MTC 14!

Using $\quad I_\alpha \overset{(5.1)}{=} \sum_\beta g_{\alpha\beta} V_\beta \quad , \quad 0 \overset{(5.2)}{=} \sum_\beta g_{\alpha\beta} \,,$

we get from

$$0 = I_3 = g_{31}V_1 + g_{32}V_2 + \overbrace{g_{33}}^{= -(g_{31}+g_{32}+\cancel{g_{34}})} V_3 + \cancel{g_{34}} V_4$$

$$0 = I_4 = g_{41}V_1 + g_{42}V_2 + \cancel{V_{43}} V_3 + \underbrace{g_{44}}_{-(g_{41}+g_{42}+\cancel{g_{43}})} V_4$$

small

$$V_3 = \frac{V_1 g_{31} + V_2 (g_{32} + g_{31} - g_{31})}{g_{31}+g_{32}} = V_2 + (V_1 - V_2)\frac{g_{31}}{g_{31}+g_{32}}$$

$$V_4 = \frac{V_1 g_{41} + V_2 (g_{42} + g_{41} - g_{41})}{g_{41}+g_{42}} = V_2 + (V_1 - V_2)\frac{g_{41}}{g_{41}+g_{42}}$$

$$V_3 - V_4 = (V_1 - V_2)\left[\frac{g_{31}}{g_{31}+g_{32}} - \frac{g_{41}}{g_{41}+g_{42}}\right] = (14.2)$$

1.5.3 Beam splitters

9.5.2010 MTC15

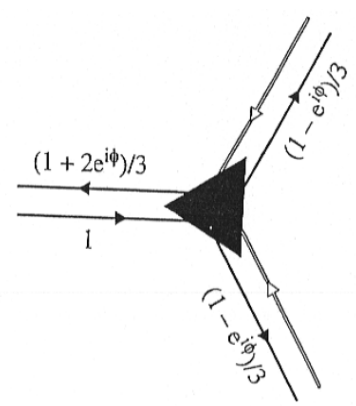
3-channel beam splitter has 3×3 $\hat{S}$-matrix

$\Rightarrow$ parametrized by 5 real numbers.

fully symmetric beam splitter:

all diagonal elements must be the same

" off-diagonal " " " " "



This fixes unitary 3×3 matrix up to a phase:

$$\hat{S} = \frac{1}{3}\begin{bmatrix} 1+2e^{i\phi} & 1-e^{i\phi} & 1-e^{i\phi} \\ 1-e^{i\phi} & 1+2e^{i\phi} & 1-e^{i\phi} \\ 1-e^{i\phi} & 1-e^{i\phi} & 1+2e^{i\phi} \end{bmatrix} \qquad (1)$$

reflection in same lead: $\quad R = \frac{1}{9}|1+2e^{i\phi}|^2 = \frac{1}{9}(5+4\cos\phi)\,, \qquad (2)$

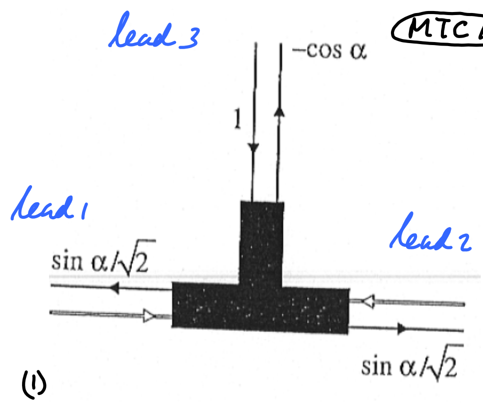
transmission into other lead: $\quad T = \frac{1}{9}|1-e^{i\phi}| = \frac{1}{9}(2-2\cos\phi) \qquad (3)$

$\phi = 0: \quad R = 1 \,, \quad T = 0 \quad \Rightarrow$ complete reflection

$\phi = \pi: \quad R = 1/9 \,, \quad T = 4/9 \quad \Rightarrow$ maximal transmission $\qquad (4)$

