Relation between $\langle\langle Q^2 \rangle\rangle$ and Current noise power $S(\omega)$:

CE17

$$S(\omega) \equiv \int_{-\infty}^{\infty} dt\, e^{i\omega(t-t')} \Big[\underbrace{\langle \hat{I}(t)\hat{I}(t') + \hat{I}(t')\hat{I}(t) \rangle}_{\text{symmetrized current correlator}} - 2 \langle \hat{I}(t) \rangle \langle \hat{I}(t') \rangle \Big] \quad (1)$$

symmetrized current correlator : $[\hat{I}(t), \hat{I}(t')] \neq 0$

$$\simeq \lim_{\Delta t \to \infty} \frac{1}{\Delta t} \int_{-\Delta t/2}^{\Delta t/2} dt' \int_{-\Delta t/2}^{\Delta t/2} dt \; \ldots \quad (2)$$

Charge passed in time Δt: $\hat{Q} = \int_{-\Delta t/2}^{\Delta t/2} dt\, \hat{I}(t)$ (3)

In limit of large Δt: $S(0) = \frac{1}{\Delta t} 2 \Big[\underbrace{\langle \hat{Q}\hat{Q} \rangle - \langle \hat{Q} \rangle \langle \hat{Q} \rangle}_{\langle\langle Q^2 \rangle\rangle} \Big]$ (4)

$\Rightarrow$ $\boxed{\langle\langle Q^2 \rangle\rangle = \frac{\Delta t}{2} S(0)}$ (5)

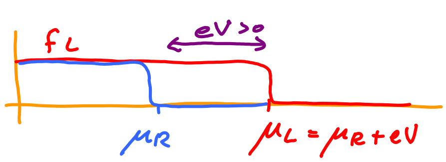
2nd. Charge cumulant $\propto$ zero frequency noise power

In equilibrium: $S(0) \overset{(16.5)}{=} 4 G k_B T$ = "Johnson-Nyquist noise" (6)

$\langle\langle Q^2 \rangle\rangle$ in "shot noise" limit : $eV \gg k_B T$

CE18

↑ electrons arrive at scatterer one by one



$f_L = \begin{cases} 1 \\ 0 \end{cases}$ for $E \lessgtr \mu_L$ (1a)

$f_R = \begin{cases} 1 \\ 0 \end{cases}$ for $E \lessgtr \mu_R$ (1b)

$$\langle\langle Q^2 \rangle\rangle \overset{(15.5)}{=} G_Q \Delta t \int dE \sum_p \Big\{ T_p \Big[\underbrace{f_L(1-f_L)}_{=0} + \underbrace{f_R(1-f_R)}_{=0} + T_p(1-T_p)\underbrace{(f_L - f_R)^2}_{\int dE \,|\, = eV} \Big] \Big\}$$

$$= G_Q \Delta t \; eV \sum_p T_p(1-T_p) \quad (2)$$

$$\boxed{S(0) \overset{(17.5)}{=} \frac{2}{\Delta t} \langle\langle Q^2 \rangle\rangle = 2 G_Q eV \sum_p T_p(1-T_p)} \quad (3)$$

fluctuations are nonzero, due to discreteness of electron charge: electrons arrive one by one!

Noise and current contain different information about T_p's !!

Compare to current: $\langle I \rangle = \frac{\langle Q \rangle}{\Delta t} = G_Q V \sum_p T_p$ (4)

Limiting cases for shot noise: $\quad S(0) \overset{(8.3)}{=} 2 G_Q eV \sum_p T_p(1-T_p) \quad (1)$ CE19

1. Tunneling limit: $T_p \ll 1$

$$S(0) \xrightarrow{(1)} 2eV \underbrace{G_Q \sum_p T_p}_{\langle I \rangle} = 2e\langle I \rangle \qquad \text{"Schottky formula"} \quad (2)$$

$\frac{S(0)}{2\langle I \rangle} = e$ { can be used to measure effective quasi-particle charge!

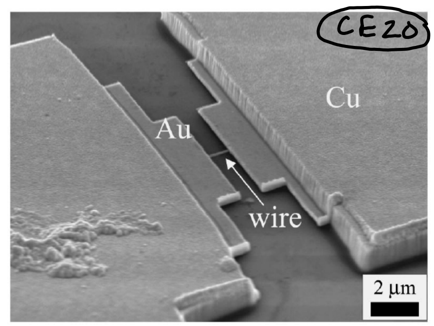
[(2) also follows from Poisson distribution $P_N \overset{(8.3)}{=} \frac{\tilde{N}^N}{N!} e^{-\tilde{N}}$ in $\langle N^2 \rangle - \langle N^2 \rangle$]

For larger T_p, $S(0) < S_{\text{Schottky}}(0)$, due to $(1-T_p)$ factors in (1).

