductivity $\Rightarrow$ Josephson junctions! Restores coherence through sample!



Chapter 4: Randomness and interference

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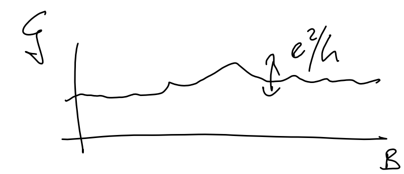
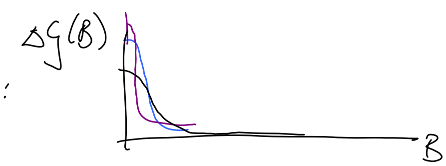
$G \ll G_Q$ and $E \simeq \delta_s$: "level statistics" (fluctuations of discrete energy levels)

$G \gg G_Q$ and $E < E_{Th}$:

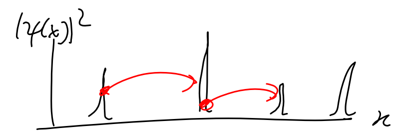


fluctuations in transmission eigenvalues "conductance fluctuations".

interference corrections: weak localization :



$G \ll G_Q$ and $E > E_{Th}$: strong localization of wave functions, electron hopping



Chapter 5: Qubits and Quantum Dots

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need discrete energy levels $\Rightarrow$ $G \ll G_Q$, $E \ll \delta_s$

quantum information & manipulation.

Chapter 6: Interaction, relaxation, decoherence

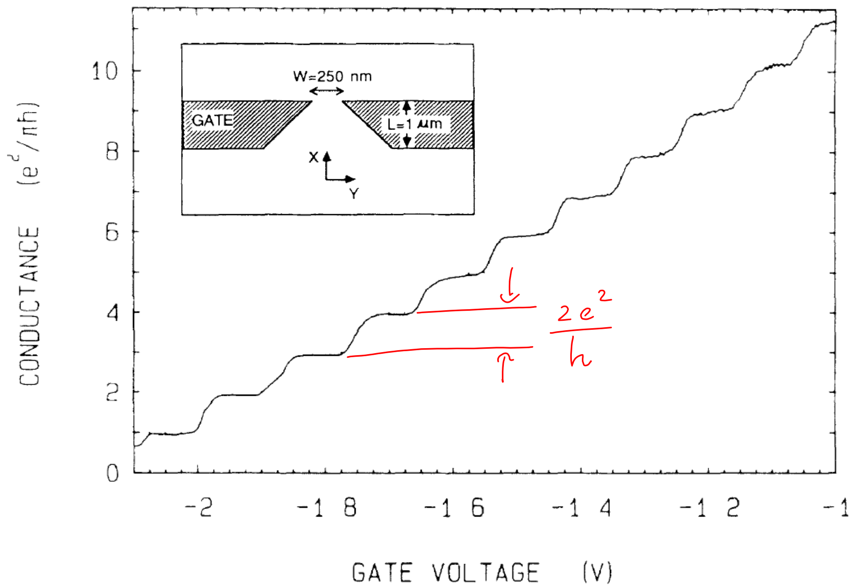
(effects beyond Coulomb blockade)

- dissipative quantum mechanics
  - $G \ll G_Q$: effect of environment on tunneling
  - $G \gtrsim G_Q$: interaction effects at higher conductance (Kondo effect, T_K depends exponentially on G).
- dephasing for qubits and electrons (at mesoscopic boundary)

1.2 Quantum Point Contacts : Conductance quantization

(11)

van Wees, B. J.; van Houten, H.; Beenakker, C. W. J.; Williamson, J. G.; Kouwenhoven, L. P.; van der Marel, D. & Foxon, C. T. Quantized conductance of point contacts in a two-dimensional electron gas Phys. Rev. Lett., 60, 848-850 (1988)

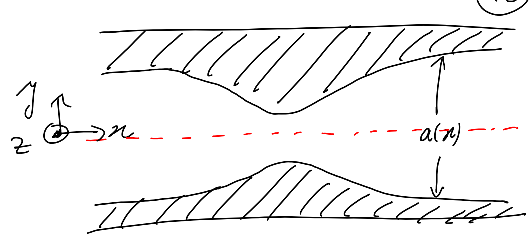


(12)

