(10.3) in (7.2): AS11

$$\begin{pmatrix} -i\hbar v_F \partial_x & \Delta(x) e^{i\varphi} \\ \Delta(x) e^{-i\varphi} & i\hbar v_F \partial_x \end{pmatrix} \begin{pmatrix} \tilde{\psi}_e(x) \\ \tilde{\psi}_h(x) \end{pmatrix} = E \begin{pmatrix} \tilde{\psi}_e(x) \\ \tilde{\psi}_h(x) \end{pmatrix} \quad (1)$$

For normal metal ($x<0$, $\Delta=0$):

$$\tilde{\psi}(x<0) = \begin{pmatrix} e^{ixE/v_F\hbar} \\ r_A\, e^{-ixE/v_F\hbar} \end{pmatrix} \begin{matrix} \text{incident electron} \\ \text{reflected hole} \end{matrix} \quad (2)$$

r_A: Andreev reflection amplitude

For SC ($x>0$, $\Delta \neq 0$):

$E<\Delta$ has only evanescent solution:

$$\tilde{\psi}(x>0) = C \begin{pmatrix} f_e \\ f_h \end{pmatrix} e^{-x\kappa}, \quad \text{with } \kappa = \frac{\sqrt{\Delta^2-E^2}}{\hbar v_F} \quad (3)$$

$$|f_e|^2 + |f_h|^2 = 1$$

Check: $(+i\hbar v_F \kappa - E)(-i\hbar v_F \kappa - E) - \Delta^2 = 0.$

$$\Rightarrow (\hbar v_F)^2\kappa^2 + E^2 - \Delta^2 = 0 \quad \Rightarrow \kappa = \pm\sqrt{\Delta^2-E^2}/\hbar v_F \quad (4)$$

AS12

Scale of penetration into SC:

$$\xi = \frac{1}{\kappa} = \frac{\hbar v_F}{\sqrt{\Delta^2-E^2}} \quad (1)$$

$$\begin{cases} \gg \lambda_F = \frac{\hbar v_F}{E_F} & \text{for } E = 0 \\ \to \infty & \text{for } E \to \Delta \end{cases} \quad (2)$$

Match solutions (11.2) & (11.3) at $x=0$: $\tilde{\psi}(x\to 0^-) = \tilde{\psi}(x \to 0^+)$

($\partial_x\psi$ does not have to be continuous, since (11.1) contains only 1st. derivatives)

One finds: $$r_A = e^{-i\varphi}\left(\frac{E}{\Delta} - i\frac{\sqrt{\Delta^2-E^2}}{\Delta}\right) \equiv e^{i\chi} \quad (3)$$

$$\boxed{\chi = -\arccos(E/\Delta) - \varphi} \quad (4)$$

$$\Rightarrow |r_A|^2 = 1 \quad (5)$$

$\Rightarrow$ electron is fully reflected as a hole, with relative phase shift χ

Similarly: incident hole is reflected as electron, with $|r_A| = 1$

$$\boxed{\tilde{\chi} = -\arccos(E/\Delta) + \varphi} \quad (6)$$

Derivation of (12.3), (12.4): (AS13)

$\tilde\psi(x\to 0^-) = \tilde\psi(x\to 0^+) \Rightarrow$ (1)

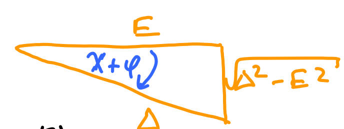
$\begin{pmatrix}1\\ r_A\end{pmatrix} \overset{(11.2)\,=\,(11.3)}{=} c\begin{pmatrix}f_e\\ f_h\end{pmatrix} \Longrightarrow c = 1/f_e \Rightarrow \boxed{r_A = \frac{f_h}{f_e}}$ (2)

Find f_h/f_e from eigenvalue eq. for $\tilde\psi(x\to 0^+)$: (11.3) into (11.1):

$\begin{pmatrix} i\hbar v_F\kappa & \Delta e^{i\varphi} \\ \Delta e^{-i\varphi} & -i\hbar v_F\kappa \end{pmatrix}\begin{pmatrix}f_e\\ f_h\end{pmatrix} = E\begin{pmatrix}f_e\\ f_h\end{pmatrix}$  (3)

