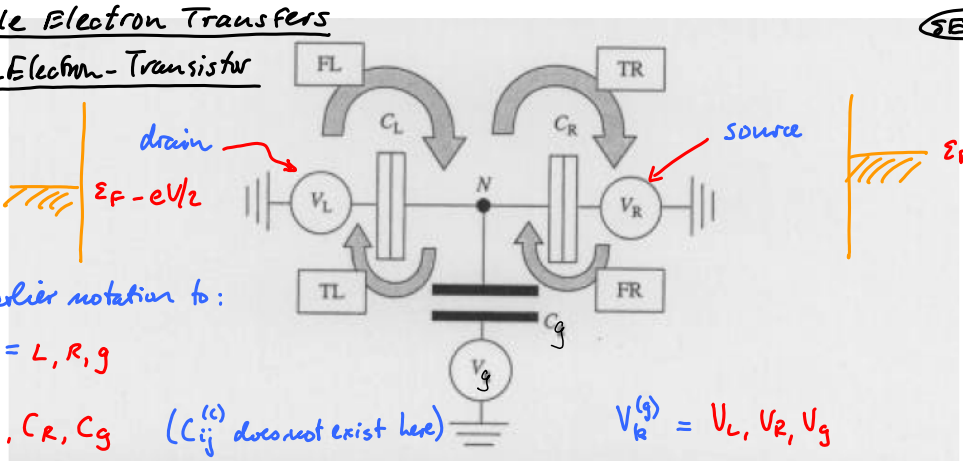
3.2 Single Electron Transfers

3.2.1 Single-Electron-Transistor

SET 1



Single-electron transistor (SET). The electron transport from left to right may be influenced by the gate voltage V_g. Arrows denote four possible single-electron transfers in the SET (FL = from the left; FR = from the right; TL = to the left; TR = to the right).

Specialize earlier notation to:

$i = 1$, $k = L, R, g$

$C_{ik}^{(g)} = C_L, C_R, C_g$ ($C_{ij}^{(c)}$ does not exist here) $\qquad V_k^{(g)} = V_L, V_R, V_g$

$$C_{ii} \overset{(CB13.5)}{=} \sum_i C_{ii}^{(c)} + \sum_k C_{ik}^{(g)}$$

$$= C_L + C_R + C_g \equiv C_\Sigma \qquad (1)$$

C_{ij} does not exist

$$\hat{C} = C_\Sigma \quad (\text{just a number, not a matrix}) \qquad (2)$$

$$E_{el} \overset{(CB14.5)}{=} E_c (N - q/e)^2 , \qquad (3)$$

$$E_c \overset{(CB14.5)}{=} \frac{e^2}{2C_\Sigma} , \qquad (4)$$

$$q \overset{(CB13.31)}{=} -(C_R V_R + C_L V_L + C_g V_g) \qquad (5)$$

SET 2

Equilibrium if $V_L = V_R$ ($= 0$ in discussion of charging, where $V^S = 0$)

No current flows, charging as in (CB4-6).

Non-equilibrium if $V_L \neq V_R$ (current flows!)

Source and drain electrodes are regarded as part of the "system".

Energy = (Energy island) + (Energy source) + (energy drain)

$$= E_{el}(N) + eV_R N_R + eV_L N_L \qquad (1)$$

Extra energy costs: $\pm eV_i$ for {adding / extracting} electron {to / from} electrode $i = L, R$ (2)

SET2

Non-equilibrium if $V_L \neq V_R$ (current flows!)

Source and drain electrodes are regarded as part of the "system".

Energy = (Energy island) + (Energy source) + (energy drain)

$$= E_{el}(N) + eV_R N_R + eV_L N_L \quad (1)$$

Extra energy costs: $\pm eV_i$ for {adding / extracting} electron {to / from} electrode $i = L, R$ (2)

Energy differences for tunneling processes, with initial charge state = N:

from the left: $\Delta E_{FL}(N) = E_{el}(N+1) - E_{el}(N) - eV_L$ (3)

to the left: $\Delta E_{TL}(N) = E_{el}(N-1) - E_{el}(N) + eV_L$ (4)

from the right: $\Delta E_{FR}(N) = E_{el}(N+1) - E_{el}(N) - eV_R$ (5)

to the right: $\Delta E_{TR}(N) = E_{el}(N-1) - E_{el}(N) + eV_R$ (6)

SET3

Electrostatic energy change upon adding/extracting charge to/from island:

$$E_{el}(N \pm 1) - E_{el}(N) \overset{(1.3)}{=} E_c(N \pm 1 - q/e)^2 - E_c(N - q/e)^2 = \left[\pm 2(N - q/e) + 1\right] E_c \quad (1)$$

↑ periodic in q.