T-beam splitter

symmetric w.r.t. lead 1 ⟷ lead 2



MTC 16

$$\hat{S} = \begin{bmatrix} -\sin^2\alpha/2 & \cos^2\alpha/2 & \frac{1}{\sqrt{2}}\sin\alpha \\ \cos^2\alpha/2 & -\sin^2\alpha/2 & \frac{1}{\sqrt{2}}\sin\alpha \\ \frac{1}{\sqrt{2}}\sin\alpha & \frac{1}{\sqrt{2}}\sin\alpha & -\cos\alpha \end{bmatrix} \quad (1)$$

α parametrizes coupling between lead 3 and leads 1-2

$\alpha = 0$: 3 is uncoupled, $\left\{ \begin{matrix} 1 \to 2 \\ 2 \to 1 \end{matrix} \right\}$ without any scattering: $\hat{S}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ (2)

$\alpha = \pi/2$:

$$\hat{S} = \begin{bmatrix} -1/2 & 1/2 & 1/\sqrt{2} \\ 1/2 & -1/2 & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix}$$

Ideal beamsplitter:
electron incident from 3 is equally divided between 1 & 2
no back-reflection into 3

$\hat{S}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ (3)

1.5.4 Counting statistics & noise for Multiterminal device:

MTC 17

$P(Q_1, Q_2, \ldots, Q_N)$ = prob. that Q_α passes from scattering region into lead α in time Δt

$$\Lambda(\{\chi_\alpha\}) = \sum_{Q_1,\ldots,Q_N} P(\{N_\alpha\})\, e^{i(\chi_1 Q_1 + \ldots \chi_N Q_N)/e} \quad (1)$$

current conservation: $\sum_N Q_N = 0 \Rightarrow \Lambda$ depends only on differences $\chi_\alpha - \chi_\beta$

Multiterminal Levitov formula:

$$\ln(\Lambda\{\chi_\alpha\}) = 2_s \Delta t \int \frac{dE}{2\pi\hbar} \mathrm{Tr} \ln\left\{1 + \hat{f} + \hat{f}\hat{S}^\dagger\hat{\tilde{S}}\right\} \quad (2)$$

$$\tilde{S}_{\alpha\beta} = S_{\alpha\beta} e^{i(\chi_\alpha - \chi_\beta)}, \quad \hat{f}_{\alpha\beta} = \delta_{\alpha\beta} f_\alpha(E), \quad \mathrm{Tr} \text{ is } \underbrace{\sum_\alpha}_{\text{leads}} \underbrace{\sum_n}_{\text{channels}} \quad (3)$$

Exercise: for 2-terminal device, with

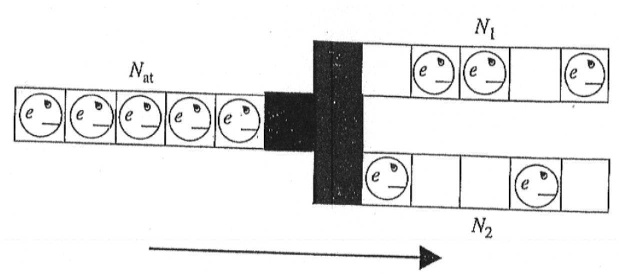
$$\hat{S} = \begin{bmatrix} \sqrt{R}\, e^{i\theta} & \sqrt{T}\, e^{i\eta} \\ \sqrt{T}\, e^{i\eta} & -\sqrt{R}\, e^{i(2\eta-\theta)} \end{bmatrix}, \qquad \text{(see p. SM14)} \qquad (1)$$

show that (17.2) reproduces 2-terminal Levitov-formula (CE9.3):

$$\ln\Lambda(\chi) = \frac{g_Q \Delta t}{e^2}\int dE \sum_p \ln\left\{1 + T_p\left(e^{i\chi}-1\right) f_L(1-f_R) + T_p\left(e^{-i\chi}-1\right) f_R[1-f_L]\right\} \qquad (2)$$

where $\chi = \chi_L - \chi_R$. $\quad$ [Hint: $\mathrm{Tr}\log M = \log(\det M)$] $\qquad (3)$

Reflectionless T-beam splitter:



$T = 0, \quad V_1, V_2 = 0, \quad V_3 = V > 0$.

$$\Rightarrow \quad \Lambda(\chi_1, \chi_2, \chi_3) = \left(\tfrac{1}{2} e^{i\chi_2} + \tfrac{1}{2} e^{i\chi_3}\right)^{N_{at}} e^{-i N_{at}\chi_3} \qquad (1)$$

$N_{at} = \Delta t\, g_Q V/e$ = # of electrons coming from lead 3 in time Δt.

