1.9 Spin-dependent scattering

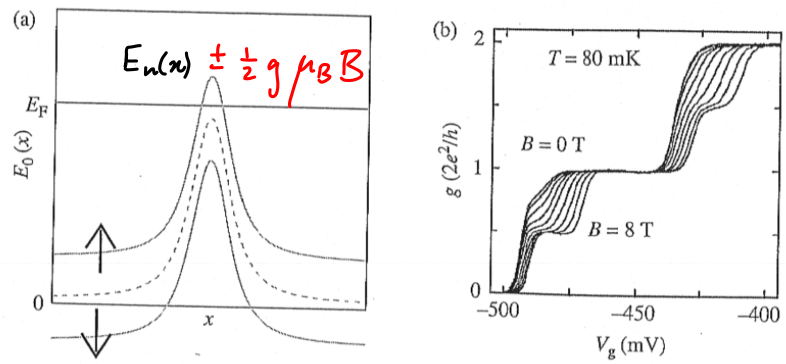
SDS 1

With spin: $\psi(\vec{r}) = \begin{pmatrix} \psi_\uparrow(\vec{r}) \\ \psi_\downarrow(\vec{r}) \end{pmatrix}$ spin operator: $\hat{S} = \frac{\hbar}{2}\vec{\sigma}$ (1)

Zeeman splitting: $\hat{H}_z = g\mu_B \vec{B}\cdot\hat{\sigma}/2$ $\mu_B = e\hbar/2mc$ (2)

$g = \begin{cases} 2 \text{ in vacuum} \\ -0.44 \text{ in GaAs} \end{cases}$

($\lesssim 10$ meV for available fields)



QPC as a spin filter. (a) Electrons see the constriction as a spin-dependent potential barrier (only E_0 is shown). Dotted line: E_0 at zero magnetic field. (b) QPC conductance quantization upon increasing magnetic field. Adapted from Ref. [39].

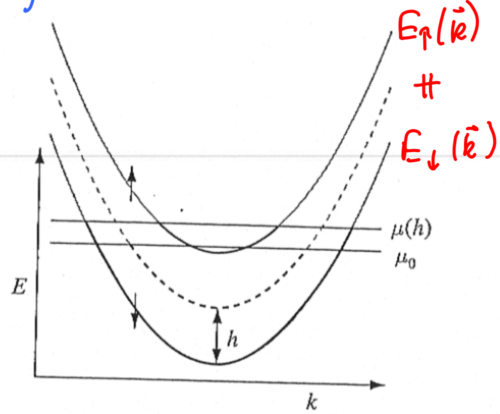
Ferromagnets

SDS 2

Spontaneous breaking of time reversal symmetry produces magnetization $\vec{m}$,

$\Rightarrow$ exchange field $\vec{h} \parallel \vec{m}$ $\Rightarrow$ splitting of $\uparrow$ and $\downarrow$ bands:

$$\left.\begin{array}{l} E_\uparrow(\vec{k}) \\ E_\downarrow(\vec{k}) \end{array}\right\} = E(\vec{k}) \pm h(\vec{k}) \quad \text{(exchange field)}$$



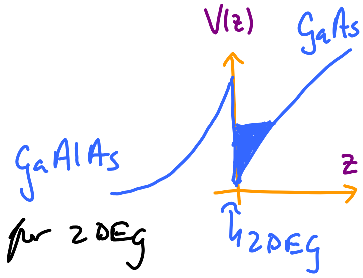
Energy bands in a ferromagnet. Spin-splitting results in different numbers of spin-down and spin-up electrons. Thicker lines: occupied states; dotted line: $h = 0$.