$\Rightarrow (+i\hbar v_F\kappa - E)f_e = \underline{-\Delta e^{i\varphi}} f_h$ with $\kappa \overset{(11.3)}{=} \frac{\sqrt{\Delta^2-E^2}}{\hbar v_F}$ (4)

$r_A = \frac{f_h}{f_e} = e^{-i\varphi}\frac{1}{\Delta}\left[E - i\sqrt{\Delta^2-E^2}\right] \equiv e^{i\chi}$ (5)

$|r_A|^2 = \frac{1}{\cancel{\Delta^2}}\left[\cancel{E^2} + \cancel{\Delta^2} - \cancel{E^2}\right] = 1$ (6) 

$\chi + \varphi = -\arccos(E/\Delta) \quad (\in[-\pi/2, 0])$ (7)

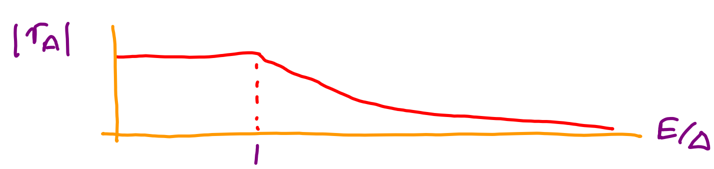
For $E > \Delta$: (AS14)

$\tilde\psi(x>0) = c\begin{pmatrix}f_e\\ f_h\end{pmatrix}e^{ikx}$, with $k = \sqrt{E^2+\Delta^2}/v_F\hbar$ (1)

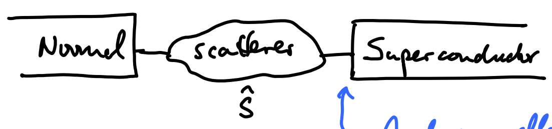
$\Rightarrow \quad r_A = e^{-i\varphi}\left(\frac{E}{\Delta} - \frac{\sqrt{E^2-\Delta^2}}{\Delta}\right)$ (2)

$|r_A|^2 < 1 \Longrightarrow$ single electrons can penetrate SC (3)

$r_A \to e^{-i\varphi}\,\frac{1}{2}\,\frac{\Delta}{E}$ for $E/\Delta \gg 1$ (4)



1.8.2 Andreev conductance

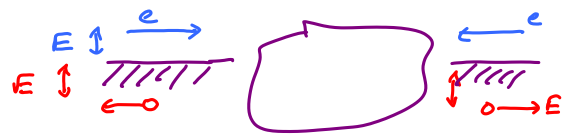


AS 16

Andreev reflection occurs upon hitting this boundary

Scatterer in normal state described by scattering matrix

$$\hat{S}(E) = \begin{pmatrix} r(E) & t'(E) \\ t(E) & r'(E) \end{pmatrix}$$



Scattering matrix for electrons with $E > 0$: $\hat{S}_e(E) = \hat{S}(E)$ (1)

[connects $(\hat{c}_E)_{in}$ to $(\hat{c}_E)_{out}$] $\Rightarrow$ $r_e = r(E)$, $t_e = t(E)$ (2)

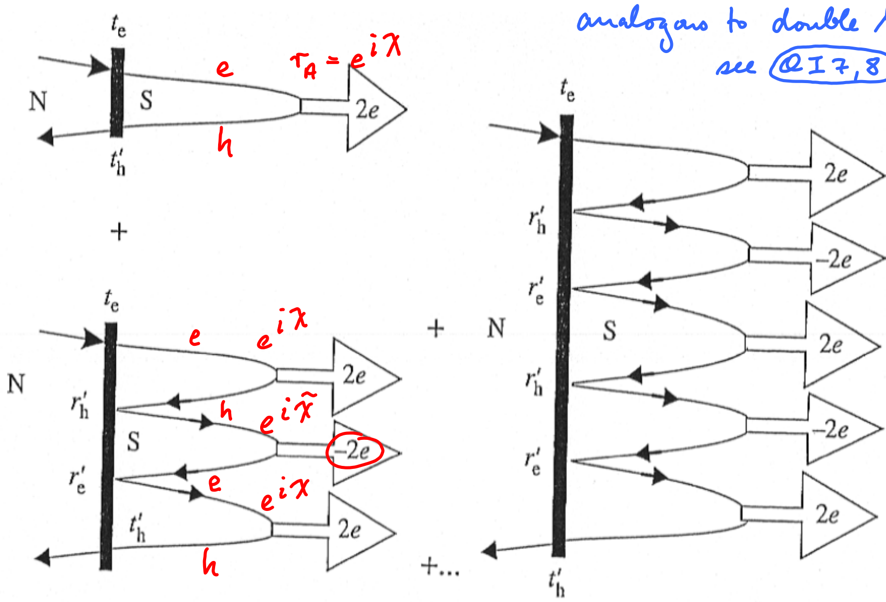
Scattering matrix for holes with $E > 0$: $\hat{S}_h(E) = \hat{S}^*(-E)$ (3)