Distribution of charges $Q_i = e N_i$:

$$P_{N_1,N_2,N_3} = \underbrace{\delta(N_3 + N_{at})}\ \underbrace{\delta(N_1 + N_2 - N_3)} \binom{N_{at}}{N_1} \underbrace{\left(\tfrac{1}{2}\right)^{N_1}\left(\tfrac{1}{2}\right)^{N_2}} \qquad (2)$$

[current from 3 does not fluctuate: every attempt to inject an electron from 3 is successful] (no reflection)

[Every injected electron from 3 exits either at 1 or 2, with equal probability of ½ each]

$\Rightarrow$ fluctuations of N_1 and N_2 are opposite, since $N_1 + N_2 = N_3$ = fixed.

MTC20

More generally, it can be shown for arbitrary beam splitter that current correlations at different terminals are <u>negative</u> at $\omega = 0$:

$$S_{\alpha\beta}(\omega) = \left\langle\left\langle \hat{I}_\alpha(t)\,\hat{I}_\beta(t') + \hat{I}_\beta(t')\,\hat{I}_\alpha(t) - 2\langle I_\alpha(t)\rangle\langle I_\beta(t')\rangle \right\rangle\right\rangle_\omega$$

$$\langle\ \rangle_\omega = \int dt\, e^{i\omega t}$$

Then
$$S_{\alpha\beta}(0) = \underbrace{-2 g_0 \int dE}_{<\,0 \text{ strictly!}}\ \underbrace{\mathrm{Tr}\Big[\overbrace{\sum_\gamma \underbrace{\hat{s}^{\dagger}_{\gamma\alpha}\,\hat{s}_{\beta\gamma}}_{s^*_{\alpha\gamma}\, s^{*\dagger}_{\gamma\beta}}\, f_\gamma}^{M_{\alpha\beta}}\Big]\Big[\overbrace{s^{\dagger}_{\delta\beta}\, s_{\alpha\delta} f_\delta}^{M^*_{\alpha\beta}}\Big]}_{\geq\,0 \text{ strictly!}}$$

<u>Anti</u>correlations in noise at T-beam splitter reflects fact that electrons are fermions: electrons come in <u>one</u> by <u>one</u>, each is either reflected or transmitted!

MTC21

1.5.5 Multi-terminal scattering in operator formalism

Straightforward generalization of 2-terminal case.

Final result for t-averaged current operator in terminal α:

(compare SFo 9.3)

$$\langle \hat{I}_\alpha \rangle_t = -\frac{e}{2\pi\hbar}\left(\frac{2\pi\hbar}{T}\right)^2 \sum_{n\sigma}\sum_E \Big[\overset{\text{incoming}}{\hat{a}^{\dagger}_{\alpha n\sigma}(E)\,\hat{a}_{\alpha n\sigma}(E)} - \overset{\text{outgoing}}{\hat{b}^{\dagger}_{\alpha n\sigma}(E)\,\hat{b}_{\alpha n\sigma}(E)} \Big] \quad (1)$$

$$= -\frac{e}{2\pi\hbar}\left(\frac{2\pi\hbar}{T}\right)^2 \sum_{n\sigma}\sum_{\beta\gamma}\sum_{\ell\ell'}\sum_E \hat{a}^{\dagger}_{\beta\ell\sigma}(E)\,\hat{a}_{\gamma\ell'\sigma}(E)\ \times$$

$$\Big[\delta_{\alpha\beta}\,\delta_{\alpha\gamma}\,\delta_{n\ell}\,\delta_{n\ell'} - s^{*}_{\beta\ell,\alpha n}(E)\, s_{\alpha n,\gamma\ell'}(E)\Big] \quad (2)$$

with $$\langle \hat{a}^{\dagger}_{\alpha n\sigma}(E)\,\hat{a}_{\alpha' n'\sigma'}(E)\rangle = \delta_{\alpha\alpha'}\,\delta_{nn'}\,\delta_{\sigma\sigma'}\,\delta(E-E') \quad (3)$$