Consider $T = 0$ $\Rightarrow$ electron transfer can only occur if $\Delta E < 0$ (2)

(energy can only be dissipated, not gained!)

$\Rightarrow$ transport occurs only in certain range of parameter space

Transport regime A: Coulomb blockade regime: all ΔE's > 0

$\Delta E_{FL} > 0$ $\Delta E_{TL} > 0$ $\Delta E_{FR} > 0$ $\Delta E_{TR} > 0$ (3)

all single-electron processes are forbidden! NO TRANSPORT

SET 5

Transport regime B: one-by-one FL–TR

Only two processes are allowed:

first:  $\Delta E_{FL}(N) < 0$ then  $\Delta E_{TR}(N+1) < 0$

[but not  $\Delta E_{FL}(N+1) > 0$ to ensure that electrons tunnel one by one.]

or:

Transport regime C: one-by-one TR–FL

first:  $\Delta E_{TR}(N) < 0$ then  $\Delta E_{FL}(N-1) < 0$

[but not  $\Delta E_{TR}(N-1) > 0$ to ensure that electrons tunnel one by one.]

SET 6

3.2.2 Coulomb Diamonds in a SET

Where does Coulomb blockade (CB) of single-electron transport occur?

To reduce number of parameters, take:

- $C_L = C_R$,
- $V_L = -V_R = V/2$,

$\Rightarrow q \overset{(1.5)}{=} -C_g V_g$ ↩ (conveniently independent of V!) (1)

Consider initial state with $N = 0$: $E_{el}(\pm 1) - E(0) \overset{(3.1)}{=} E_c(\mp 2q/e + 1)$ (2)

$\Delta E_{FL}(N) \overset{(2.3-2.6)}{=} E_c(-2q/e + 1) - eV/2 > 0$

$\Delta E_{TL}(N) = E_c(+2q/e + 1) + eV/2 > 0$ (3)

$\Delta E_{FR}(N) = E_c(-2q/e + 1) + eV/2 > 0$

$\Delta E_{TR}(N) = E_c(+2q/e + 1) - eV/2 > 0$

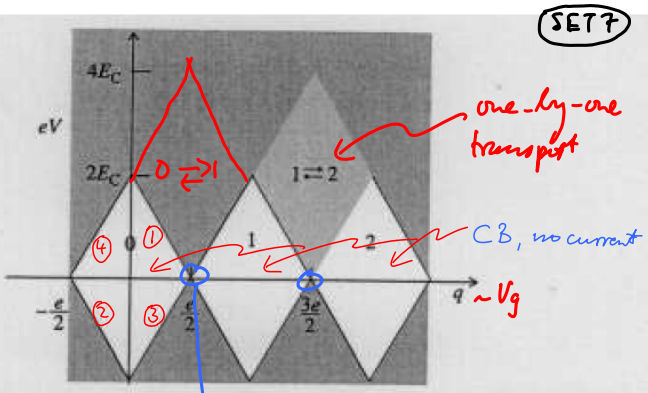
SET7

FL: $eV < 2E_C[1 - 2q/e]$ ①

TL: $eV > -2E_C[1 + 2q/e]$ ②

FR: $eV > -2E_C(1 - 2q/e)$ ③

TR: $eV < 2E_C(1 + 2q/e)$ ④



Coulomb diamonds in a SET. The transport is blocked inside white diamonds. The light gray region denotes where the one-by-one transport cycle takes place.

- Diamond pattern is e-periodic in q
- Diamonds touch at V = 0 ; there

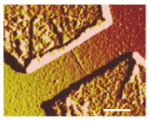
$\Delta E(N) = 0$, $\Delta E(N+1) = 0$ } charge degeneracy points

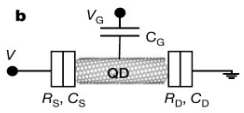
- At charge degeneracy point, CB is lifted already by a small bias ⇒ current can flow.

