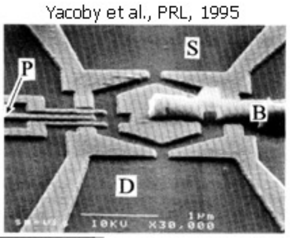
5.4 Quantum Dots

Typically realized in 2-DEGS.

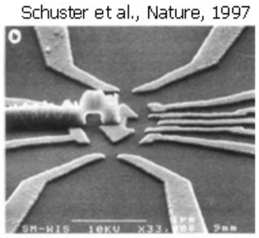
Yacoby et al., PRL, 1995



Moty Heiblum

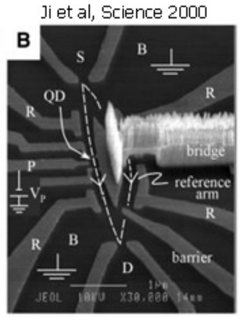
N ≈ 200 phase coherence!

Schuster et al., Nature, 1997



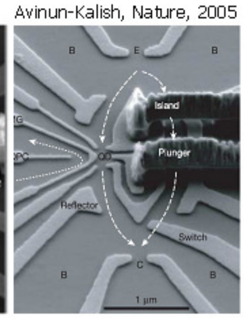
N ≈ 200 universal behavior

Ji et al, Science 2000

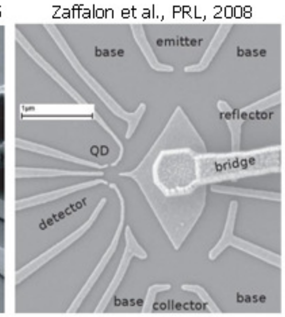


N ≈ 30 – 50 many levels, Kondo

Avinun-Kalish, Nature, 2005



Zaffalon et al., PRL, 2008



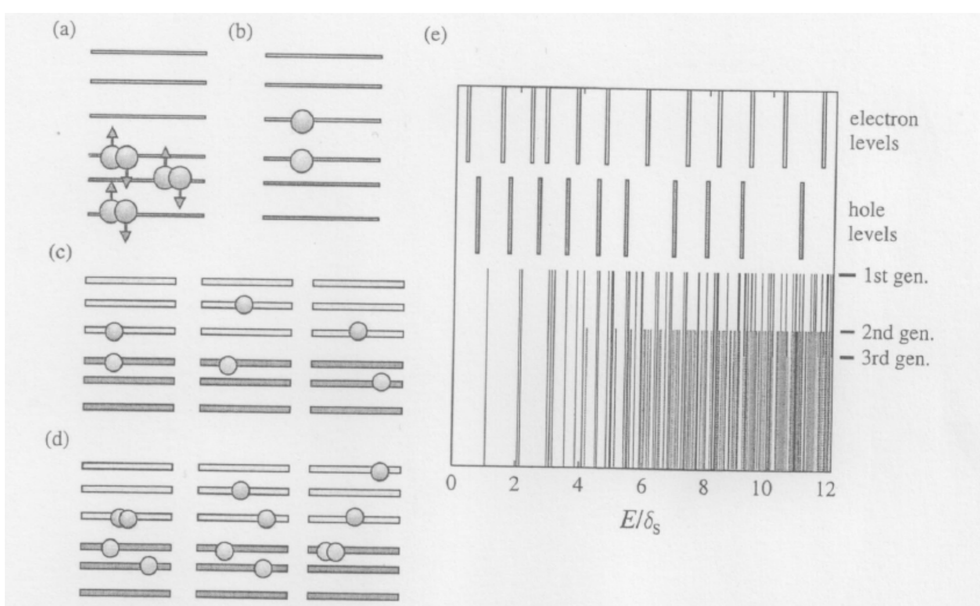
N = 1 pure Kondo

For small dots: new energy scale: <u>finite</u> level spacing

5.4.1 From levels to states

Single-particle levels: $\{E_k\}$

Occupation numbers: $\{n_k\}$, $n_k = 0, 1$

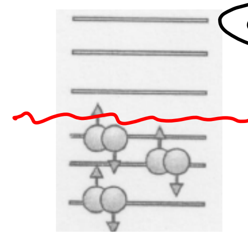


Levels and states in quantum dots. (a) The ground state of the dot with $N = 6$ electrons. (b) The first excited state. (c) Some states of the first generation. (d) Some states of the second generation. (e) The number of states exponentially increases with energy.