[connects $(\hat{c}^\dagger_{-E})_{in}$ to $(\hat{c}^\dagger_{-E})_{out}$] $\Rightarrow$ $r_h = r^*(-E)$, $t_h = t^*(-E)$ (4)

Multiple Andreev reflections are possible:

AS 16

analogous to double barrier, see QI 7,8



Andreev conductance. The amplitude of the Andreev equation to the normal (N) lead from the nanostructure adjacent to a superconductor (S) is contributed by the processes that differ in the number of electron trips between the nanostructure and superconductor.

Andreev reflection for $E < \Delta$

AS17

amplitude for 0 intermediate Andreev reflections:

$$A_0 = t_e e^{i\chi} t'_h = r_{A0}$$

phase factor from Andreev reflection of electron or of hole

$$1 \quad : \quad A_1 = t_e e^{i\chi} r'_h e^{i\tilde{\chi}} r'_e e^{i\chi} t'_h \tag{2}$$

$$m \quad : \quad A_m = t_e t'_h e^{i\chi} \left(r'_h r'_e e^{i(\chi+\tilde{\chi})} \right)^m \tag{3}$$

total amplitude for Andreev reflection:

$$r_A = \sum_{m=0}^{\infty} A_m = t_e t'_h e^{i\chi} \sum_{m=0}^{\infty} \left(r'_h r'_e e^{i(\chi+\tilde{\chi})} \right)^m \tag{4}$$

$$= \frac{t_e t'_h e^{i\chi}}{1 - r'_h r'_e e^{i(\chi+\tilde{\chi})}} \tag{5}$$

total phase accumulated

Simplifying assumptions:

AS18

- neglect E-dependence : $\hat{S}(E) \equiv \hat{S}$, then $\begin{cases} t_e = t_h^* \equiv t \\ r_e = r_e^* \equiv r \end{cases}$, etc. (1)

- low voltage, $eV \ll \Delta$: then $\chi + \tilde{\chi} \overset{(12.4,6)}{=} \underbrace{-2 \arccos \underbrace{E/\Delta}_{\simeq 0}}_{\pi/2} = -\pi$ (2)

$$\Rightarrow \quad r_A = \frac{t(t')^* e^{i\chi}}{1 - \underbrace{r' r'^*}_{R = 1-T} e^{-i\pi}} = \frac{t t'^* e^{i\chi}}{2 - T} \tag{3}$$

Andreev reflection coefficient: (hole scatters back)

$$\boxed{R_A = |r_A|^2 = \frac{T^2}{(2-T)^2}} \tag{4}$$

Normal reflection coefficient (electron scatters back) :

$$R_N = 1 - R_A \tag{5}$$

In limit of ideal contact ($T=1$), we recover $R_A = 1$ $\quad R_N = 0$ (cf. 12.5)

AR19

Andreev conductance:

Fraction R_N of normally reflected electrons do not contribute to current.