SET8

Electron–hole symmetry in a semiconducting carbon nanotube quantum dot

Pablo Jarillo-Herrero[1,2], Sami Sapmaz[1], Cees Dekker[1], Leo P. Kouwenhoven[1,2] & Herre S. J. van der Zant[1]





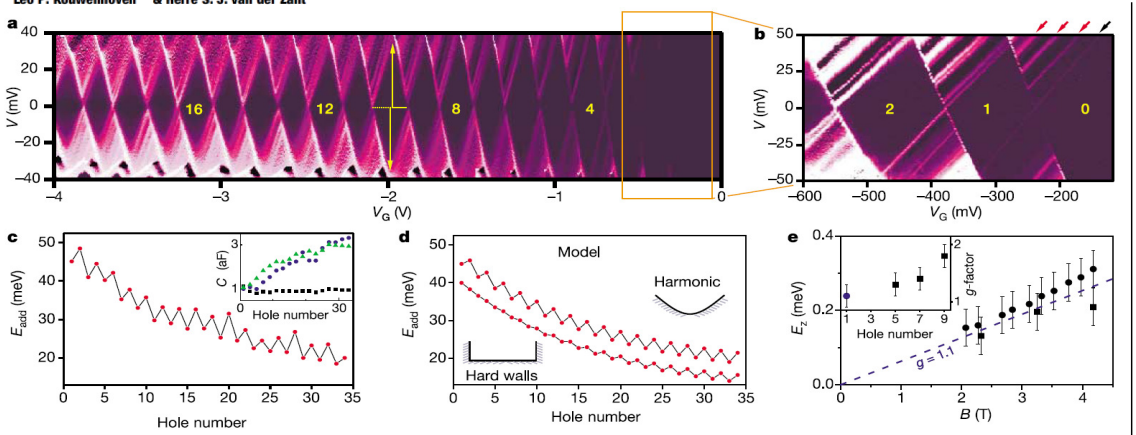


Figure 2 Few-hole semiconducting nanotube. **a**, Two-dimensional colour plot of the differential conductance, d*I*/d*V*, versus *V* and negative V_G at $T = 4$ K (black is zero, white is 3 μS). In the black diamond-shaped regions, the number of holes (indicated) is fixed by Coulomb blockade. **b**, Zoom-in, taken at 0.3 K of the region with 0, 1 and 2 holes (white represents d*I*/d*V* > 10 nS). Lines outside the diamonds running parallel to the edges correspond to discrete energy excitations (the black arrow points at the one-electron ground state; the red arrows at the one-electron excited states). **c**, Addition energy, E_{add}, as a function of hole number. E_{add} is deduced from the diamond size for positive and negative *V* (that is, half the sum of the yellow arrows in **a**). Inset, the capacitances C_S (green), C_D (blue) and C_G (black) versus hole number. **d**, Calculation of the addition energy spectrum for a semiconducting nanotube (as an example we have taken a zigzag (35,0), with $E_{gap} \approx 259$ meV, $m_{eff} = 0.037\, m_e$; ref. 3) for a harmonic potential (top) and a hard-wall potential (bottom). The parameters for the harmonic potential are: $V(x = \pm 135\,\text{nm}) = E_{gap}/2$, where *x* is the distance from the centre of the nanotube (see Supplementary Information). **e**, Zeeman splitting energy, E_Z, versus magnetic field, *B*, for the one-hole orbital states. The data result from two different types of measurements: (1) individual gate voltage traces at fixed bias (circles) and (2) stability diagrams (squares, see also Supplementary Information). Inset, *g*-factor as a function of hole number. The point for $N = 1$ is the average of the data in Fig. 2e. The points for $N = 5$, 7 and 9 are obtained from co-tunnelling (see Supplementary Information).

3.2.3 Coulomb Shards

SET 9

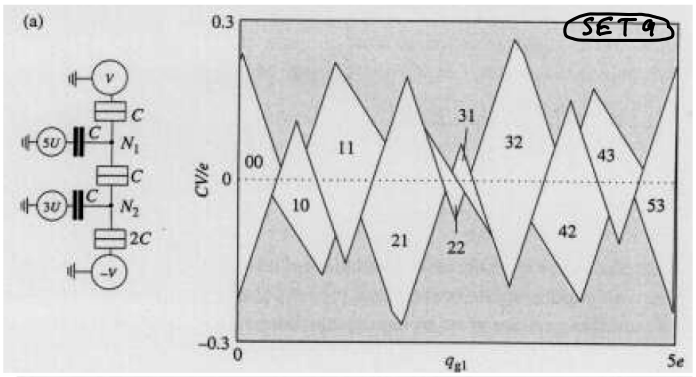
Multi-dot arrays show more complicated patterns in $V_g - V_{sd}$ diagrams.