Point contact ≃ adiabatic wave guide :

"adiabatic" = changes smoothly :

$$\left|\frac{da}{dx}\right| \ll 1 \quad , \quad a(x)\left|\frac{d^2a}{dx^2}\right| \ll 1 \qquad (1)$$



Potential : $$V(x,y,z) = \begin{cases} 0 & \text{if } |y| < a(x)/2 \,,\ |z| < b(x)/2 \\ \infty & \text{otherwise.} \end{cases} \qquad (2)$$

Schrödinger eq : $$\left[-\frac{\hbar^2}{2m}\vec{\nabla}^2 + V(x,y,z) - E\right]\psi(x,y,z) = 0 \qquad (3)$$

variables separate locally : $$\psi_n(x,y,z) = \psi(x)\,\Phi_n(a(x), b(x); y, z) \qquad (4)$$

$$\left(-\frac{\hbar^2\partial_x^2}{2m} + E_n(x) - E\right)\psi(x) = 0 \qquad (5)$$

effective potential for 1D motion in x-direction.

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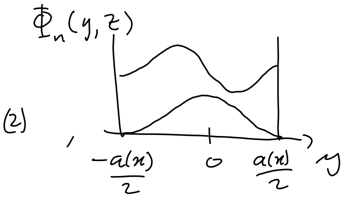
$$\left[-\frac{\hbar^2}{2m}\left(\partial_y^2+\partial_z^2\right) + V(y,z) - E_n(x)\right] \Phi_n\left(a(x), b(x), y, z\right) = 0 \qquad (1)$$

transverse modes are standing waves:

$$\Phi_n(a,b;y,z) = \frac{2}{\sqrt{ab}} \sin\left(k_y^{(n)}(y-a/2)\right) \sin\left(k_z^{(n)}(z-b/2)\right) \qquad (2)$$



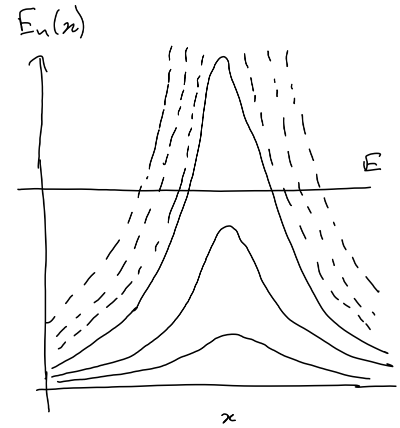
mode index: $n \equiv (n_y, n_z)$ (3)

transverse momenta:

$$k_y^{(n)} = \frac{\pi n_y}{a}, \qquad k_z^{(n)} = \frac{\pi n_z}{b} \qquad (4)$$

transverse energy:

$$E_n\left(a(x), b(x)\right) = \frac{\pi^2\hbar^2}{2m}\left(\frac{n_y^2}{a^2(x)} + \frac{n_z^2}{b^2(z)}\right) \qquad (5)$$



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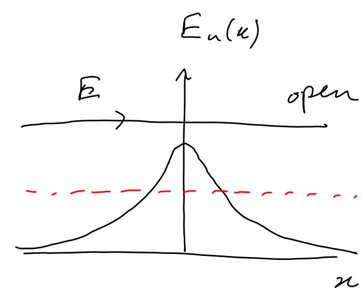
For given total energy E and given n; we have 1-d transmission problem:

If $E \underset{<}{>} \underbrace{E_n(0)}_{\text{barrier height}}$, channel is open (blocked)



open: Transmission $T \simeq 1$

closed: $T \simeq 0$

in general: $T_n(E) \in [0,1]$ (tunneling)

$\Longrightarrow$ there are only finite number of open channels!