Fraction R_A of Andreev-reflected electrons each contribute charge $2e$ to current

$\Rightarrow$ Andreev conductance : $G_A = 2 G_Q R_A$ (1)

Counting statistics for Andreev junction: $e \to 2e$, $T \to R_A$ (2)

For many channels: $G_A = 2 G_Q \sum_p (R_A)_p \overset{(18.4)}{=} 2 G_Q \sum_p \frac{T_p^2}{(2-T_p)^2}$ (3)

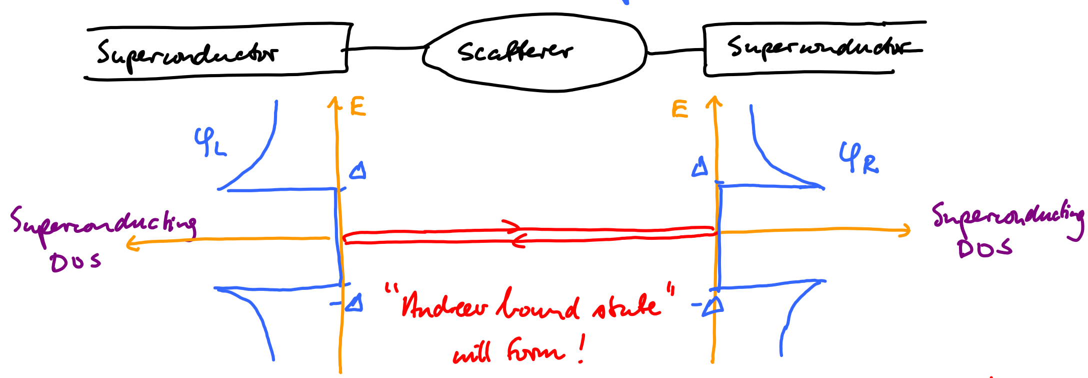
For tunnel junction ($T_p \ll 1$) : $G_A = \frac{1}{2} G_Q \sum_p T_p^{(2)}$ need transmission of electron and hole (3)

For ideal contact ($T_p \simeq 1$) : $G_A = 2 G_Q \sum_p 1 = 2 G_N$ (4)

2 electrons transferred at a time

AR20

1.8.3 Andreev bound states (normal or superconductor)



Assume scatterer so short that dwell time T_d satisfies $T_d < \frac{\hbar}{\Delta}$ (1)

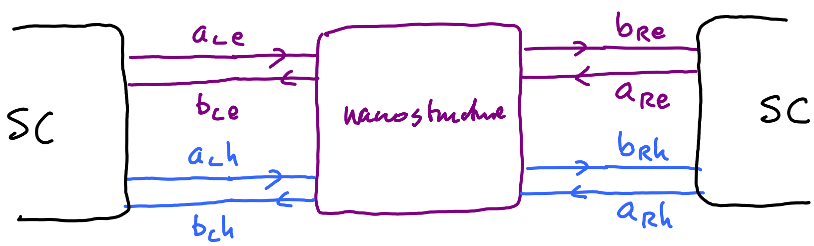
Then electrons spend so little time in scatterer that they do not "notice" whether it is normal or superconducting. $\Rightarrow$ $\hat{S} = \hat{S}_{normal}$ (2)

At each junction, electrons and holes will be Andreev reflected, i.e., they "cannot get out" $\Rightarrow$ "Andreev bound states"

with binding energy $E = \Delta \left[1 - T \sin^2(\varphi_L - \varphi_R)/2\right]^{1/2}$ (3)

AR21

Single channel:



$\hat{S}$ matrix relates in and out states:

electrons: $$\underline{b}_e = \begin{pmatrix} b_L \\ b_R \end{pmatrix}_e = \hat{S} \begin{pmatrix} a_L \\ a_R \end{pmatrix}_e = \hat{S}\, \underline{a}_e , \qquad (1)$$

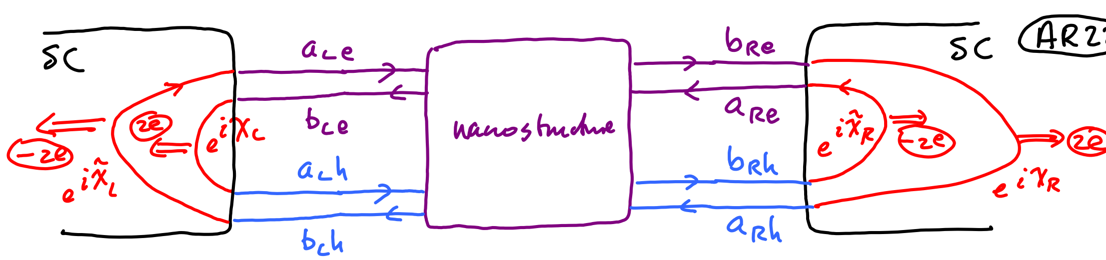
holes: $$\underline{b}_h = \begin{pmatrix} b_L \\ b_R \end{pmatrix}_h = \hat{S}^* \begin{pmatrix} a_L \\ a_R \end{pmatrix}_h = \hat{S}^*\, \underline{a}_h \qquad (2)$$

combined S-matrix for nanostructure:

[Block-diagonal, since it does not convert electrons to holes]

$$\boxed{\begin{pmatrix} \underline{b}_e \\ \underline{b}_h \end{pmatrix} = \begin{pmatrix} \hat{S} & 0 \\ 0 & \hat{S}^* \end{pmatrix} \begin{pmatrix} \underline{a}_e \\ \underline{a}_h \end{pmatrix}} \qquad (3)$$

AR22



Andreev reflection:

electrons ← holes: $\tilde{\chi}_{L,R} \overset{(12.6)}{=} \varphi_{L,R} - \arccos E/\Delta$

$$\underline{a}_e = \begin{pmatrix} a_{Le} \\ a_{Re} \end{pmatrix} = \begin{pmatrix} e^{i\tilde{\chi}_L} & 0 \\ 0 & e^{i\tilde{\chi}_R} \end{pmatrix} \begin{pmatrix} b_{Lh} \\ b_{Rh} \end{pmatrix} = \hat{S}_{eh}\, \underline{b}_h , \qquad (1)$$

holes ← electrons: $\chi_{L,R} \overset{(12.4)}{=} -\varphi_{L,R} - \arccos E/\Delta$

$$\underline{a}_h = \begin{pmatrix} a_{Lh} \\ a_{Rh} \end{pmatrix} = \begin{pmatrix} e^{i\chi_L} & 0 \\ 0 & e^{i\chi_R} \end{pmatrix} \begin{pmatrix} b_{Le} \\ b_{Re} \end{pmatrix} = \hat{S}_{he}\, \underline{b}_e , \qquad (2)$$

Combined:

$$\boxed{\begin{pmatrix} \underline{a}_e \\ \underline{a}_h \end{pmatrix} = \begin{pmatrix} 0 & \hat{S}_{eh} \\ \hat{S}_{he} & 0 \end{pmatrix} \begin{pmatrix} \underline{b}_e \\ \underline{b}_h \end{pmatrix}} \qquad (3)$$

AR23

Insert (22.3) into (21.3) to eliminate $\underline{a}$:

$$\begin{pmatrix} \underline{b}_e \\ \underline{b}_h \end{pmatrix} \overset{(21.3)}{=} \begin{pmatrix} \hat{s} & 0 \\ 0 & \hat{s}^* \end{pmatrix} \begin{pmatrix} \underline{a}_e \\ \underline{a}_h \end{pmatrix} \overset{(22.3)}{=} \underbrace{\begin{pmatrix} \hat{s} & 0 \\ 0 & \hat{s}^* \end{pmatrix} \begin{pmatrix} 0 & \hat{s}_{eh} \\ \hat{s}_{he} & 0 \end{pmatrix}}_{\equiv} \begin{pmatrix} \underline{b}_e \\ \underline{b}_h \end{pmatrix} \qquad (1)$$

$$b = \hat{\Pi}\, b \qquad \hat{\Pi} \equiv \begin{pmatrix} 0 & \hat{s}\,\hat{s}_{he} \\ \hat{s}^*\,\hat{s}_{eh} & 0 \end{pmatrix} \leftarrow \left\{ \begin{array}{l} \text{must have one} \\ \text{eigenvalue} = 1 \end{array} \right.$$

$$\Rightarrow \quad 0 = \det(\hat{\Pi} - 1) \qquad \text{(this eq. determines allowed values of } E) \qquad (2)$$

$$= \det \begin{pmatrix} -1 & \hat{s}\,\hat{s}_{eh} \\ \hat{s}^*\,\hat{s}_{he} & -1 \end{pmatrix} = (-1)\det\left(-1 + \hat{s}^*\,\hat{s}_{he}\,\hat{s}\,\hat{s}_{eh}\right) \qquad (3)$$

$$\left[\text{we used: } \det\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \det A \,\det(D - CA^{-1}B) \right] \qquad (4)$$