Spin-orbit interaction (does not break time-reversal symmetry) SDS3

$$\hat{H}_{SO} = \frac{\mu_B}{2c}\, \vec{\sigma} \cdot (\vec{E} \times \vec{v}) \qquad (1)$$

relativistic interpretation: when moving relative to $\vec{E}$-field, it gets a $\perp$ $\vec{B}$-field component

velocity operator: $\vec{v} = -(i\hbar/m)\,\vec{\nabla}$



$\vec{E}$: electric field from
- crystal lattice potential
- defects $(\sim Z^4)$
- confinement in z-direction for 2DEG

In perfect lattice with inversion symmetry, $\hat{H}_{SO}$ is not noticed:

$$E_\downarrow(\vec{k}) \underset{\substack{\uparrow \\ \text{time-reversal symmetry}}}{=} E_\uparrow(-\vec{k}) \overset{\text{inversion symmetry}}{=} E_\uparrow(\vec{k})$$

SDS4

SO-Scattering of defect: changes momentum, can also change spin,

with probability $(v/c)^4 Z^4 \simeq \begin{cases} \simeq 0 & \text{for light atoms} \\ 10^{-4} & \text{middle of periodic table} \\ 1/2 & \text{for heavy atoms} \end{cases} \qquad (1)$

Spin flip time $\tau_{sf} \gg \tau$ (time for changing momentum)

2DEG from GaAs, (with $z \perp$ 2DEG plane):

$$\hat{H}_{SO} = \underbrace{\alpha(\hat{\sigma}_x k_y - \hat{\sigma}_y k_x)}_{\substack{\text{Rashba interaction,} \\ \text{from asymmetry of} \\ \text{confinement in z-direction}}} + \underbrace{\beta(\hat{\sigma}_x k_x + \hat{\sigma}_y k_y)}_{\substack{\text{Dresselhaus interaction,} \\ \text{from lack of inversion symmetry}}}$$

1.9.1 Scattering matrix with spin

SDS5

$\hat S$ needs spin indices: $$S_{p'\alpha', p\alpha} = \underbrace{S^{(0)}_{p',p}}_{\hat S^{(0)}} \delta_{\alpha'\alpha} + \underbrace{\vec S_{p'p}}_{\hat{\vec S}} \cdot \vec\sigma_{\alpha'\alpha} \qquad (1)$$

$\alpha = \uparrow, \downarrow$

(^ means: matrix in channel indices)

Constraints due to unitarity & time-reversal symmetry (TR) (i.e. with SO int.)

$$\psi = \begin{pmatrix}\psi_\uparrow \\ \psi_\downarrow\end{pmatrix} \xrightarrow{TR} \begin{pmatrix} -\psi_\downarrow^* \\ \psi_\uparrow^* \end{pmatrix} = \hat g \psi^* \equiv \psi^{tr} \qquad (2)$$

$$\hat g = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

This ensures that $\vec S \to -\vec S$

$$S_z = \psi_\uparrow^* \psi_\uparrow - \psi_\downarrow^* \psi_\downarrow \longrightarrow (-\psi_\downarrow)(-\psi_\downarrow^*) - (\psi_\uparrow \psi_\uparrow^*) = -S_z$$

$$S_x = \psi_\uparrow^* \psi_\downarrow + \psi_\downarrow^* \psi_\uparrow \longrightarrow (-\psi_\downarrow)\psi_\uparrow^* + \psi_\uparrow(-\psi_\downarrow^*) = -S_x \qquad (3)$$

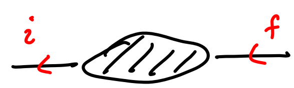
$$S_y = (-i)\left[\psi_\uparrow^* \psi_\downarrow - \psi_\downarrow^* \psi_\uparrow\right] \longrightarrow (-i)\left[(-\psi_\downarrow)\psi_\uparrow^* - \psi_\uparrow(-\psi_\downarrow^*)\right] = -S_y$$

SDS6

If $\psi_f = \hat S \psi_i$

then time-reversed process must be characterized by same amplitude: (1)

$$\psi_i^{tr} = \hat S \psi_f^{tr} \qquad (2)$$



$$\cancel{\hat g}\, \psi_i^* = -\hat g \hat S \hat g \psi_f^* \qquad (3)$$

$$\hat g^{-1} = -\hat g$$

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = 1$$

$$\psi_i = \underbrace{-\hat g \hat S^* \hat g}_{\hat S^{-1} = \hat S^\dagger = \hat S^{*T}} \psi_f \qquad (4)$$

(unitarity)

$$\Rightarrow \quad \hat S^T = -\hat g \hat S \hat g \qquad (5)$$

Pauli notation: $$S^{(0)}_{p'p} = S^{(0)}_{pp'}, \qquad \vec S_{p'p} = -\vec S_{pp'} \qquad (6)$$

(without spin structure, S is symmetric, as we have used so far) (7)

SDS7

If spin scattering arises due to magnetic field or exchange field, then $\hat{S}$ gets extra $(-)$.