"Fano Factor": $\quad \boxed{F \equiv \frac{S(0)}{2e\langle I \rangle} \overset{(1)}{=} \frac{\sum_p T_p(1-T_p)}{\sum_p T_p}, \quad 0 \le F \le 1} \quad (3)$

2. Quantum point contact: $\quad T_p = 0 \text{ or } 1 \;\; \forall p \;\Rightarrow\; \boxed{F_{QPC} = 0} \quad (4)$

3. Diffusive wires CE20



Henny et al, PRB 59, 2871 (1999)

Distribution of transmission eigenvalues:

$$P_{\text{diffusive}}(T) \overset{(SM\,19.1)}{=} \frac{\langle G \rangle}{2G_Q} \frac{1}{T(1-T)^{1/2}} \quad (1)$$

Recall (p. SM17): $\quad \left\langle \sum_p f(T_p) \right\rangle_{\text{dis}} \to \int_0^1 dT\, f(T)$

average over realizations of disorder

Calculate Fano factor: $\quad F \overset{(19.3)}{=} \frac{\left\langle \sum_p T_p(1-T_p) \right\rangle_{\text{dis}}}{\left\langle \sum_p T_p \right\rangle_{\text{dis}}} = \frac{2/3}{2} = \frac{1}{3}$

[prefactor $\frac{\langle G \rangle}{2G_Q}$ from (1) drops out]

$$\left\langle \sum_p T_p \right\rangle_{\text{dis}} \to \int_0^1 dT \frac{T}{T(1-T)^{1/2}} = \int_0^1 dT \frac{1}{(1-T)^{1/2}} = -2(1-T)^{1/2}\Big|_0^1 = 2$$

$$\left\langle \sum_p T_p(1-T_p) \right\rangle_{\text{dis}} \to \int_0^1 dT \frac{T(1-T)}{T(1-T)^{1/2}} = \int_0^1 dT \sqrt{1-T} = -\frac{2}{3}(1-T)^{3/2}\Big|_0^1 = \frac{2}{3}$$

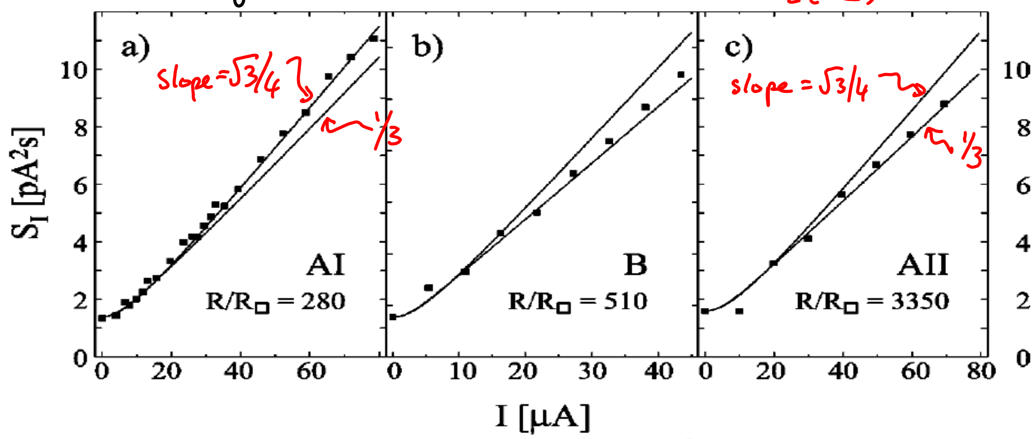
CE21

Result: diffusive wire without heating: $F = 1/3$

Can be shown: " " with heating: $F = \sqrt{3}/4$

(e-e interactions must be accounted for)

Experiment: Henny et al, PRB 59, 2871 (1999) $F = \frac{S(0)}{2e\langle I\rangle}$



Sample AI : thin leads ⇒ heating ⇒ $F = \sqrt{3}/4$

Sample AII : thick leads ⇒ no heating ⇒ $F = 1/3$

CE22

<u>Third cumulant C_3</u> : (first measured in 2003)

Equilibrium $(V=0)$: $C_3 = 0$, because for $f_L = f_R$, (1)

$\ln\Lambda(\chi)$ of (9.3) is <u>even</u> function of χ ⇒ all odd cumulants vanish.

Shot noise regime $(eV \gg k_B T)$:

$$C_3 = \langle\langle Q^3\rangle\rangle \overset{(4.4)}{=} e^3 \frac{\partial^3}{\partial^3(i\chi)} \ln\Lambda(\chi)\Big|_{\chi=0} \tag{2}$$

$$\overset{(14.6),(23.3)}{=} e^2 V g_Q \Delta t \sum_p T_p(1-T_p)(1-2T_p) \tag{3}$$

$$\langle\langle Q^3\rangle\rangle = \begin{cases} e^2\langle I\rangle\Delta t & \text{for tunnel junction} \quad (4a)\\ 0 & \text{for QPC} \quad (4b)\\ e^2\langle I\rangle\Delta t/15 & \text{for diffusive wire} \quad (4c)\end{cases}$$

Derivation of (22.3): CE23

$$\partial^3_{i\chi} \ln\{\ \} \overset{(14.6)}{=} \frac{T_p e^{i\chi} f_L(1-f_R) - T_p e^{-i\chi} f_R(1-f_L)}{\{\ \}} - 3\frac{(\ )[\ ]}{\{\ \}^2} + 2\frac{[\ ]^3}{\{\ \}^3} \quad (1)$$

$$= T_p \underbrace{(f_L - f_R)}_{=1} - \overbrace{3\,T_p\Big(\underbrace{f_L[1-f_R]}_{=1} + \underbrace{f_R[1-f_L]}_{=0}\Big)}^{(14.5)} \overbrace{T_p\underbrace{(f_L - f_R)}_{=1}}^{(14.4)} + 2\,\overbrace{T_p^3\underbrace{(f_L-f_R)^3}_{=1}}^{(14.4)^3} \quad (2)$$

Fermi function combinations are nonzero only for $E \in [\mu_R, \mu_L]$:

$$(2) = T_p - 3T_p^2 + 2T_p^3 = T_p(1 - 3T_p + 2T_p^3) = T_p(1-T_p)(1-2T_p) \quad (3)$$