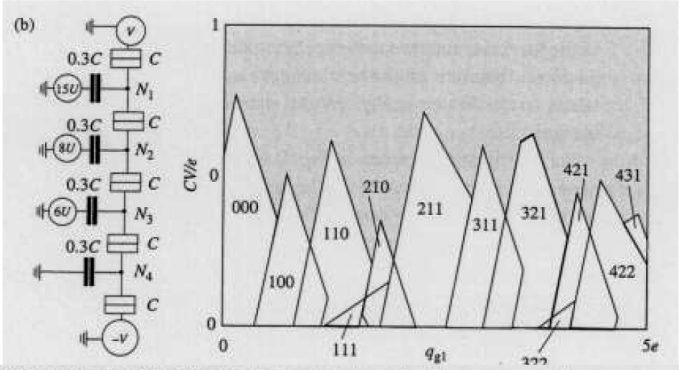
Irregularity arises from making low-dimensional cut through a structure which is regular & periodic in $(n+1)$-dimensional space, (n = # of islands)

cf.: CB 16.4:

$E_{el}(N_1, N_2) =$

$\sum_{ij} (\hat{E}_{ij})\,(N_i - q_i/e)(N_j - q_j/e)$





Coulomb shards in more complicated Coulomb nanostructures. The figures present the parameter regions (white) where a certain charge state is blocked for (a) two- and (b) four-island arrays biased as shown. There is some single-electron current in the gray-shaded regions. For a four-junction array, $N_4 = 0$ for all blocked states in the parameter region shown.

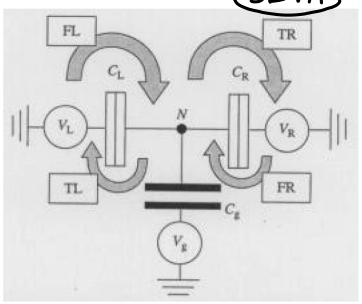
3.2.4 Master Equation

SET 10

- In CB regime, system can be characterized by the numbers $\{N_i\}$ of electrons on each island. "Definite charge state" = $|\{N_i\}\rangle$.
- These are not eigenstates of full Hamiltonian, that includes tunneling. But if tunneling is weak, coherence between states $|\{N_i\}\rangle$ and $|\{N_i'\}\rangle$ is negligible.
- Then a classical description of transport is possible, using master equation for probability $P_{\{N_i\}}(t)$ to be in state $|\{N_i\}\rangle$
- Allowed tunnel processes (with $\Delta E < 0$) occur randomly, with a rate $\Gamma(\{N_i\})$ that depends on charge state

SET11

Master equation for SET



$P_N(t)$ = probability to have N electrons on island at time t

Normalization: $\sum_N P_N(t) = 1 \quad \forall t$.

$$\frac{d}{dt}P_N(t) = -\left[\Gamma_F^{N\to N-1}(N) + \Gamma_T^{N\to N+1}(N)\right] P_N(t) + \Gamma_T^{N-1\to N}(N-1)\, P_{N-1}(t) + \Gamma_F^{N+1\to N}(N+1)\, P_{N+1}(t) \quad \text{"master eq."} \qquad (1)$$

Total rate of going from island: $\Gamma_F = \Gamma_{FL} + \Gamma_{FR}$ (2a)

" " " " to " $\Gamma_T = \Gamma_{TL} + \Gamma_{TR}$ (2b)

If SET-parameters (V, V_g, C's) are time-independent, then so are Γ's.

Stationary solution $P_N^{(0)}$ exists, obtained from setting $\frac{d}{dt}P_N(t) = 0$ in (1), imposing normalization, $\sum_N P_N^{(0)} = 1$, and solving (1) numerically for $P_N^{(0)}$'s.