If $\hat{S}^{(o)} \gg \hat{S}$, we may write:

$$\hat{S} = \hat{S}^{(so)} + \hat{S}^{(m)} \quad , \text{ with } \begin{cases} S^{(so)}_{p'p} = - S^{(so)}_{p'p} \\ S^{(m)}_{p'p} = S^{(m)}_{pp'} \end{cases} \qquad (1)$$

For case that there is only one favored direction in nanostructure (e.g. all ferromagnetic reservoirs are colinear), we may write $\vec{S}^{(m)}_{p'p} \sim \vec{m}$,

and obtain two separate scattering matrices for spins parallel and antiparallel to $\vec{m}$:

$$\hat{S} = \hat{S}_\uparrow \left(1 + \frac{\vec{\sigma}\cdot\vec{m}}{2}\right) + \hat{S}_\downarrow \left(1 - \frac{\vec{\sigma}\cdot\vec{m}}{2}\right) \qquad (2)$$

Electron polarized along $\vec{m}$ direction experiences no spin flip

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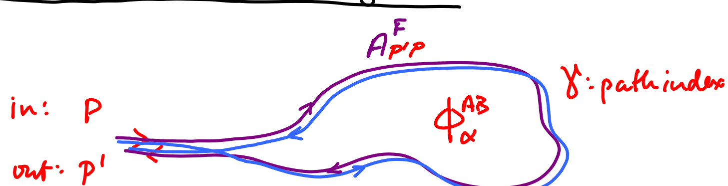
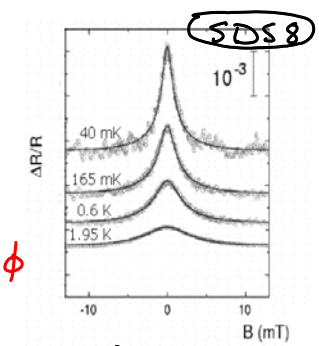
SDS8

<u>Reminder: weak localization</u>





$$A^B_{p'p} \simeq A^F_{pp'} \overset{(6.7)}{=} A^F_{p'p} \equiv A^o_{p'p} e^{i\phi}$$

$$\Rightarrow \boxed{A^B = (A^F)^T = A^F}$$

Return probability: $$P_{p'p}(\gamma) = \left| A^F_{p'p}\, e^{i\phi^{AB}} + A^B_{p'p}\, e^{-i\phi^{AB}} \right|^2 \qquad (1)$$

(drop $p'p$ indices, to simplify notation)
$$= \left[|A^o|^2 + |A^o|^2 + 2\,\mathrm{Re}\left(A^o A^{o*} e^{2i\phi^{AB}}\right) \right]$$

$$= \left[\underbrace{2|A^o(\gamma)|^2}_{\equiv P_{classical}} + \underbrace{2|A^o(\gamma)|^2 \cos 2\phi^{AB}(\gamma)}_{\equiv P_{interference}} \right] \qquad (2) \quad (3)$$

P_{int} enhances backscattering if $\phi^{AB}(\gamma) \ll 1$

but: $\sum_\gamma P_{int}(\gamma) \simeq 0$ if $\phi^{AB}(\gamma) \gtrsim 2\pi$ $\Longrightarrow$ Resistance $\downarrow$ for magnetic field $\uparrow$

Effect of spin-orbit scattering on weak localization

SDS 9

With spin-orbit scattering, A^F, A^B, which already are matrices in channel space (indices $p'p$), also become 2×2 matrices in spin space (with spin indices $\alpha'\alpha$)

$$(\hat A^F)_{p'\alpha',p\alpha} = A^0_{p'p}\,(\hat{\mathbb{1}})_{\alpha'\alpha} + \vec A_{p'p}\cdot\hat{\vec\sigma}_{\alpha'\alpha} \quad \text{(1a)}$$

etc.