QD3

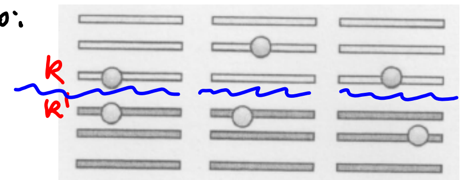
Ground state: Filled Fermi sea:

$$|g\rangle = \prod_{E_k < E_F} c^\dagger_{k\uparrow} c^\dagger_{k\downarrow} |F\rangle \,, \quad \text{set } E_g = 0$$



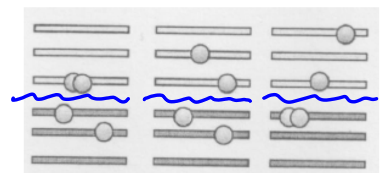
Excitations relative to ground state:

1. generation: single particle (k) - hole (k') excitations:



$$|s\rangle = c^\dagger_k c_{k'} |g\rangle \,, \quad \text{energy: } E_s = E_k - E_{k'} > 0$$

2. generation: double particle (k) - hole (k') excitations:



$$|s'\rangle = c^\dagger_k c_{k'} \, c^\dagger_{\bar{k}} c_{\bar{k}'} |g\rangle \,,$$

$$E_s = E_k + E_{\bar{k}} - E_{k'} - E_{\bar{k}'}$$

n.th generation: $E_s = \sum_k E_k - \sum_{k'} E_{k'}$, with $E_k - E_{k'} \geq 0$ for all pairs

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QD4

Claim: density of e-h-states grows rapidly: $\frac{dN}{dE} \simeq \frac{1}{\delta_s} e^{\pi\sqrt{2E/3\delta_s}}$

$$Z = \sum_s e^{-E_s/k_BT} \qquad (1)$$

count both electron and hole levels

$$\ln Z = \ln \prod_{k=1}^{\infty} \left(1 + e^{-E_k/k_BT}\right)^2 = 2 \underbrace{\sum_k}_{\frac{1}{\delta_s}\int dE_k} \ln(\quad) \qquad (2)$$

for uniform level spacing

$$\ln Z = \frac{2}{\delta_s} \int_0^\infty dE \, \ln\left(1 + e^{-E/k_BT}\right) \overset{x = E/k_BT}{=} \frac{k_BT}{\delta_s} \underbrace{2\int_0^\infty dx \, \ln(1 + e^{-x})}_{\pi^2/6} \qquad (3)$$

Alternatively:

$$Z = \int_0^\infty dE_s \underbrace{\frac{dN(E_s)}{dE_s}}_{\frac{1}{\delta_s} e^{\pi\sqrt{2E/3\delta_s}}} e^{-E_s/k_BT} \simeq e^{\frac{k_BT}{\delta_s}\pi^2/6} \qquad (4)$$

$y = \sqrt{E_s/\delta_s}$

$dy = \frac{1}{2}\frac{1}{y}\frac{dE_s}{\delta_s}$

Saddle-point treatment

$$= \int_0^\infty dy \, 2y \, e^{\pi\sqrt{2/3}\,y} \, e^{-\delta_s/T \, y^2} \simeq e^{\frac{\pi^2}{6} T/\delta_s} \,. \; \# \qquad (5)$$