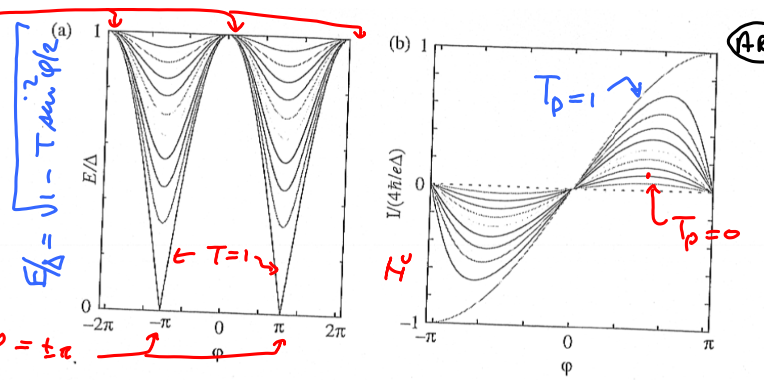
Condition (3) ultimately yields:
(see below, 26.4)

$$\boxed{\begin{array}{l} E = \Delta\sqrt{1 - T\sin^2 \varphi/2} \\ \text{where} \quad \varphi = \varphi_L - \varphi_R = \end{array}} \qquad (5)$$

phase difference between L, R superconductor.

AR24

- If $\varphi = 0$,
  for any T_p
  bound states are at edge of continuum



- Minimum of E_p at $\varphi = \pm\pi$

(a) The energies of Andreev bound states versus φ for T_p, ranging from 0.1 (upper curve) to 1 (lowest curve) with step 0.1. (b) Corresponding superconducting currents (the upper curve at positive φ corresponds to $T_p = 1$).

For many channels, there is a bound state for each channel, with energy:
$$E_p = \Delta\sqrt{1 - T_p \sin^2 \varphi/2} \qquad (1)$$

So far, we considered bound states with $E_p > 0$: excitations. (2)

There are also " " " $E_p < 0$: filled, in ground state

They contribute to ground state energy:
$$E_g = \varphi\text{-independent part} - \sum_p E_p(\varphi) \qquad (3)$$

<u>Derivation of (23.5)</u> **AR25**

$$\hat S^* \hat S_{he}\, \hat S\, \hat S_{eh} = \begin{pmatrix} r^* & t'^* \\ t^* & r'^* \end{pmatrix} \overset{(22.2)}{\begin{pmatrix} e^{i\chi_L} & 0 \\ 0 & e^{i\chi_R} \end{pmatrix}} \begin{pmatrix} r & t' \\ t & r' \end{pmatrix} \overset{(22.1)}{\begin{pmatrix} e^{i\tilde\chi_L} & 0 \\ 0 & e^{i\tilde\chi_R} \end{pmatrix}} \qquad (1)$$

$$= \begin{pmatrix} r^* e^{i\chi_L} & t'^* e^{i\chi_R} \\ t^* e^{i\chi_L} & r'^* e^{i\chi_R} \end{pmatrix} \begin{pmatrix} r e^{i\tilde\chi_L} & t' e^{i\tilde\chi_R} \\ t e^{i\tilde\chi_L} & r' e^{i\tilde\chi_R} \end{pmatrix} \qquad (2)$$

$$= \begin{vmatrix} (R e^{i\chi_L} + t'^* t e^{i\chi_R}) e^{i\tilde\chi_L} & (r^* t' e^{i\chi_L} + t'^* r' e^{i\chi_R}) e^{i\tilde\chi_R} \\ (t^* r\, e^{i\chi_L} + r'^* t e^{i\chi_R}) e^{i\tilde\chi_L} & (t^* t' e^{i\chi_L} + R e^{i\chi_R}) e^{i\tilde\chi_R} \end{vmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \qquad (3)$$

$$0 = \det(\hat\Pi - 1) = -\det\left(-1 + \hat S^* \hat S_{he}\, \hat S\, \hat S_{eh}\right) = -\begin{vmatrix} a-1 & b \\ c & d-1 \end{vmatrix} = -(ad+1) + a + d + bc \qquad (4)$$