$$\hat A^B \simeq (\hat A^F)^T \overset{(1a)}{=} (A^0)^T\mathbb{1} + \vec A^T\cdot\hat{\vec\sigma} \quad \text{(1b)}$$

$$(6.6):\ = A\,\hat{\mathbb{1}} - \vec A\cdot\hat{\vec\sigma}$$

Then, for $P_{p'p}$ (with $\underbrace{p',p}_{\text{not shown below}}$ fixed) we must sum over spin indices $\alpha'\alpha$:

$$P_{cl} = \sum_{\alpha'\alpha}\left[\left|(A^0\mathbb{1} + \vec A\cdot\hat{\vec\sigma})_{\alpha'\alpha}\right|^2 + \left|(A^0\,\mathbb{1} - \vec A\cdot\vec\sigma)_{\alpha'\alpha}\right|^2\right] = 2\cdot 2\left(|A^0|^2 + \vec A\cdot\vec A^*\right) \quad (2)$$

$$P_{int} = \sum_{\alpha'\alpha} 2\,\mathrm{Re}\left[(A^0 + \vec A\cdot\hat{\vec\sigma})_{\alpha'\alpha}\,(A^0 - \vec A\cdot\vec\sigma)^*_{\alpha'\alpha}\right] = 2\cdot2\left(|A^0|^2 - \vec A\cdot\vec A^*\right)\cos 2\phi^{AB} \quad (3)$$

SDS 10

Perform spin sums using:

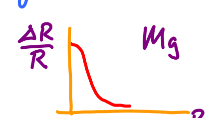
$$\sum_{\alpha'\alpha}(a^0\hat{\mathbb{1}} + \underbrace{\vec a\cdot\hat{\vec\sigma}}_{\sum_i a^i\hat\sigma^i})_{\alpha'\alpha}\ \underbrace{(b^0\hat{\mathbb{1}} + \vec b\cdot\vec\sigma)^*_{\alpha'\alpha}}_{=(b^{0*}\mathbb{1} + \vec b^*\vec\sigma)_{\alpha'\alpha}} \quad (1)$$

(since $\vec\sigma = \vec\sigma^\dagger$)

$$= a^0b^{0*}\underbrace{\mathrm{Tr}(\hat{\mathbb{1}})}_{2} + \sum_{i=1,2,3}(a^ib^{0*} + a^0b^{i*})\underbrace{\mathrm{Tr}(\hat{\mathbb{1}}\cdot\hat\sigma^i)}_{0} + \sum_{ij\in(1,2,3)} a^ib^{j*}\underbrace{\mathrm{Tr}(\hat\sigma^i\hat\sigma^j)}_{2\delta_{ij}} = 2(a^0b^{0*} + \vec a\cdot\vec b^*) \quad (2)$$

P_{cl} vs. P_{int} depends of length of loop (L) vs. spin-orbit scattering length L_{so}:

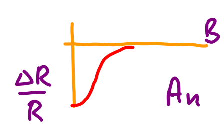
For $L \ll L_{so}$: $|A_0| \gg |A_i|$ $\Rightarrow$ $P_{cl} \overset{(9.2),(9.3)}{=} P_{int} = 4|A^0|^2$ (3)



$\Rightarrow P(\phi^{AB}=0) = P_{cl} + P_{int} = 2P_{cl} > \underbrace{P(\phi^{AB}\geq 2\pi)}_{P_{cl}}$ (4) $\Longrightarrow$ weak localization

For $L \gg L_{so}$: $|A_0| \simeq |A_i|$ $\Rightarrow$ $P_{cl} \overset{(9.2)}{=} 16|A_0|^2$, $P_{int} \overset{(9.3)}{=} -8|A_0|^2$ (5)

(spin is forgotten) $\Rightarrow$ (scattering with or without spin flip has same weight)



$\Rightarrow P(\phi^{AB}=0) = P_{cl} + P_{int} = \frac{1}{2}P_{cl} < \underbrace{P(\phi^{AB}\geq 2\pi)}_{P_{cl}}$ (6) $\Longrightarrow$ weak anti-localization