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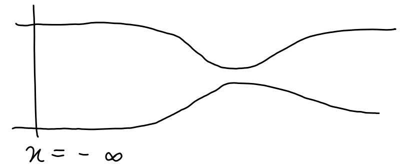
Calculating the current:



$$I = \int dy \int dz \underbrace{j(x=-\infty, y, z)}_{\text{current density, indep. of } y,z \text{ at } x=-\infty} \qquad (1)$$

$$= ab\, j = ab \underset{\uparrow\,\text{spin}}{2_s} \int \frac{d^3\vec{k}}{(2\pi)^3} \underset{\uparrow\,\text{charge}}{e}\, \underset{\uparrow\,\text{velocity}}{\vec{v}(\vec{k})}\, \underset{\uparrow\,\text{occupation function}}{f(\vec{k})} \qquad (2)$$

(13.4)

$$= ab \int \frac{dk_x}{2\pi} \frac{1}{ab} \underset{n}{\overset{\text{transverse modes}}{\sum}} e\, v_x(k_x)\, f_n(k_x) \qquad (3)$$

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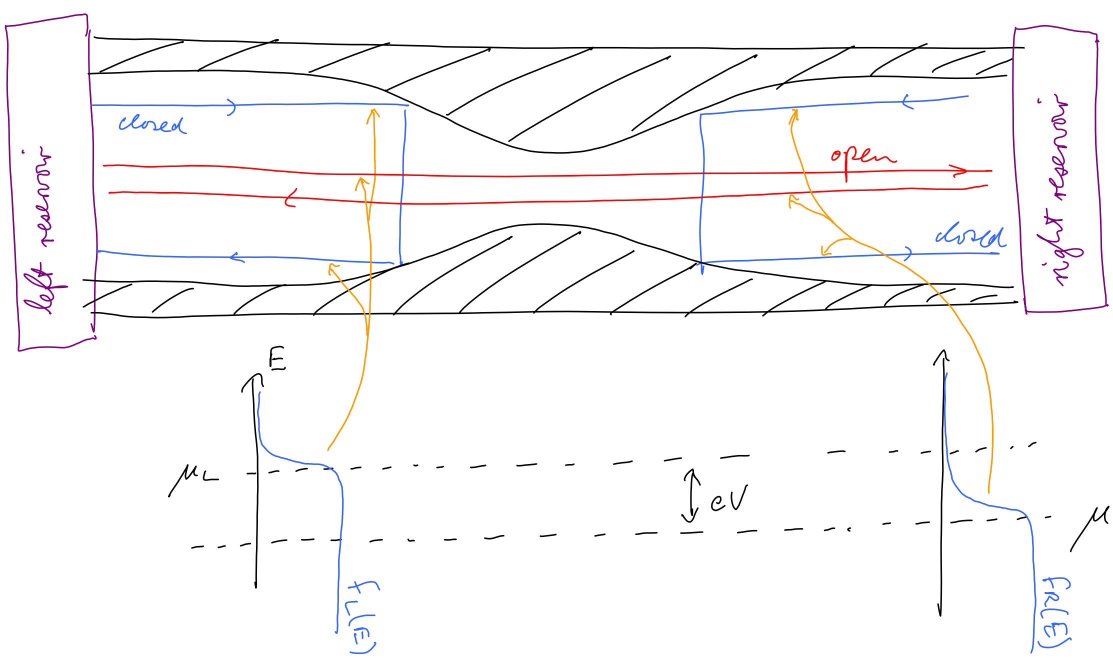

For closed channels ; $\quad f_n(k_x) = f_n(-k_x)$

no contribution, since : $\quad v_x(k_x) = -v_x(-k_x) \qquad (4)$

For open channels:

$k_x > 0$ come from left reservoir, with $\left.\right\}$ $f_L(E(k_x)) = f_F(E(k_x) - \mu_L)$ (1)

$k_x < 0$ right $f_R(E(k_x)) = f_F(E(k_x) - \mu_R)$ (2)

change variables: $\int dk_x \underbrace{v_x(k)}_{= \frac{1}{\hbar}\frac{\partial E(k_x)}{\partial k_x}} = \frac{1}{\hbar}\int dE$ (3)

(17.3) in (15.3) : $I = \frac{2s}{2\pi}\frac{e}{\hbar} \underbrace{\sum_{n \in \text{open}}}_{N_{open}} \underbrace{\int dE \left[f_L(E) - f_R(E)\right]}_{eV = \mu_L - \mu_R}$

$\hbar = \frac{h}{2\pi}$ $= \underbrace{2s\frac{e^2}{h}}_{G_Q} N_{open} V$

Conductance: $G = \frac{I}{V} = G_Q N_{open} \Longrightarrow$ conductance quantization! independent of material properties!!