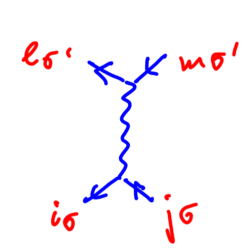
$$e^{-\delta_s/T\left(y^2 - \frac{\pi}{\sqrt{6}} T/\delta_s\right)^2} e^{\frac{\pi^2}{6} T/\delta_s}$$

constant $O(\sqrt{T/\delta_s})$

Interactions

QD5

$$H_0 = \sum_{i\sigma} E_i \, a^\dagger_{i\sigma} a_{i\sigma} \qquad (1)$$



$$H_u = \int d\vec{r}_1 \int d\vec{r}_2 \quad \rho(\vec{r}_1)\, U(|\vec{r}_1 - \vec{r}_2|)\, \rho(\vec{r}_2) \qquad (2)$$

$$\rho(\vec{r}_1) = \sum_\sigma \psi^\dagger_\sigma(\vec{r}_1)\, \psi_\sigma(\vec{r}_1) = \sum_\sigma \varphi^*_i(\vec{r}_1)\, \varphi_j(\vec{r}_1)\, a^\dagger_{i\sigma} a_{j\sigma} \qquad (3)$$

$$\psi_\sigma(\vec{r}_1) = \sum_j \varphi_j(\vec{r}_1)\, a_{j\sigma}$$ destroys particle at position $\vec{r}_1$ (4)

$$H_u = \sum_{ijlm} \sum_{\sigma\sigma'} (a^\dagger_{i\sigma} a_{j\sigma})\, U_{ij,lm}\, (a^\dagger_{l\sigma'} a_{m\sigma'}) \qquad (5)$$

$$U_{ij,lm} = \int dr_1 \int dr_2\, \varphi_i(r_1)\, \varphi^*_j(r_1)\, U(|\vec{r}_1 - \vec{r}_2|)\, \varphi_l(\vec{r}_2)\, \varphi^*_m(\vec{r}_2) \qquad (6)$$

"orthodox model"

QD6

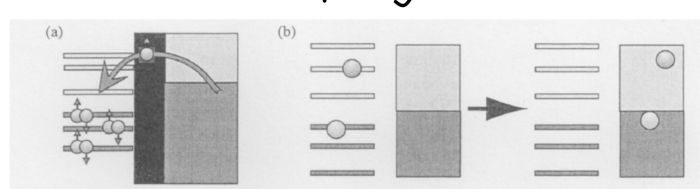
$$H_{charging} = E_{ch}(\hat{N}) = E_c \left(\hat{N} - \frac{C_g V_g}{e}\right)^2 \qquad (1)$$

$$N = \sum_{i\sigma} a^\dagger_{i\sigma} a_{i\sigma} = \text{total charge} \qquad (2)$$

Coulomb blockade as for large islands, but level spacing matters, too.

