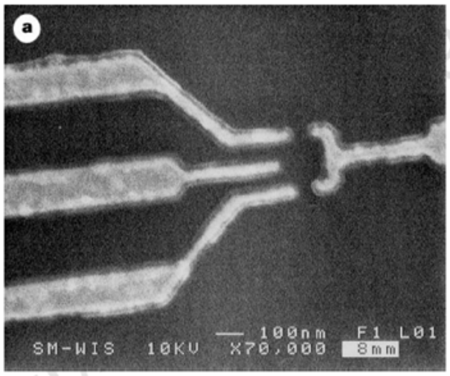
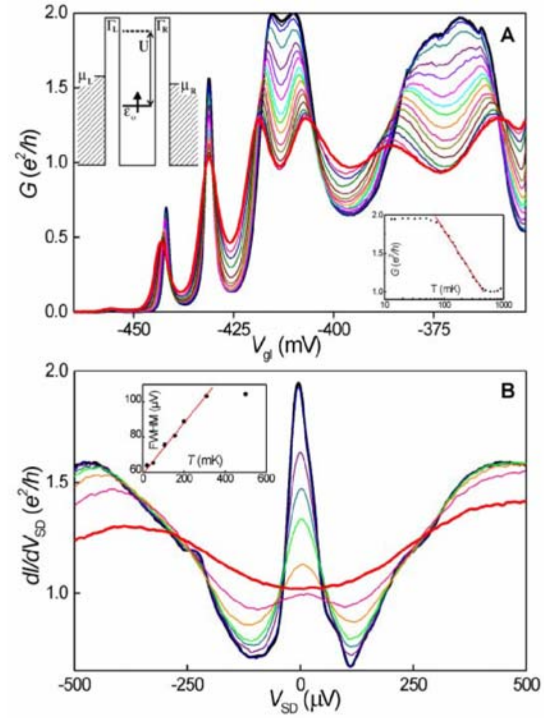
Kondo Effect in Metals and Quantum Dots

$\boxed{\text{KD}}$



Goldhaber-Gordon et al., Nature **391**, 156 (1998)

Cronenwett et al., Science **281**, 540 (1998)
Simmel et al., PRL **83**, 804 (1999)
van der Wiel et al., Sience 289, 2105 (2000)



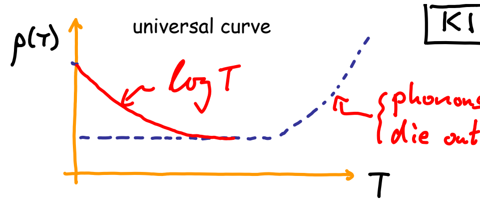
van der Wiel et al., Sience 289, 2105 (2000)

Lecture 1: Kondo model

[Kondo, Phys Rev 1964]

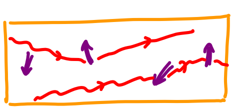
$\boxed{\text{K1}}$

anomalous resistivity minimum in dilute magnetic alloys
(localized spins scatter conduction electrons)



Kondo Model:

$$H_{Kondo} = H_{band} + H_{scat}$$



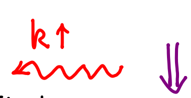
$$H_{band} = \sum_{k\sigma} \varepsilon_k c^\dagger_{k\sigma} c_{k\sigma} \qquad (1)$$

$$H_{scat} = J \sum_{kk'\sigma\sigma'} \Big(\underbrace{c^\dagger_{k\sigma} \tfrac{1}{2}\vec{\sigma}_{\sigma\sigma'} c_{k'\sigma'}}_{=:\vec{s}} \Big) \cdot \vec{S} \qquad (2)$$

conduction electron spin

magnetic impurity

Spin-flip scattering:

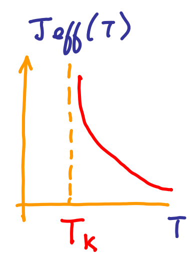


final state:

initial state:

turns out to be enhanced at low temperatures:

$$T \lesssim T_K = D\, e^{-\frac{1}{\nu J}} \qquad (3)$$



T = 0: Fermi liquid theory

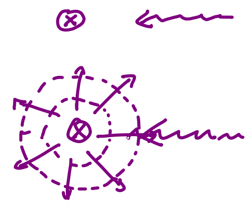
ground state = spin singlet $(\Downarrow\uparrow) = k\uparrow \otimes k'\uparrow$

$$\frac{e^{i(kr-\delta_\sigma)}}{r} \longleftarrow \frac{e^{-ikr}}{r}$$

Scattering states and T-matrix

$$H = H_0 + H_1 \quad (1)$$

K2



Free state: $H_0 |k\sigma\rangle = \varepsilon_k |k\sigma\rangle$ (2)

Scattering state: $H |\widetilde{k\sigma}\rangle = \varepsilon_k |\widetilde{k\sigma}\rangle$ (3)

Ansatz: $|\widetilde{k\sigma}\rangle = |k\sigma\rangle + \frac{1}{\varepsilon_k - H_0 + i\eta} H_1 |\widetilde{k\sigma}\rangle$ (4)

Check: $\cancel{(\varepsilon_k - H_0 + i\eta)}^{=0} |\widetilde{k\sigma}\rangle = \cancel{(\varepsilon_k - H_0 + i\eta)}^{=0\,(2)}|k\sigma\rangle + H_1 |\widetilde{k\sigma}\rangle$ (5)

$(\varepsilon_k - H_0 - H_1) |\widetilde{k\sigma}\rangle \overset{(3)}{=} 0$

Iterate (4): $|\widetilde{k\sigma}\rangle = \left[1 + \frac{1}{\varepsilon_k - H_0 + i\eta} H_1 + \frac{1}{\varepsilon_k - H_0 + i\eta} H_1 \frac{1}{\varepsilon_k - H_0 + i\eta} H_1 + \dots\right]$

$$= \left[1 + \frac{1}{\varepsilon_k - H_0 + i\eta}\, T\right] |k\sigma\rangle \quad (6)$$

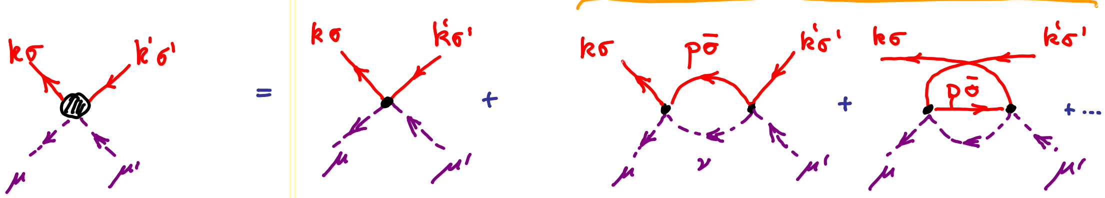
T-matrix: $T = H_1 + H_1 \frac{1}{\varepsilon_k - H_0 + i\eta} H_1 + H_1 \frac{1}{(\ )} H_1 \frac{1}{(\ )} H_1 + \dots$ (7)

Matrix elements of T

$\langle k\sigma| \otimes \langle\mu|\ T\ |k'\sigma'\rangle \otimes |\mu'\rangle$

K3

pert. exp: $T^{\mu\mu'}_{k\sigma, k'\sigma'} = T^{(1)} + \overbrace{T^{(2)}}^{} + \dots$ (1)