SET12

Current:

Through left junction: $I_L = e\sum_N \left[\Gamma_{FL}(N) - \Gamma_{TL}(N)\right] P_N$ (1)

Through right junction: $I_R = e\sum_N \left[\Gamma_{TR}(N) - \Gamma_{FR}(N)\right] P_N$ (2)

Charge conservation implies: $I_R = I_L$ (3)

In stationary case, this follows from (11.2) (can be shown)

General master equation (beyond SET)

SET 13

label charge states by $\alpha \equiv \{N_i\}$:

$$\frac{dp_\alpha}{dt} = -\sum_\beta \Gamma_{\alpha\to\beta} p_\alpha + \sum_\beta \Gamma_{\beta\to\alpha} p_\beta \equiv \sum_\beta \tilde{\Gamma}_{\alpha\beta} p_\beta \qquad (1)$$

("out of α" / "into α")

$$\tilde{\Gamma}_{\alpha\beta} = -\delta_{\alpha\beta} \sum_{\bar\beta} \Gamma_{\alpha\to\bar\beta} + \Gamma_{\beta\to\alpha} = \begin{pmatrix} -\sum_\beta \Gamma_{1\to\beta} & \Gamma_{2\to1} & \\ \Gamma_{1\to2} & -\sum_\beta \Gamma_{2\to\beta} & \\ & & \ddots \end{pmatrix} =$$

Stationary solution satisfies : $\sum_\beta \tilde{\Gamma}_{\alpha\beta} p_\alpha^{(0)} = 0$ (2)

↑ eigenvector of $\tilde{\Gamma}$ with eigenvalue 0

SET 14

For each junction i — C — j there are two rates

Forward ($i \to j$) : $\Gamma_f^{(c)}(\alpha)$, backward ($j \to i$) : $\Gamma_b^{(c)}(\alpha)$ (1)

↑ depend on initial state α

Final state is determined by direction of transfer

$$\left.\begin{aligned} N_i^{(\beta)} &= N_i^{(\alpha)} \mp 1 \\ N_j^{(\beta)} &= N_j^{(\alpha)} \pm 1 \end{aligned}\right\} \text{ for } \begin{matrix} f \\ b \end{matrix} \text{ process} \qquad (2)$$

$$N_k^{(\beta)} = N_k^{(\alpha)} \quad \text{for } k \neq i, j$$

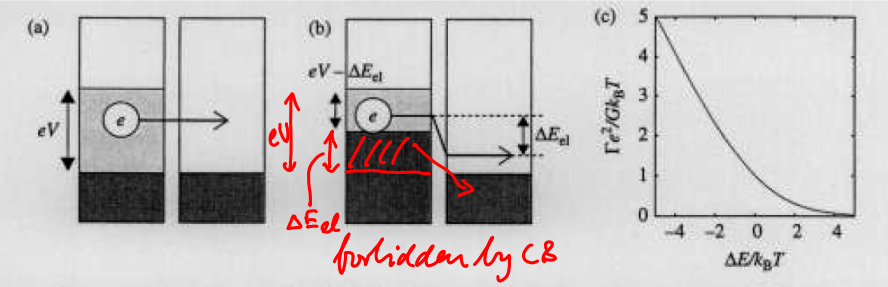
Similarly for transfer from island to transparent electrode. i — K
except that charge state $\{N_i\}$ does not keep track of k.

Current across junction (c) : $I^{(c)} = \sum_\alpha \left(\Gamma_f^{(c)}(\alpha) - \Gamma_b^{(c)}(\alpha)\right) p_\alpha^{(0)}$ (3)

3.2.6 Tunneling rates

SET15

Γ's depend only on macroscopic parameters (resistances, charging energies, capacitances)



Tunneling rates. (a) Without Coulomb blockade, an electron preserves its energy when tunneling. (b) With Coulomb blockade, it has to pay charging energy ΔE_{el}, this reduces the energy strip available for tunneling by a corresponding amount. (c) Energy dependence of the rate at finite temperature.

Single junction without CB: $\Gamma = I/e = \frac{eVG}{e^2}$

Forward rate: $\Gamma_f = \frac{eVG}{e^2}\,\Theta(eV)$

Backward rate: $\Gamma_b = -\frac{eVG}{e^2}\,\Theta(-eV)$

SET16

With CB:

tunneling from lead into SET, causing energy of SET to change by ΔE_{el}

Electron performs work to charge capacitors of SET, so its energy changes

initial state → final state (must be empty)

$E \longrightarrow E - \Delta E_{el}$

for $\Delta E_{el} > 0$ (< 0) width of strip of available states decreases/increases

$$\Gamma = \underbrace{(eV - \Delta E_{el})}_{-\Delta E} \frac{G}{e^2}\,\Theta\underbrace{(eV - \Delta E_{el})}_{-\Delta E}$$ (recall 6.3)

At T=0: $$\Gamma = \frac{G}{e^2}(-\Delta E)\,\Theta(-\Delta E)$$