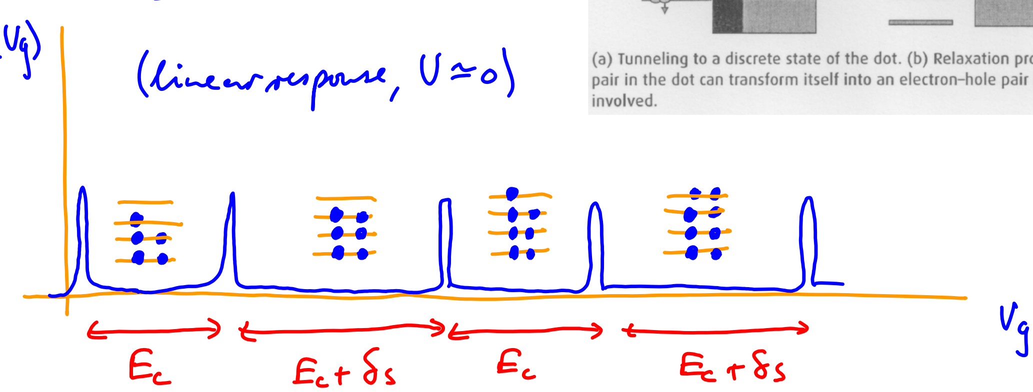
for small dots, typically

$$E_c/\delta_s \simeq 2 \text{ to } 10 \qquad (3)$$



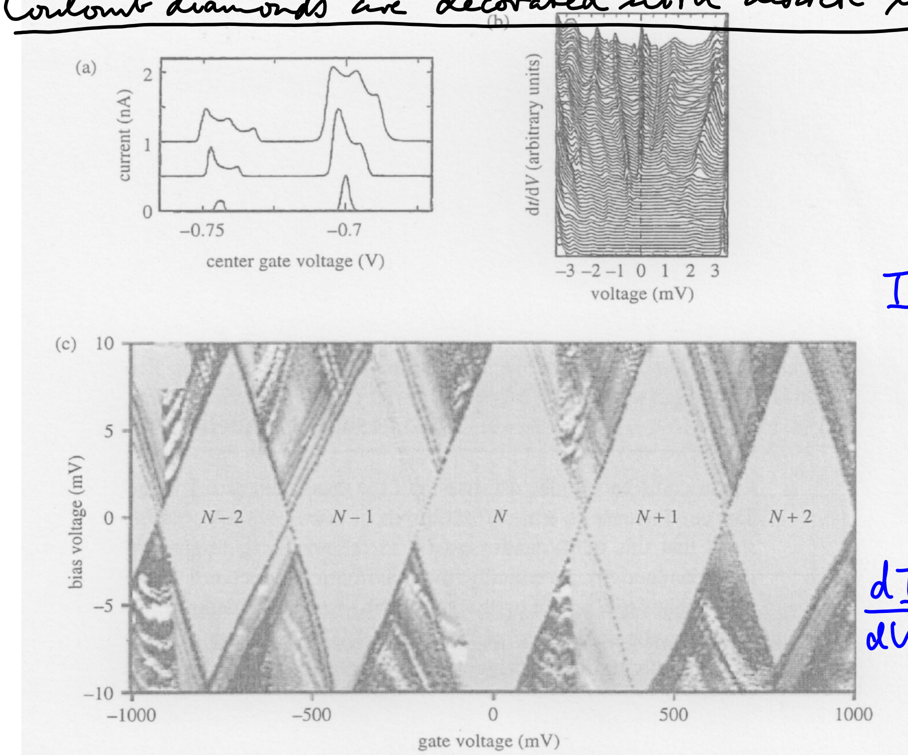
(a) Tunneling to a discrete state of the dot. (b) Relaxation process in the dot: the electron-hole pair in the dot can transform itself into an electron-hole pair in a nearby lead. No tunneling is involved.

$g(V_g)$ (linear response, $U \simeq 0$)



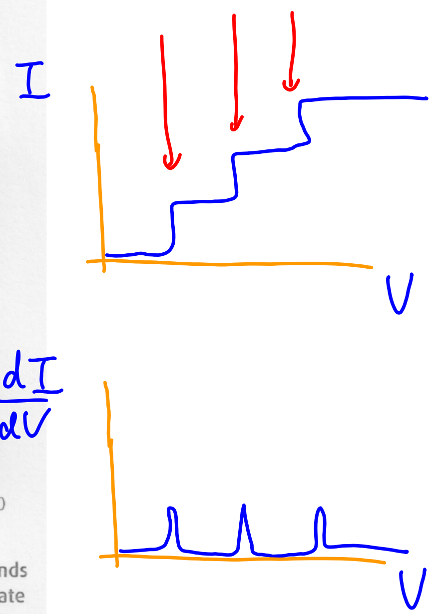
Coulomb diamonds are decorated with discrete lines

QD7



Discrete states in quantum dots: experiments. (a) Current versus gate voltage for three diamonds at increasing bias voltage. The highest trace shows a current step associated with a discrete state [123]. (b) Positions of conductance peaks and dips indicate the energies of the discrete states. The evolution of these energies in a magnetic field is shown [124]. (c) Overall picture of the discrete states across five diamonds [125].

new discrete levels become accessible.