(2)

$$T^{(1)\,\mu\mu'}_{k\sigma,k'\sigma'} = \tfrac{1}{2} J\, \vec{\sigma}_{\sigma\sigma'} \cdot \vec{S}_{\mu\mu'} \quad (3)$$

$$T^{(2)\,\mu\mu'}_{k\sigma,k'\sigma'} = J^2 \sum_{p\bar\sigma,\nu} \frac{(\tfrac{1}{2}\vec{\sigma}_{\sigma\bar\sigma} \cdot \vec{S}_{\mu\nu}) \cdot (\tfrac{1}{2}\vec{\sigma}_{\bar\sigma\sigma'} \cdot \vec{S}_{\nu\mu'})}{\varepsilon_k - \varepsilon_p + i\eta} [1 - f(\varepsilon_p)] \quad (4a)$$

relative minus: $\overbrace{1 - f(\varepsilon_p)}$

final state is $c^\dagger_k \overbrace{c_p c^\dagger_p} c_{k'} |\widetilde{k\sigma}\rangle$

versus $\underbrace{c^\dagger_p c_{k'}}_{f(\varepsilon_p)} c^\dagger_k c_p |\widetilde{k\sigma}\rangle$

$c^\dagger_k$, $c_{k'}$ act in opposite order

$$\ominus J^2 \sum_{p\bar\sigma,\nu} \frac{(\tfrac{1}{2}\vec{\sigma}_{\bar\sigma\sigma'} \cdot \vec{S}_{\mu\nu}) \cdot (\tfrac{1}{2}\vec{\sigma}_{\sigma\bar\sigma} \cdot \vec{S}_{\nu\mu'})}{\varepsilon_k - (\varepsilon_{k'} - \varepsilon_p + \varepsilon_k) + i\eta} f(\varepsilon_p) \quad (4b)$$

K4

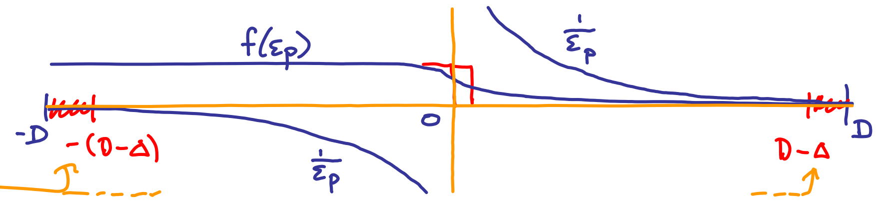
$$T^{(2)} \overset{(3.6)}{=} J^2 \sum_{\substack{aa'=\\ xyz}} (S^a S^{a'})_{\mu\mu'} \left[\nu \int_{-D}^{D} d\varepsilon_p \, \frac{1}{4} \left\{ (\sigma^a \sigma^{a'})_{\sigma\sigma'} \frac{f(\varepsilon_p)-1}{\varepsilon_p - \varepsilon_k - i\eta} - (\sigma^{a'} \sigma^{a})_{\sigma\sigma'} \frac{f(\varepsilon_p)}{\varepsilon_p - \varepsilon_{k'} + i\eta} \right\} \right] \quad (1)$$

Consider $\varepsilon_k \simeq \varepsilon_{k'} \simeq \varepsilon_F$:

$$\frac{1}{4}[\sigma^a, \sigma^{a'}]_{\sigma\sigma'} \, \nu \int_{-D}^{D} d\varepsilon_p \frac{f(\varepsilon_p)}{\varepsilon_p} \propto \int_{-D}^{T} d\varepsilon_p \frac{1}{\varepsilon_p} = \ln T/D \xrightarrow{T \to 0} \infty \quad (2)$$

↳ performing entire integral yields log. divergence

Anderson "poor man's scaling" idea: split off contribution from strips near band edges:



$$T^{(2)}_{(D)} = T^{(2)}_{(D-\Delta)} + \delta T^{(2)} \quad (3)$$

$$\delta T^{(2)} = J^2 \underbrace{\sum_{aa'} (S^a S^{a'})_{\mu\mu'} \frac{1}{4} \overbrace{[\sigma^a, \sigma^{a'}]}^{\frac{1}{2} i \varepsilon_{aa'b} \sigma^b}_{\sigma\sigma'}} \nu \left[\int_{D-\Delta}^{D} d\varepsilon_p + \int_{-D}^{-(D-\Delta)} d\varepsilon_p \right] \frac{f(\varepsilon_p)}{\varepsilon_p} \quad (4)$$

check algebra yourself!