$$\Longrightarrow \quad \underline{(bc - ad) + a + d = 1}$$

Recall: $\begin{pmatrix} \hat r & \hat t' \\ \hat t & \hat r' \end{pmatrix} = \begin{pmatrix} \sqrt{R}\, e^{i\theta} & \sqrt{T}\, e^{i\eta} \\ \sqrt{T}\, e^{i\eta} & -\sqrt{R}\, e^{i(2\eta - \theta)} \end{pmatrix} \Rightarrow \left.\begin{aligned} r r^* &= r' r'^* = R \\ t t'^* &= t'^* t = T \\ r r' t^* t'^* &= -RT \\ r^* r'^* t t' &= -RT \end{aligned}\right\} \quad (5)$

AR26

$$ad = \left[(T^2 + R^2) e^{i(\chi_L + \chi_R)} + RT\,(e^{2i\chi_L} + e^{2i\chi_R})\right] e^{i(\tilde\chi_L + \tilde\chi_R)} \qquad (1)$$

$\Updownarrow$ cancel $\Updownarrow$

$$bc = \left[(-RT - RT)\, e^{i(\chi_L + \chi_R)} + RT\,(e^{2i\chi_L} + e^{2i\chi_R})\right] e^{i(\tilde\chi_L + \tilde\chi_R)} \qquad (2)$$

$$a + d = R\left[e^{i(\chi_L + \tilde\chi_L)} + e^{i(\chi_R + \tilde\chi_R)}\right] + T\left[\underbrace{e^{i(\chi_R + \tilde\chi_L)}}_{\sim e^{i\varphi}} + \underbrace{e^{i(\chi_L + \tilde\chi_R)}}_{\sim e^{-i\varphi}}\right] \qquad (3)$$

$$\tilde\chi_{L,R} \overset{(12.6)}{=} \varphi_{L,R} - \arccos E/\Delta \quad , \qquad \chi_{L,R} \overset{(12.4)}{=} -\varphi_{L,R} - \arccos E/\Delta \qquad (4)$$

$$\left.\begin{aligned} \chi_L + \hat\chi_L \\ \chi_R + \tilde\chi_R \end{aligned}\right\} = -\underbrace{2\arccos E/\Delta}_{\equiv\theta} \qquad \begin{aligned} \tilde\chi_L + \chi_R &= \overbrace{\varphi_L - \varphi_R}^{\equiv\varphi} - 2\arccos E/\Delta \\ \tilde\chi_R + \chi_L &= \underbrace{\varphi_R - \varphi_L}_{-\varphi} - 2\arccos E/\Delta \end{aligned} \qquad (5)$$

$$e^{i(\chi_L + \hat\chi_L)} = e^{-2i\arccos E/\Delta} = e^{-i2\theta}$$

$$ad - bc = \underbrace{(T^2 + R^2 + 2RT)}_{(T+R)^2 = 1} \underbrace{e^{i(\chi_L + \tilde\chi_L)} e^{i(\chi_R + \tilde\chi_R)}}_{e^{-2i\theta}} \qquad (6)$$

$$1 - \cos\varphi = 2\sin^2 \varphi/2$$

$$a + d = \left[(1 - T)\, 2 + 2T\cos\varphi\right] e^{-i\theta} = 2\left(1 - 2T\sin^2 \varphi/2\right) e^{-i\theta} \qquad (7)$$

AR26

$$0 \overset{(25.4)}{=} ad - bc - (a+d) + 1 = e^{-2i\theta} - 2\left(1 - 2T\sin^2\varphi/2\right)e^{-i\varphi} + 1 \tag{1}$$

$$\Rightarrow \quad \cos\theta = 1 - 2T\sin^2\varphi/2 \tag{2}$$

$$1 - T\sin^2\varphi/2 = \tfrac{1}{2}(\cos\theta + 1) = \cos^2\theta/2 = \left(\cos(\arccos E/\Delta)\right)^2 = \frac{E}{\Delta} \tag{3}$$

$$\Longrightarrow \quad \boxed{E = \Delta\sqrt{1 - T\sin^2\varphi/2}} \qquad \square \tag{4}$$

1.8.5 Josephson effect : Supercurrent flows without voltage

AR28

Relation between potential and s.c. phase, for bulk s.c.