Double quantum dots as charge qubits

QD8

charge states: $(N_L, N_R) = \begin{cases} (1,0) \equiv |0\rangle, & \text{Energy } E_1 \\ (0,1) \equiv |1\rangle & E_2 \end{cases}$

$$\hat{H} = \frac{1}{2}\begin{pmatrix} \varepsilon & T \\ T & -\varepsilon \end{pmatrix}$$

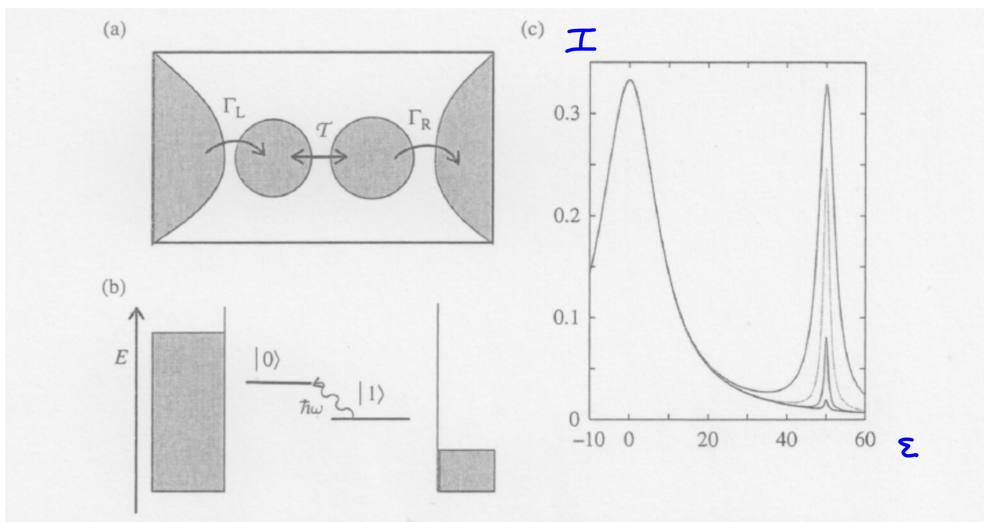
$\varepsilon = E_1 - E_2$

T = tunneling

splitting

$\hbar\Omega = \sqrt{\varepsilon^2 + T^2}$

Modulate resonantly: ε or T



Double-dot charge qubit. (a) Double dot between two leads. The arrows show the tunnel processes: coherent ($\mathcal{T}$) and incoherent ($\Gamma_{R,L}$). (b) Energy diagram of the effective qubit. Irradiation enables manipulation. (c) Current versus energy mismatch ε at different irradiation intensities: $\tilde{\varepsilon}/\varepsilon\Gamma_R\hbar = 1, 3, 9, 27$ from lower to upper curves. $\mathcal{T}/\hbar\Gamma_R = 10$, $\hbar\omega = 5\mathcal{T}$. The broad peak (see Eq. (5.36)) is the qubit "leakage." The narrow peak (see Eq. (5.38)) is the result of the resonant irradiation growing with intensity.

QD9

Transport cycle: coherent transitions

$$(0,0) \longrightarrow \underset{|0\rangle}{(1,0)} \longleftrightarrow \underset{|1\rangle}{(0,1)} \longrightarrow (0,0) \qquad (1)$$

initialization

readout : only of state $|1\rangle$

here we can manipulate : apply pulses to switch $|0\rangle \to |1\rangle$

With driving $\varepsilon(t) = \varepsilon + \tilde{\varepsilon}\cos\omega t$, $\omega \simeq \Omega$, (2)

one finds:

$$I/e = \frac{\tau^2 \Gamma_R}{\tau^2(2+\Gamma_R/\Gamma_L) + \hbar^2\Gamma_R^2 + 4\varepsilon^2} \qquad (3)$$

Near resonance of 2nd peak:

$$I/e = I_{max}\,\frac{1}{1+(\delta\omega/W)^2} \qquad (4)$$

$$W = \text{width of peak} = \frac{\tau^2\tilde{\varepsilon}^2(2+\Gamma_R/\Gamma_L)}{16\hbar^2\varepsilon^2} + \frac{\Gamma_R^2}{4} \qquad (5)$$