$$= J^2 \left(-\frac{1}{2} \vec{S}_{\mu\mu'} \cdot \vec{\sigma}_{\sigma\sigma'}\right) \nu\Delta \begin{cases} \frac{0}{D} + \frac{1}{-D} & \text{for } T \ll D \quad (5a) \\ \frac{1/2}{D} + \frac{1/2}{-D} \simeq 0 & \text{for } D \ll T \quad (5b) \end{cases}$$

K5

Integrated-out strips yield term of same form as bare vertex:

$T^{(1)} \overset{(3.3)}{=} \frac{1}{2} J \, \vec{\sigma} \cdot \vec{S}$

$$\delta T^{(2)}(J) \overset{(4.5a)}{=} \underbrace{J^2 \frac{\nu\Delta}{D}}_{\delta J} \frac{1}{2} \vec{\sigma} \cdot \vec{S} = T^{(1)}(\delta J) \quad (1)$$

Scaling of T-matrix under band-width reduction:

$$T(D,J) \overset{(3.2)}{=} T^{(1)}(J) + T^{(2)}(D,J) + O(J^3) \quad (2)$$

$$\overset{(4.4)}{=} T^{(1)}(J) + T^{(2)}(D-\Delta, J) + \underbrace{\delta T^{(2)}(J)}_{\overset{(1)}{=} T^{(1)}(\delta J)} \quad (3)$$

$$\overset{(1)}{=} T^{(1)}(J+\delta J) + T^{(2)}(D+\delta D, J+\delta J) + O(\cancel{J^3}) \quad (4)$$

$$= T(D', J') \quad (5)$$

So, reducing bandwidth $D \to D' = D + \delta D$, $\delta D = -\Delta$ (6)

generates increase in coupling constant: $J \to J' = J + \delta J$, $\underbrace{\delta J \nu}_{\delta g} \overset{(1)}{=} -\underbrace{J^2 \nu^2}_{g^2} \frac{\delta D}{D}$ (7)

Scaling eq. for dimensionless coupling: $g(D) := \nu J$

$$\boxed{\frac{dg}{d(\ln D)} = -g^2} \quad (8)$$

Flow to strong coupling; Kondo temperature

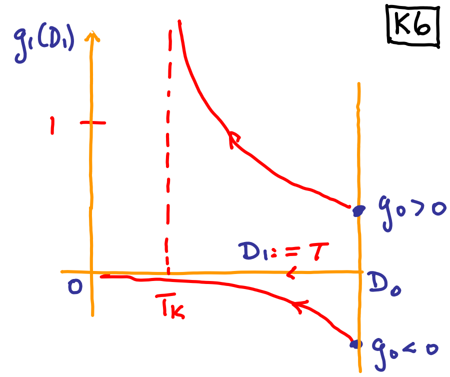
K6

Scaling equation:

$$-\int_{g_0}^{g_1} \frac{dg}{g^2} \overset{(5.8)}{=} \int_{D_0}^{D_1} d(\ln D) \qquad (1)$$

$$\frac{1}{g_1} - \frac{1}{g_0} = \ln D_1/D_0 \qquad (2)$$

$$g_1(D_1) = \frac{1}{\frac{1}{g_0} - \underbrace{\ln(D_0/D_1)}_{>0\text{, grows as } D_1 \to 0}} \qquad (3)$$



Reduce bandwidth until $D_1 = T$ [because by (4.5b), renormalization of coupling stops for $D_1 \ll T$; lecture 3 will illustrate this in more detail !]

and use as effective coupling constant at temperature T:

$$g_{eff}(T) = g_1(D_1 = T) \overset{(3)}{=} \frac{1}{\frac{1}{g_0} - \ln(D/T)} = \frac{1}{\ln(T/T_K)} \qquad (4)$$

for $T > T_K$

For $g_0 > 0$, scaling approach eventually breaks down: eff. coupling diverges at a scale T_K :

$$\ln D/T_K := \frac{1}{g_0} \Rightarrow \boxed{T_K = D e^{-\frac{1}{g_0}}} \qquad (5)$$