Consider: $\hat{H}_{BCS} \rightarrow \hat{H}_{BCS} - eV\hat{N}$ constant uniform potential (1)

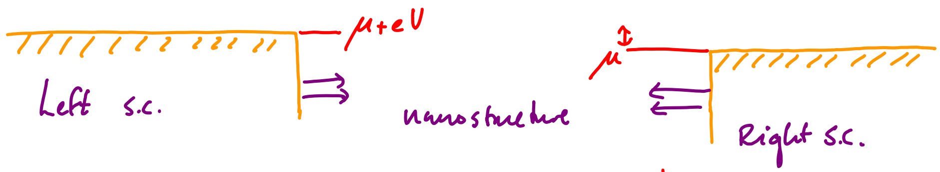
single-particle wave functions: $\psi(t) \rightarrow \psi(t)\, e^{ieVt/\hbar}$ (2)

Gap: $\Delta(t) \propto \langle \hat{\psi}(t)\,\hat{\psi}(t)\rangle \rightarrow \Delta(t)\, e^{i2eVt/\hbar}$ (3)

$$|\Delta(t)|\, e^{i\varphi(t)} \rightarrow |\Delta(t)|\, e^{i(\varphi(t) + 2eVt/\hbar)} \tag{4}$$

$\Longrightarrow$ Constant potential induces changing phase: $\boxed{\dot{\varphi} = 2eV/\hbar}$ (5)

Consider slight bias eV_0 between left and right s.c.: AR29



Phase difference changes: $\dot{\varphi} = \dot{\varphi}_L - \dot{\varphi}_R \overset{(28.5)}{=} 2eV/\hbar$ (1)

Binding energy of all Andreev bound states changes $\dot{E}_A = \sum_p \dot{E}_p(\varphi) = \sum_p \frac{\partial E_p(\varphi)}{\partial \varphi} \underbrace{\dot{\varphi}}_{2eV/\hbar}$ (2)

Power is dissipated because Super current flows: $IV = P = \dot{E}_g \overset{(24.3)}{=} - \dot{E}_A = -\frac{2eV}{\hbar} \sum_p \frac{\partial E}{\partial \varphi}$ (3)

Super current: (Flows even for $V \to 0$!) $$\boxed{I_s = -\frac{2e}{\hbar} \sum_p \frac{\partial E_p(\varphi)}{\partial \varphi} \overset{(24.1)}{=} \frac{\pi \Delta}{2e} G_Q \sum_p \frac{T_p \sin\varphi}{\sqrt{1 - T_p \sin^2 \varphi/2}}}$$ (4)

Josephson effect: phase difference $\varphi_L - \varphi_R \neq 0$ induces flow of supercurrent

---

- I_s is odd, periodic function of φ, $I_s(\varphi=0) = 0$ AR30

- Historically, term "Josephson junction" referred to

Tunnel junction: (with $T_p << 1$) : $I_s(\varphi) = \underbrace{\left(\frac{\pi\Delta}{2e} G_Q \sum_p T_p\right)}_{I_c} \sin\varphi$ (1)

conductance of normal junction

Critical current: $I_c = \frac{\Delta\pi}{2e} \underbrace{G_Q T}_{G_N} = \frac{\Delta\pi}{2e} G_N$ (2)

Josephson energy: (defined such that $I_s = \frac{2e}{\hbar}\partial_\varphi E_J(\varphi)$): $E_J(\varphi) = -\left(\frac{\hbar}{2e}\right) I_c \cos\varphi$ (3)

Maximum supercurrent : $\varphi = \pi/2$ (4)

- More generally, any nanostructure between two s.c. is a Josephson device:

- For point contact ($T=1$): $I_c(\varphi) = \underbrace{\frac{\pi\Delta}{e} G_Q \sum_p \cancel{T_p}}\frac{1}{2}\frac{\sin\varphi}{\sqrt{1-\sin^2\varphi/2}}$ (5)

Maximum at $\varphi = \pi$ $= I_c \sin\varphi/2$, $I_c = \frac{\pi\Delta}{e} G_N$ (6)